

Statistical Physics Exam

3rd March 2014

Name	Student Number

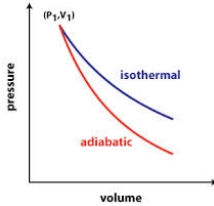
Task 1	Task 2	Task 3	Task 4	Total	Percentage	Mark

Useful constants

gas constant R	$8.31 J / (K \cdot mol)$
Boltzmann constant k_B	$1.38 \cdot 10^{23} J / K$
Avogadro number N_A	$6.02 \cdot 10^{23} mol^{-1}$
speed of light c	$300 \cdot 10^3 m / s$

Problem 1 (25P)

- Sketch the p,V-diagramm for the expansion of an ideal gas from V_1 to V_2 for (3P)
 - isothermic
 - adiabatic expansion



- Is the temperature after adiabatic expansion lower, higher, or equal to that after isothermic expansion? lower
- Which of the following quantities are extensive: Density, total mass, temperature, heat capacity, molar volume (1P) ? Total mass, heat capacity
 - What is the triple point of a pure substance (1P)? coexistence of solid, liquid, gas phase at certain p,T
 - (2P) Write down the multinomial distribution $P_N(\{m; p\})$ for random variables $\{m_1, m_2, \dots, m_k\}$ with their probabilities $\{p_1, p_2, \dots, p_k\}$, where $\sum_{j=1}^k m_j = N$ and $\sum_{j=1}^k p_j = 1$.

$$P_N = \frac{N!}{m_1! \dots m_k!} p_1^{m_1} \dots p_k^{m_k}. \quad (1)$$

- Consider the phase space $\mathbf{\Gamma} \equiv \{q_1, \dots, p_1, \dots\}$ with the total phase volume v , where $\rho(\{q, p\}; t)$ is the density function. The dimensionless Hamiltonian is denoted by $\mathcal{H}(\{q, p\})/(k_B T) \equiv \beta \mathcal{H}(\{q, p\})$ with the Boltzmann's constant k_B and the absolute temperature T .
 - (2P) Write down the condition for the density function ρ to yield the stationary ensemble which represents a system in equilibrium.

$$\frac{\partial \rho}{\partial t} = 0. \quad (2)$$

- (2P) Write down the condition for the density function ρ to yield the micro-canonical ensemble. Express the ensemble average $\langle f \rangle$ of a certain physical quantity f .

$$\rho = \text{constant}. \quad (3)$$

$$\langle f \rangle = \int f dv / v. \quad (4)$$

- (c) **(2P)** Write down the condition for the density function ρ to yield the canonical ensemble. Express the ensemble average $\langle g \rangle$ of a certain physical quantity g .

$$\rho \sim e^{-\beta\mathcal{H}}. \quad (5)$$

$$\langle g \rangle = \frac{\int g e^{-\beta\mathcal{H}} dv}{\int e^{-\beta\mathcal{H}} dv}. \quad (6)$$

6. Consider a system of 3 non-interacting particles. Denoting partition functions of each particle by z_1, z_2 and z_3 , express the total partition function for,

- (a) **(1P)** distinguishable particles.

$$z_1 z_2 z_3. \quad (7)$$

- (b) **(1P)** indistinguishable particles.

$$\frac{1}{3 \cdot 2 \cdot 1} z_1 z_2 z_3, \quad (8)$$

or

$$\frac{1}{3 \cdot 2 \cdot 1} (z_i)^3, \quad (9)$$

for $i = 1, 2$, or 3 .

7. **(3P)** In which situations does quantum statistics become important?

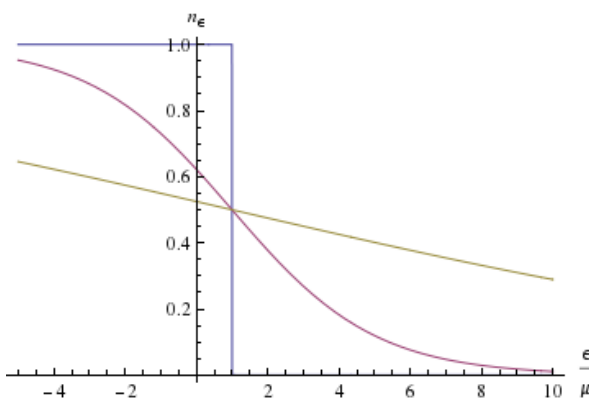
Quantum statistics becomes important when the wave functions of the individual particles start to overlap. This can happen at low temperature and/or high density and/or low mass.

