A 5)

$$R_T = \frac{c}{4}\int_0^\infty \rho(\nu)\, d\nu$$

$$= \frac{c}{4}\int_0^\infty \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/kT}-1}\, d\nu$$

$$q = h\nu/kT \qquad dq/d\nu = h/kT$$

$$\nu = kTq/h \qquad d\nu = kT/h\, dq$$

$$= \frac{2\pi k^4 T^4}{c^2 h^3}\int_0^\infty \frac{q^3}{e^q - 1}\, dq$$

$$= \frac{2\pi^5 k^4 T^4}{15 c^2 h^3} = \sigma T^4 \qquad \text{q.e.d.}$$

A 6) $\rho(\lambda_1, T) = 0{,}2\, \rho(\lambda_{max}, T) \qquad \lambda_1 < \lambda_{max}$

$\rho(\lambda_2, T) = 0{,}2\, \rho(\lambda_{max}, T) \qquad \lambda_2 > \lambda_{max}$

$T = 3\,K$

ges.: λ_1, λ_2

Lös.: λ_{max} aus Wienschen Verschiebungsgesetz

$$\lambda_{max} = hc/4{,}965\,kT = 9{,}66\cdot 10^{-4}\,m = 0{,}966\,mm$$

$$\Rightarrow \rho(\lambda_{max}, T) = \frac{8\pi ch}{\lambda_{max}^5} \frac{1}{e^{hc/kT\lambda_{max}}-1} \approx 4{,}2\cdot 10^{-11}\, J/m^4$$

$$\frac{\rho(\lambda, T)}{\rho(\lambda_{max}, T)} = \frac{8\pi ch}{\lambda^5} \frac{1}{e^{hc/kT\lambda}-1} / \rho_{max}$$

$\Rightarrow$ plotten und λ_1 / λ_2 graphisch bestimmen

$\Rightarrow \lambda_1 \approx 5\cdot 10^{-4}\,m = 0{,}5\,mm$

$\lambda_2 \approx 2{,}5\cdot 10^{-3}\,m = 2{,}5\,mm$

A7) geg.: $T_1 = 50°C$
$T_2 = 10°C$
$T_0 = 0°C$

$(m = 1kg \Rightarrow a = 0{,}1 m \Rightarrow A = 0{,}06 m^2)$

$c_{H2O} = 4187 \; J/kgK$

ges.: t mit $\int_0^t P_{Str.} \, dt = \Delta W(T_1, T_2)$

Lös.: $\Delta W(T) = c_{H2O} \, m \, (T(t) - T_1)$

$P_{Str.}(T) = A \sigma (T^4 - T_0^4)$

$P_{Str.} = \frac{d}{dt} \Delta W(T(t))$

$A \sigma (T^4(t) - T_0^4) = c_{H2O} \, m \frac{dT}{dt}$

$dt = \frac{c_{H2O} m}{A \sigma} \frac{1}{(T^4(t) - T_0^4)} dT$

$t = \int_0^t dt = \int_{T_1}^{T_2} \frac{c_{H2O} m}{A \sigma} \frac{1}{(T^4 - T_0^4)} dT$

$\Rightarrow t \approx 5{,}94$

A8) ges.: $\lambda = 0{,}1\,m,\ 500\,nm,\ 0{,}1\,nm$
$T = 20\,°C$

ges.: W_{spont} / W_{ind}

Lös.: $W_{spont} = A$

$W_{ind,e} = B\varrho$

$W_{ind,a} = B\varrho$

thermodyn. GG.: $A N_2 + B\varrho N_2 = B\varrho N_1$

$$A N_2 = B\varrho (N_1 - N_2)$$

$$\varrho = \frac{A}{B} \frac{1}{N_1/N_2 - 1}$$

Boltzmann: $N_2/N_1 = e^{-\Delta E/kT} = e^{-h\nu/kT}$

$$\Rightarrow \varrho = \frac{A}{B} \frac{1}{e^{+h\nu/kT} - 1}$$

$$\frac{W_{spont}}{W_{ind}} = \frac{A N_2}{B \frac{A}{B} \frac{1}{e^{+h\nu/kT} - 1} N_2} = e^{+h\nu/kT} - 1 = e^{+hc/kT\lambda} - 1$$

$$\frac{W_{spont}}{W_{ind}} (0{,}1\,m) = 4{,}9 \cdot 10^{-4}$$

$$\text{"} \quad (500\,nm) = 4{,}4 \cdot 10^{42}$$

$$\text{"} \quad (0{,}1\,nm) = 3{,}9 \cdot 10^{213254}$$