Problem sheet 2

Please hand in your solutions before the lecture on Wednesday, 28th of October.

Problem 1 - Different distributions

(a) A container of volume V holds N randomly distributed non-interacting particles. Consider the subvolume v and assume that the probability of finding a particular particle in this subvolume is given by v/V.

  (1) Give the probability p_n of finding n particles within v. *(2 points)*

  (2) Show with the help of the Stirling formula that p_n corresponds approximately to a Gaussian distribution when N and n are large. *(3 points)*

  (3) Show in the limit $v/V \to 0$ and $V \to \infty$ with $N/V = const.$ the p_n approches a poisson distribution. *(3 points)*

(b) Suppose that the number of inquiries arriving at a certain interactive system follows a Poisson distribution with arrival rate of 12 inquiries per minute. Find the probability of exactly 10 inquiries arriving

  (1) in a 1-minute interval; *(1 points)*

  (2) in a 3-minute interval; *(1 points)*

  (3) What is the expectation and the variance of the number of arrivals during each of these intervals? *(2 points)*

Problem 2 - Directed random walk

The motion of a particle (a random walker) in three dimensions is a series of independent steps of length l. Each step makes an angle θ with the z axis. The corresponding probability density is $p(\theta) = 2cos^2(\theta/2)/\pi$. The distance of the particle from the origin is $a = const$ and the movement in ϕ direction is uniformely distributed from 0 to 2π. (Note that the solid angle factor of $sin\theta$ is already included in the definition of $p(\theta)$, which is correctly normalized to unity.) The particle (walker) starts at the origin and makes a large number of steps N. [*Hint*, a,θ,ϕ are spherical coordinate components and x, y, z are their relevant cartesian coordinate components]

(a) Calculate the expectation values $\langle z \rangle$,$\langle x \rangle$,$\langle y \rangle$,$\langle z^2 \rangle$,$\langle x^2 \rangle$, and $\langle y^2 \rangle$, and the covariance $\langle xy \rangle$, $\langle xz \rangle$, and $\langle yz \rangle$. *(9 points)*

(b) Use the central limit theorem to estimate the probability density $p(x, y, z)$ for the particle to end up at the point (x, y, z). *(4 points)*

Problem 3 - Correlation

Consider a set of variables that are shown below. (Hint, the relashionship between variables are: $x^2 + y^2 = 1$, $z = 0.1x + 0.5$, and w is arbitrary random variable)

x	0.54	0.00	-0.54	0.91	-0.98	0.74	-0.25	-0.30	0.77	-0.99	0.89	-0.50	-0.04	0.58	-0.93	0.97	-0.71	0.21
y	0.84	-0.99	0.83	-0.40	-0.15	0.67	-0.96	0.95	-0.62	0.10	0.45	-0.86	0.99	-0.81	0.36	0.20	-0.70	0.97
z	0.55	0.50	0.44	0.59	0.40	0.57	0.47	0.46	0.57	0.40	0.58	0.44	0.49	0.55	0.40	0.59	0.42	0.52
w	0.69	0.31	0.95	0.03	0.43	0.38	0.76	0.79	0.18	0.48	0.44	0.64	0.70	0.75	0.27	0.67	0.65	0.16

(a) Find correlation coefficients between these variables. ($R(x, y)$,$R(x, z)$,$R(y, z)$,$R(x, w)$,$R(y, w)$) *(3 points)*

(b) Between variables x and y, as (they are) points on a circumference, it is expected that the value of the correlation will be considerable but this value shows inconsiderable amount. Which explanation do you have for this problem? *(2 points)*

[*Hint*, To calculate the correlation coefficient for two random variables, you would use the correlation formula: $R(x, y) \equiv Corr(x, y) = \dfrac{Cov(x, y)}{\sigma_x \sigma_y}$; $Cov(x, y)$ is the covariance of the variable x and y; σ_x and σ_y are the deviations (fluctuations) of variable x and y, respectively.]