8. **(4P)** Write down the Fermi-Dirac distribution function and sketch the average occupation as a function of the single particle energy for

- (a) temperature $T = 0$.

- (b) small temperature T .

The Fermi-Dirac distribution gives the average occupation of an energy level with energy ϵ : $n_\epsilon = \frac{1}{e^{\beta(\epsilon-\mu)} + 1}$.



- (c) very large temperature T . Which classical distribution is approached?
For large temperatures the Boltzmann distribution $e^{-\beta\epsilon}$ is approached.
9. **(3P)** What happens to an ideal quantum gas in the limit temperature $T \rightarrow 0$
- (a) if the particles are fermions? Fermions will occupy the lowest energy levels in pairs up to the Fermi energy.
 - (b) if the particles are bosons? A macroscopic number of bosons will occupy the ground state (condensate).
 - (c) if the particles are photons (which are also boson, but without mass)? Since the number of photons is not conserved, all photons will disappear (up to vacuum fluctuations).

Problem 2 (25P)

Consider molecules coexist in gas-solid phases, at the temperature $T = 1/(k_B\beta)$. The molecules have the same mass m . Assume that gas molecules (indistinguishable) of the number N_g and the volume V_g are ideal, and solid molecules (distinguishable) of the number N_s and V_s are three-dimensional harmonic oscillators with the frequency ω . (i.e. the Einstein solid model with the potential $m\omega^2 q^2/2$).

- a) **(5P)** Calculate the grand partition function Ξ_g for the gas molecules, where the thermal wavelength is $\lambda = h\beta^{1/2}/\sqrt{2\pi m}$, and the chemical potential is μ_g .

The grand partition function Ξ_g :

$$\begin{aligned}\Xi_g &= \sum_{N_g=0}^{\infty} \frac{1}{N_g!} \left(\frac{V_g}{\lambda^3}\right)^{N_g} e^{\beta\mu_g N_g} \\ &= \exp\left[\frac{V_g}{\lambda^3} e^{\beta\mu_g}\right].\end{aligned}\tag{10}$$

- b) **(5P)** Calculate the average number $\langle N_g \rangle$ and the average energy U_g of the gas molecules. For U_g , express it in terms of $\langle N_g \rangle$ and $k_B T$.

The average number $\langle N_g \rangle$:

$$\begin{aligned}\langle N_g \rangle &= \frac{1}{\beta} \left[\frac{\partial}{\partial \mu_g} \ln(\Xi_g) \right]_{V_g, T} \\ &= \frac{V_g}{\lambda^3} e^{\beta\mu_g}.\end{aligned}\tag{11}$$

The average energy U_g :

$$\begin{aligned}U_g &= - \left[\frac{\partial}{\partial \beta} \ln(\Xi_g) \right]_{e^{\beta\mu_g}, V_g} \\ &= -e^{\beta\mu_g} V_g \left[\frac{\partial}{\partial \beta} \lambda^{-3} \right] \\ &= -e^{\beta\mu_g} V_g \left[\frac{\partial}{\partial \beta} (2\pi m)^{3/2} h^{-3} \beta^{-3/2} \right] \\ &= -e^{\beta\mu_g} V_g (2\pi m)^{3/2} h^{-3} \left[\frac{\partial}{\partial \beta} \beta^{-3/2} \right] \\ &= \frac{3}{2} \underbrace{e^{\beta\mu_g} V_g (2\pi m)^{3/2} h^{-3} \beta^{-3/2}}_{\langle N_g \rangle} \beta^{-1} \\ &= \frac{3}{2} \langle N_g \rangle k_B T.\end{aligned}\tag{12}$$

- c) **(5P)** Calculate the grand partition function Ξ_s for the solid molecules, where the Planck constant is h , and the chemical potential is μ_s .

