Sheet 1

Please hand in your solutions before the Monday lecture at 10:15.

Problem 1 : Poisson Distribution (10 points)

a) Consider the binomial distribution $W_N(m;p)$ and the Poisson distribution $W(m;Np)$.
   1) (3 points) Find values of $W_{1000}(m;0.01)$ for $m = 5, 6, \cdots, 10$.
   2) (3 points) Find values of $W(m;Np)$ using the same values of N and p used in 1), for $m = 5, 6, \cdots, 10$.

b) (4 points) The variance of the Poisson distribution is $\Delta m^2 = \lambda = Np$. Show that it can be obtained from the variance of the binomial distribution, under a limiting condition.

Problem 2 : Characteristic Function (20 points)

a) Calculate the characteristic function $G(k)$ and the first two moments of the following probability density functions $w(x)$. Here a is a positive real number.
   1) (5 points) $w(x) = \frac{1}{2a} \exp\left(-\frac{|x|}{a}\right)$.
   2) (5 points) $w(x) = \sqrt{\frac{2}{\pi}} \frac{x^2}{a^3} \exp\left(-\frac{x^2}{2a^2}\right)$, $x \geq 0$.

b) If we measured, from experiments, $\langle x^m \rangle = a^{2m+1}$ where a is a real number,
   1) (4 points) What would be $w(x)$ and $G(k)$ in functional forms, respectively?
   2) (4 points) Find the value of variance $\langle (x - \langle x \rangle)^2 \rangle$.
   3) (2 points) Briefly explain (in one sentence) the physical meaning of this variance in relation with fluctuations and probability density.

Problem 3 : Normal distribution (20 points)

A factory that produces brake elements for cars has developed a new family of brake disks. The most important feature for these disks to accomplish their purpose is the thickness. The development department has established that the thickness of every disk should be between 9.98 and 10.0 mm. To produce these disks, a new machine has been bought and, to certify it, it has to be proven that more than 99.73% (mean ± 3 times the standard deviation) of the produced disks lay inside the requested data. In order to certify the new machine, 100 measures have been done (data shown in the table).

a) (5 points) Calculate the probability density for the disk thickness and plot it.

b) (5 points) Calculate the mean and the standard deviation.

c) (5 points) Prove the capacity of the machine (i.e. all the disks with a length = (mean ± 3 times the standard deviation) lay inside the requested data).

d) (5 points) What is the probability to have disks thicker than 9.989 mm? And disks between 9.988 and 9.990 mm?

Thickness (mm)	Number of disks
9.985	4
9.986	10
9.987	23
9.988	25
9.989	22
9.990	10
9.991	6