Statistical Mechanics WS 2013/14 Sheet 5

Please hand in your solutions (in pairs) before the Monday lecture.

Problem 1 : Isothermal / Isobaric Process (20 points)

Suppose that a system follows $E(S,V,N) = \alpha S^{n+2}/(N^m V^n)$, where $\{n,m\} \in \{\text{positive integers}\}$, and α is a real number.

a) (4 points) Find the value of m. [Hint: $E(S,V,N)$ is the extensive quantity].

b) (4 points) Find the temperature $T(S,V,N)$.

c) (4 points) Find the pressure $P(T,V,N)$.

d) (4 points) Express the equation of state in terms of P, T, N/V, α and n.

e) (4 points) The volume of the system is increased isothermally from V to vV at T. The temperature then is increased isobarically from T to θT. Find a factor of the corresponding entropy change.

Problem 2 : Carnot Cycle (20 points)

a) (5 points) Draw a typical Carnot cycle (reversible) of a gas with isobaric temperature changes $(T_0, 5T_0)$ and isothermal volume changes $(V_0, 10V_0)$, in P vs. V diagrams and S vs. T diagrams, respectively.

b) (5 points) Calculate the efficiency η of the above Carnot engine.

c) (5 points) Calculate the work $\mathcal{W}$ done by one mole of the ideal gas.

d) (5 points) Suppose that FUB-Physics building is heated by the Carnot engine from a heat source at the temperature T_0, consuming power W and losing the heat at a rate $\alpha(T - T_0)$. Derive the equilibrium temperature T_{eq} in the building.

Problem 3 : Static Piston (20 points)

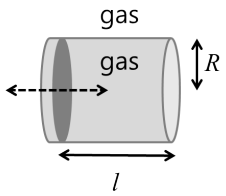

Fig. 1: The gas is inside the cylinder as well as outside.

Consider a cylinder of a radius R. At the cylinder's open-side there is a movable piston (the grey disk in Fig. 1.) while the other end is capped. The piston can be moved in the cylinder's longitudinal direction (the dashed arrow in Fig. 1.), by applying a constant force f. The ideal gas is inside the cylinder as well as outside, separated by the piston. Inside the cylinder there are N gas molecules, and the temperature is T.

a) (10 points) Find the pressure difference between gas inside and outside.

b) (10 points) Find the equilibrium length l_0 denoting pressure of outer gas by P_{out}.

Problem 4 : Oscillating Piston (20 points)

Now the above piston starts to oscillate with its longitudinal position $l(t) = l_0 + A\sin(\omega t)e^{-b\omega t}$, where $0 < A \ll l_0$ and $0 < A\omega \ll \bar{v}$, and $\bar{v}$ is the mean velocity.

a) (4 points) Calculate the velocity v of the piston.

b) (8 points) Calculate the relative energy change $\mathrm{d}E/E_0$ during one-cycle.

c) (8 points) Calculate the relative energy change $\mathrm{d}E/E_0$ in total process.