Sheet 8

Please hand in your solutions before the Monday lecture at 10:15.

text in red is a correction to a previously uploaded version of this problem sheet.

Problem 1 : State Functions (12 points)

Check whether the following functions are state functions, by comparing the mixed second derivatives (you can use the ideal gas law).

a) (3 points) Helmholtz free energy $A(V, T)$

b) (3 points) Gibbs free energy $G(p, T)$

c) (3 points) Pressure $p(V, T)$

d) (3 points) Work done by the system $W(p, V)$

Problem 2 : Hiking (8 points)

You are hiking in the mountains. Your initial state is at altitude $h = 200$ m (valley), and your final state will be at altitude $h = 1000$ m (top). There are two hiking trails leading from the valley to the top. The "tourist way" with a gentle slope takes is 8 km long. The "adventure trail" is rather steep but is only 2 km long. In addition to the work to "fight" earth gravitation $W_G = \int mgdh$, there is a force F_{step} required per step $dL = 50$ cm taken (e.g. to lift the legs).

a) (3 points) Calculate the work done by you hiking the "tourist way" and the one for taking the "adventure trail".

b) (2 points) The hiker climbs up to the top and returns to the valley. The change in potential energy is zero. What about the work done by the hiker?

c) (3 points) The hiker can take in energy by eating Muesli (100 g equivalent to 5000 kJ) to "heat" their muscles. How much Muesli does the hiker have to eat to compensate the work (neglect the mass of the Muesli that has to be carried).

Problem 3 : Energy Change in Ideal Gas (8 points)

Calculate the energy change in an ideal gas upon changing:

a) (2 points) temperature from T_1= 300K to T_2 = 400K

b) (2 points) volume from $V_1 = 1m^3$ to $V_2 = 8m^3$

c) (2 points) both T and V

d) (2 points) How much should the volume change to maintain energy (dE=0) for a change in temperature from T_1 to T_2 ?

Problem 4 : Quasi-static expansion (6 points)

Consider an ideal gas in a volume $L_x \cdot L_y \cdot L_z$ where we vary L_x. The microscopic state r of the N gas particles can be defined by quantum states $r = (n_1, \ldots, n_{3N})$ with the moments

$$\mathbf{p}_{\nu_{x,y,z}} = \frac{\pi\hbar}{L_{x,y,z}} n_{x,y,z} \tag{1}$$

and energy eigenvalues

$$E_r = \sum_{\nu=1}^{N} \frac{\mathbf{p}_\nu^2}{2m} = \sum_{\nu=1}^{N} \sum_{x,y,z} \frac{\pi^2\hbar^2}{2mL_{x,y,z}^2} n_{x,y,z}^2 \tag{2}$$

a) (2 points) Calculate the work $-đW_{q.s.}$ the gas does in the quasi-static expansion in terms of change in the external parameter L_x.

b) (2 points) Use the fact that energy is equi-partitioned in an ideal gas (the energy eigenvalue is the same for all directions of momenta) to express the work $-đW_{q.s.}$ in terms of (total) internal energy and volume.

c) (2 points) Use the above to show that the pressure of an ideal gas is $p = \frac{2}{3}\frac{E}{V}$.

Problem 5 : Isothermal expansion (6 points)

Show that for the isothermal expansion (T=const.) of an ideal gas from volume 1 (V_1) to volume 2 (V_2):

a) (2 points) the entropy change is $\Delta S = Nk_B \ln(V_2/V_1)$.

b) (2 points) the work is $W = T\Delta S$.

c) (2 points) how much does the energy of the ideal gas change by such a process?

Use the Sackur-Tetrode expression of the entropy in terms of number of particles, volume and temperature for an ideal monoatomic gas.