Statistical Physics I
WS 2014/15, FUB

Sheet 4

Please hand in your solutions before the Monday lecture at 10:15.

Problem 1 : Ideal Gas (20 points)

The Sackur-Tetrode equations claims to be the entropy function for the ideal gas. From this equation, obtain:

a) (6 points) the temperature as a function of the energy and the number of particles.

b) (7 points) the pressure as a function of the temperature, the volume and the number of particles. Is your result consistent with the ideal gas?

c) (7 points) the chemical potential as a function of the energy, the volume and the number of particles.. Under conditions near room temperature and atmospheric pressure, is the chemical potential of the ideal gas positive or negative?

Problem 2 : Mixing Entropy (20 points)

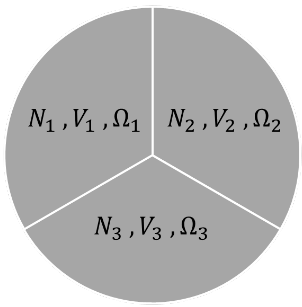

Fig. 1: A system for mixing entropy in Problem 2.

Suppose that three different species (with the index $\alpha = 1, 2, 3$) of ideal gases of the particle number N_α, the volume V_α, and microstate number Ω_α, are initially partitioned in a system at the same temperature T as depicted in Fig. 1, where $N = \sum_\alpha N_\alpha$, $V = \sum_\alpha V_\alpha$.

a) (2 points) Express the total microstate number Ω of the system, in terms of Ω_α.

b) (2 points) One can assume that the entropy S_α is an arbitrary function of Ω_α, i.e., $S_\alpha = f(\Omega_\alpha)$. Find the functional form of f from the character of the total entropy $S = \sum_\alpha S_\alpha$ and the character of Ω.

c) (6 points) Calculate the mixing entropy $(\Delta S)_{123}$ for $N_1 = N_2 = N_3 = n$ and $V_1 = V_2 = V_3 = v$. Express $(\Delta S)_{123}$ in terms of the Boltzmann constant k_B and n.

d) (2 points) Calculate the mixing entropy $(\Delta S)_{12}$ for $N_1 = N_2 = n$, $N_3 = 0$ and $V_1 = V_2 = V_3 = v$. Express $(\Delta S)_{12}$ in terms of k_B and n.

e) (6 points) Now consider that gases are all the same kind. Calculate the mixing entropy $(\Delta S)_{11}$ for $N_1 = N_2 = n$, $N_3 = 0$ and $V_1 = V_2 = V_3 = v$. Express $(\Delta S)_{11}$ in terms of k_B and n.

f) (2 points) Compare $(\Delta S)_{12}$ with $(\Delta S)_{11}$. Which one is larger?

Problem 3 : Information Entropy (20 points)

Consider the rescaled information entropy $S/k_B = \int dx w(x) \ln \{1/w(x)\}$ in a continuous one-dimensional space x, where $w(x)$ is a probability distribution (density) function. Derive the rescaled entropy of systems governed by the following distributions $w(x)$ in a), b), c) and d):

a) (3 points) The uniform distribution function for $0 \leq x \leq L$.

b) (4 points) The linear distribution function $w(x) = Ax$ (first, find A in terms of L) for $0 \leq x \leq L$.

c) (4 points) The exponential distribution function $w(x) = Be^{-x/L}$ (first, find B in terms of L) for $0 \le x < \infty$.

d) (4 points) The normal distribution function with the zero mean and the variance L^2, for $-\infty < x < \infty$. Use $\int_{-\infty}^{\infty} dy e^{-y^2} = \sqrt{\pi}$.

e) (1 point) Enumerate the entropies $S_{\text{a)}}$, $S_{\text{b)}}$, $S_{\text{c)}}$ and $S_{\text{d)}}$ obtained from the above in an ascending order.

f) (4 points) According to the U.S. Naval Observatory (*http://aa.usno.navy.mil/data/docs/MoonFraction.php*), the fraction of the moon illuminated depending on the moon phases at midnights in March 2014 was estimated as below:

Index (n)	1	2	3	4	5	6	7	8	9	10
Days (days)	3	6	9	12	15	18	21	24	27	30
Fraction (f_n)	0.01	0.05	0.11	0.16	0.19	0.19	0.16	0.10	0.04	0.00

Denoting the probability density by $w_n = f_n/\Delta d$ where $\Delta d = 3$ days, compute the rescaled information entropy S/k_B in a one-dimensional discrete space.