Problem sheet 8

Please hand in your solutions before the lecture on Wednesday, 9th of December.

Problem 1 - State functions

Check whether the following functions are state functions, by comparing the mixed second derivatives (you can use the ideal gas law).

(a) Helmholtz free energy $A(V,T)$. *(2 points)*

(b) Gibbs free energy $G(p,T)$. *(2 points)*

(c) Pressure $p(V,T)$. *(2 points)*

(d) Work done by the system $W(p,V)$. *(2 points)*

Problem 2 - Adabatic compression

In the course of pumping up a bicycle tire, a liter of air at atmospheric pressure is compressed adabatically to a pressure of 7 atm. (Treat air as an ideal gas.)

(a) What is the final volume of this gas after compression? *(2 points)*

(b) How much work is done in compressing the gas? *(3 points)*

(c) If the temperature of the gas is initially 300 K, what is its temperature after compression? *(2 points)*

(d) What is the difference in internal energy ΔE of the gas? *(2 points)*

(e) Is the change in internal energy due to a change in external parameters or probabilities? *(2 points)*

Problem 3 - Quasi-static adiabatic process

An ideal gas is contained in a large jar of volume V_0. Fitted to the jar is a glass tube of cross-sectional area A in which a metal ball of mass m fits snugly. The equilibrium pressure in the jar is slightly higher than atmospheric pressure p_0 because of the weight of the ball. The ball executes as a simple harmonic motion (neglecting friction) by moving up and down. Associated with the ball's movement, the gas undergoes a quasistatic adiabatic process in which its volume changes by dV. The moving ball stays in the glass tube and no gas leaves the jar. (Hint: pV^γ)

(a) Find the stiffness coefficient of the oscillator $k(\gamma, A, p, V)$. *(4 points)*

(b) Find the oscillation frequency $f(\gamma, m, A, V)$. *(2 points)*

Problem 4 - Quasi-static isothermal expansion

(a) How much heat is required to cause the quasi-static isothermal expansion of one mol of an ideal gas at $T = 500K$ from $P_A = 0.42atm$, $V_A = 100liters$ to $P_B = 0.15atm$.*(2 points)*

(b) What is V_B?*(1 points)*

(c) What is ΔE of this process?*(2 points)*