Statistical Mechanics WS 2013/14 Sheet 11

Please hand in your solutions (in pairs) before the Monday lecture.

Problem 1 : Two-Particle System (20 points)

Consider a system of a pair of two particles which are identical, and can have any of energy levels $\epsilon_n = n\epsilon$ with $n = 0, 1, 2$. The degeneracy of the lowest energy state $\epsilon_0 = 0$ is 2. Calculate the partition function and average energy for;

a) (5 points) Bosons.

b) (5 points) Fermions.

c) (5 points) Distinguishable Classical particles.

d) (5 points) Depict all the possible particle configurations in energy levels for a), b), and c).

Problem 2 : Bose-Einstein Condensation in Low Dimensions (20 points)

Consider an ideal gas of non-relativistic, identical bosons.

a) (10 points) Express the mean number of bosons in one-dimension with a system length L. Explain whether there occurs the Bose-Einstein condensation or not.

b) (10 points) Express the mean number of bosons in two-dimension with a system area L^2. Explain whether there occurs the Bose-Einstein condensation or not.

[Hint: The condition for the Bose-Einstein condensation is zero-chemical potential $\mu = 0$ and low temperature $T \to 0$]

Problem 3 : Photon Gas (20 points)

Consider a photon gas in a volume V at temperature T. The energy of the photon is $\epsilon = pc$ (or $= \hbar\omega$), where p is the momentum and ω is the frequency.

a) (4 points) Using canonical ensemble, calculate the average energy. Use $\epsilon = pc$.

b) (4 points) Does the above result obey the "equipartition theorem"? If not, explain what is the condition for the Hamiltonian to satisfy the equipartition theorem.

c) (4 points) Find the chemical potential of the photon.

d) (4 points) Find the integral form for the average number of photons. How does this depend on T?

e) (4 points) Find the integral form for the average energy of photons. How does this depend on T?

[Hint: For d) and e) use $\epsilon = \hbar\omega$ and make the integrals dimensionless.]

Problem 4 : The Planck Distribution (20 points)

The Planck distribution is photon's radiated power R per unit area per unit wavelength λ:

$$\frac{dR}{d\lambda} = \frac{2\pi hc^2}{\lambda^5(e^{hc/(\lambda k_B T)} - 1)}. \tag{1}$$

a) (10 points) Using the above distribution, express the number of photons per unit area per second per unit wavelength. [Hint: Energy per photon is hc/λ.]

b) (5 points) Using the sun's temperature $T = 6000$ K and wavelength range of the visible light 400 nm $\leq \lambda \leq$ 700 nm, Calculate the number of photons radiated from the sun per second per unit area [meter^2].

c) (5 points) What is number of photons radiated from the "cosmic background" per second per unit area [meter^2]? Use $T = 2.7$ K.