Solution sheet 11

Please hand in your solutions before the lecture on Wednesday, 13th of January.

Problem 1 - Joule-Thomson effect

(a) Write down the Joule-Thomson coefficient $\mu_{\rm JT}$ in terms of the thermal expansion coefficient α, T, C_p and V. What does this coefficient describe physically? Express in one sentence. *(2 points)*

(b) Calculate $\mu_{\rm JT}$ for the ideal gas. Explain the result in one sentence. *(2 points)*

(c) Calculate $\mu_{\rm JT}$ for the van der Waals gas. *(2 points)*

Problem 2 - Interacting gases

The Dietrici equation of state is defined as

$$P(v-b) = k_B T \exp(-a/k_B T v) \tag{1}$$

(a) Find the critical constants P_c, v_c, and T_c in terms of the parameters a and b of given system, and show that the quantity $k_B T_c/P_c v_c = e^2/2 \approx 3.695$.*(4 points)*

(b) At witch condition is the Dietrici equation of state is approximately the same as the van der Waals equation?*(5 points)*

(c) Suppose that we can use the Morse potential given as $\Phi(r) = \epsilon\left[e^{-2(r-\sigma)/\sigma} - 2e^{-(r-\sigma)/\sigma}\right]$ (acting between each pair of molecules) for a molecular gas (where r is the radial distance between molecules, σ is the diameter of a molecule, and ϵ denotes some units of energy), and assuming that the total mean energy per a molecule $\langle E\rangle$ is proportional to the mean energy for a pair of the molecules: $\langle E\rangle = \langle E_{\rm pair}\rangle N/2$ (here N is the number of molecules in system of voleme V). Calculate $\mu_{\rm JT}$. Use the excluded volume (V_{ex}) term from the van der Waals gas. *(6 points)*
(*Hint:* Assume that the molecules are isotropically distributed in volume V. In this case partition function of each molecule could be written as $z_1 = \dfrac{V - V_{ex}}{\lambda^3} e^{-\langle E\rangle/k_B T}$)

Problem 3 - Clausius–Clapeyron equation

(a) Consider that the latent heat along the phase boundary in liquid-gas transition is temperature dependent. Using the Clausius–Clapeyron equation find an expression of the phase boundary including this temperature dependence. (assume $v_L \ll v_g$)
[*Hint:* Use $\Delta S = \Delta H/T$ and $(\dfrac{dS}{dT}|_P = C_P/T)$ and $\dfrac{d(\Delta S)}{dT}$ with P and T independent] *(3 points)*

(b) The density of ice is $917 kg/m^3$

b-1) Use the Clausius-Clapeyron relation to explain why the slope of the phase boundary between water and ice is negative. *(2 points)*

b-2) How much pressure would you have to put on an ice cube to make it melt at $-1^0 C$? *(1 points)*

b-3) Approximately how deep under a glacier would you have to be before the weight of the ice gives the pressure you found in part (b)? *(2 points)*