The single partition function (in three dimension) z_s :

$$\begin{aligned}
z_s &= \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dqdp}{h} \exp \left\{ -\beta \left(\frac{1}{2} m \omega^2 q^2 + \frac{p^2}{2m} \right) \right\} \right]^3 \\
&= \frac{1}{h^3} \left[\underbrace{\int_{-\infty}^{\infty} dq \exp \left(-\frac{\beta}{2} m \omega^2 q^2 \right)}_{\sqrt{\frac{2\pi}{\beta m \omega^2}}} \right]^3 \left[\underbrace{\int_{-\infty}^{\infty} dp \exp \left(-\frac{\beta p^2}{2m} \right)}_{\sqrt{\frac{2\pi m}{\beta}}} \right]^3 \\
&= \left[\frac{2\pi}{\beta h \omega} \right]^3.
\end{aligned} \tag{13}$$

The grand partition function Ξ_s :

$$\begin{aligned}
\Xi_s &= \sum_{N_s=0}^{\infty} e^{\beta \mu_s N_s} \left(\frac{2\pi}{\beta h \omega} \right)^{3N_s} \ll e^{\beta \mu_s} \left(\frac{2\pi}{\beta h \omega} \right)^3 \ll 1, \\
&= \left[1 - e^{\beta \mu_s} \left(\frac{2\pi}{\beta h \omega} \right)^3 \right]^{-1}.
\end{aligned} \tag{14}$$

- d) **(5P)** Calculate the average number $\langle N_s \rangle$ and the average energy U_s of the solid molecules. For U_s , express it in terms of $\langle N_s \rangle$ and $k_B T$.

The average number $\langle N_s \rangle$:

$$\begin{aligned}
\langle N_s \rangle &= \frac{1}{\beta} \left[\frac{\partial}{\partial \mu_s} \ln (\Xi_s) \right]_T \\
&= \frac{1}{\beta} \left[\frac{\partial}{\partial \mu_s} \ln \left(\frac{1}{1 - e^{\beta \mu_s} \left(\frac{2\pi}{\beta h \omega} \right)^3} \right) \right]_T \\
&= -\frac{1}{\beta} \left[\frac{\partial}{\partial \mu_s} \ln \left(1 - e^{\beta \mu_s} \left(\frac{2\pi}{\beta h \omega} \right)^3 \right) \right]_T \\
&= \frac{e^{\beta \mu_s} \left(\frac{2\pi}{\beta h \omega} \right)^3}{1 - e^{\beta \mu_s} \left(\frac{2\pi}{\beta h \omega} \right)^3}.
\end{aligned} \tag{15}$$

The average energy U_s :

$$U_s = - \left[\frac{\partial}{\partial \beta} \ln (\Xi_s) \right]_{e^{\beta \mu_s}}$$

$$\begin{aligned}
&= \left[\frac{\partial}{\partial \beta} \ln \left(1 - e^{\beta \mu_s} \left(\frac{2\pi}{\beta h \omega} \right)^3 \right) \right]_{e^{\beta \mu_s}} \\
&= \frac{- \left(\frac{2\pi}{h \omega} \right)^3 e^{\beta \mu_s}}{1 - e^{\beta \mu_s} \left(\frac{2\pi}{\beta h \omega} \right)^3} \left[\frac{\partial}{\partial \beta} \beta^{-3} \right] \\
&= 3 \frac{\left(\frac{2\pi}{h \omega} \right)^3 e^{\beta \mu_s} \beta^{-3}}{1 - e^{\beta \mu_s} \left(\frac{2\pi}{\beta h \omega} \right)^3} \beta^{-1} \\
&= 3 \langle N_s \rangle k_B T. \tag{16}
\end{aligned}$$

e) **(5P)** Calculate and express the density $\rho_g = \langle N_g \rangle / V_g$ of the gas molecules, in terms of $\langle N_s \rangle$, β , h , ω and λ .

