Statistical Physics I
WS 2015/16, FUB

Problem sheet 3

Please hand in your solutions before the lecture on Wednesday, 4th of November.

Problem 1 - Phase Space

Sketch the potential U(q) as a function of q, and typical paths of a particle in p, q space for the following one-dimensional systems. Here p is the momentum, q is the position. The energy E and mass m are fixed.

(a) A free particle. *(2 points)*

(b) A particle trapped in a monostable quartic potential $U(q) = q^4$. *(2 points)*

(c) A particle trapped in a bistable quartic potential $U(q) = q^4 - 2q^2$. *(2 points)*

(d) A particle under an attractive central potential $U(q) = 1/q^2$. *(2 points)*

Problem 2 - Entropy

(1) Show that in the microcanonical ensemble the uniform distribution, i.e. that in which all microstates have equal probability, maximises the thermodynamic entropy. *(3 points)*

(2) Calculate the mean information in the case of throwing: *(3 points)*

- one die.
- two dice.

Problem 3 - Magnetization of a two level system

Consider a system of N levels and in each of this levels sits a spin with $S = 1/2$. Each spin will have a projection over the z-axis equal to $S^z = \pm 1/2$ and has equal probability of choosing any of the two projections.

(a) Calculate the number of accessible microstates. *(3 points)*

(b) Each spin is directly related to a magnetic moment. This means that we can define a magnetization in the direction of the z-axis as $m^z = a\Sigma_{i=1}^N S_i^z$, with a an arbitrary constant. Give an expression of the magnetization in the z-direction in terms of the number of spins up and down, $N_\uparrow$ and $N_\downarrow$.*(2 points)*

(c) Now we are able to identify the different macrostates present in the system. What is the probability, $P(m)$, of having a macrostate corresponding to a magnetization m? *(4 points)*

(d) Calculate the asociated entropy to each of these macrostates. Which one is the most probable? *(4 points)*

(e) If my system is in the maximal entropy state, and I apply an external magnetic field in the positive z-direction that forces all my spins to align with it. What is the change of entropy between the initial and final states? *(4 points)*

Problem 4 - Harmonic oscilators

We will study a system consisting of N harmonic oscillators in 1D, independent and identical. The Hamiltonian of the system is:

$$H(p_i, q_i) = \Sigma_{i=1}^N \left((\frac{1}{2m} p_i^2 + \frac{1}{2} m w^2 q_i^2) \right) \tag{1}$$

(a) Using the semi-classical hypothesis that a quantum state occupies a cell of volume h^N in phase space, calculate the phase space volume occupied by the microcanonical ensemble. *(3 points)*

$$\Omega(E) = \frac{1}{N! h^N} \int_{E < H(p,q) < E + \Delta} d^N q d^N p \tag{2}$$

(b) Calculate the entropy S as a function of the total energy E. *(4 points)*

(c) Find the joint probability $P(p, q)$ for a single oscillator. Calculate the mean kinetic energy, and mean potential energy for each oscillator. *(4 points)*