Sheet 5

Please hand in your solutions before the Monday lecture at 10:15.

Problem 1 : Argon Gas (30 points)

For a mol of Argon at a temperature $T_1 = 25°\mathrm{C}$ and volume $V_1 = 0.03\mathrm{m}^3$ (the mass of a mol of Argon is 39.95 g) and another mol of Argon at temperature T_2 and volume V_2, using the partition function of a mono-atomic ideal gas in the canonical ensemble, compute and plot the following quantities in a) $\sim$ e), as functions of the temperature $T_2 = \{25, 30, 35, \ldots, 100°\mathrm{C}\}$ at $V_1 = V_2$, and $V_2 = \{0.03, 0.04, 0.05, \ldots, 0.1\mathrm{m}^3\}$ at $T_1 = T_2$, respectively:

a) (6 points) Helmholtz free energy difference $A_1 - A_2$

b) (2 points) pressure difference $p_1 - p_2$

c) (4 points) entropy difference $S_1 - S_2$

d) (4 points) internal energy difference $E_1 - E_2$

e) (6 points) micro-canonical entropies $S_2(N, V_1, E_2(V1, T2))$ and $S_2(N, V_2, E_2(V2, T1))$ for N =one mol of Argon and the same V_1, V_2, T_1, T_2 as in a) $\sim$ e). For the computation, use the two sets of internal energies $E_2(T_2, V_1 = V_2)$ and $E_2(V_2, T_1 = T_2)$ obtained in d).

From the micro-canonical entropies obtained in e), calculate and plot

f) (4 points) temperatures as a function of E_2

g) (4 points) and pressures as a function of E_2.

Problem 2 : Gases under Potentials (20 points)

a) Consider ideal gas particles of the total number N in the (ideal) atmosphere that are in thermal equilibrium at temperature T. The energy of a gas particle with mass m can be described by its potential energy $E = mgh$ where h is the height (altitude) of the particle above ground and g is earth gravity.

1) (4 points) Show that the relation between the pressure difference dp and the height difference dh is $dp = -p\frac{mg}{k_B T}dh$. This is called the "hydrostatic pressure".
2) (2 points) Calculate the pressure $p(h)$ where $p(0) \equiv p_0$.
3) (6 points) Using the canonical ensemble calculate $p(h)$ where $p(0) \equiv p_0$.

b) Consider ideal gas particles of the total number N that are in thermal equilibrium at temperature T, interacting with an infinitely long rod via an attractive potential $U/(k_B T) = \alpha \ln(r/r_{\max})$ where α is a positive real number, and $r_{\min} \leq r \leq r_{\max}$ is a radial distance of the particle from the rod axis.

1) (3 points) Calculate the single particle partition function for this system with a cylindrical volume of a height L and a radius $r_{\min} \leq r \leq r_{\max}$. For the calculation, consider only spatial degree of freedom r and you don't have to introduce a prefactor.
2) (3 points) Find the ratio of the number of particles at r and at $r_{\min}$, $N(r)/N(r_{\min})$. Sketch a plot of $N(r)/N(r_{\min})$ as a function of r, where $r_{\min} = 1$ nm, $r_{\max} = 100$ nm, and $\alpha = 2$.
3) (2 points) Find the ratio of the pressure $p(r)/p(r_{\min})$.