Using the above results for $\langle N_g \rangle$ and $\langle N_s \rangle$ (Eq. (11) and Eq. (15)), one can equate $e^{\beta \mu_g}$ and $e^{\beta \mu_s}$, i.e., the chemical potential balance:

$$\begin{aligned}
\langle N_g \rangle &= \frac{V_g}{\lambda^3} e^{\beta \mu_g} \\
\Rightarrow e^{\beta \mu_g} &= \rho_g \lambda^3. \tag{17}
\end{aligned}$$

$$\begin{aligned}
\langle N_s \rangle &= \frac{e^{\beta \mu_s} \left(\frac{2\pi}{\beta h \omega} \right)^3}{1 - e^{\beta \mu_s} \left(\frac{2\pi}{\beta h \omega} \right)^3} \\
\Rightarrow e^{\beta \mu_s} &= \left(\frac{\beta h \omega}{2\pi} \right)^3 \frac{\langle N_s \rangle}{1 + \langle N_s \rangle}. \tag{18}
\end{aligned}$$

By equating Eq. (17) and Eq. (18), the density $\rho_g = \langle N_g \rangle / V_g$ is obtained as below

$$\rho_g = \left(\frac{\beta h \omega}{2\pi \lambda} \right)^3 \frac{\langle N_s \rangle}{1 + \langle N_s \rangle}. \tag{19}$$

Problem 3 (25P)

The energy of a relativistic electron is given by $\epsilon_{\vec{p}} = \sqrt{p^2 c^2 + m^2 c^4}$ independently of its spin. Assume that N electrons are confined to a cubic box with sides of length L and volume V with periodic boundary conditions.

- a) **(10P)** Find the Fermi momentum k_F and the Fermi energy $\epsilon_F = \mu(T = 0)$ as a function of N and V . How would you (in principle) find the chemical potential μ for $T \neq 0$?

The wave function of a free electron in a box of volume V is

$$\psi(\vec{x}) = \frac{1}{\sqrt{V}} e^{i\vec{k}\cdot\vec{x}} \quad (20)$$

where the allowed values of the wave vector are determined by the boundary conditions

$$\psi(\vec{x} + L\hat{e}_i) = \psi(\vec{x}) \quad i = x, y, z \quad (21)$$

$$e^{iLk_i} = 1 \quad (22)$$

$$\Rightarrow k_i = \frac{2\pi}{L} n_i \quad n_i = 0, 1, 2, \dots \quad (23)$$

The sum over discrete energy levels can be replaced by an integral using $d^3k = \left(\frac{2\pi}{L}\right)^3 d^3n = \frac{(2\pi)^3}{V}$:

$$\sum_{n_x=0}^{\infty} \sum_{n_y=0}^{\infty} \sum_{n_z=0}^{\infty} \approx \int \int \int dn_x dn_y dn_z = \frac{V}{(2\pi)^3} \int d^3k \quad (24)$$

The average number of fermions in the energy level \vec{k} is given by the Fermi-Dirac distribution

$$n_{\vec{k}} = \frac{1}{e^{\beta(\epsilon_{\vec{k}} - \mu)} + 1} \quad (25)$$

The chemical potential μ has to be adjusted such that the integral

$$N = 2 \frac{V}{(2\pi)^3} \int n_{\vec{k}} d^3k \quad (26)$$

equals the total number of particles. The factor of 2 is introduced because each energy level can be occupied by two fermions.

- b) At $T = 0$, the Fermi-Dirac distribution simplifies to a step function, where all energy levels below $\epsilon_F = \sqrt{\hbar^2 k_F^2 c^2 + m^2 c^4}$ are occupied and all above are empty. The Fermi energy is such that

$$N = 2 \frac{V}{(2\pi)^3} \int_0^{k_F} 4\pi k^2 dk = \frac{V}{3\pi^2} k_F^3 \quad (27)$$

equals the total number of Fermions. The Fermi energy scales as the cubic root of the number density:

$$k_F = \left(3\pi^2 \frac{N}{V}\right)^{1/3} \quad (28)$$

The Fermi energy is then

$$\epsilon_F = \sqrt{\hbar^2 \left(3\pi^2 \frac{N}{V}\right)^{2/3} c^2 + m^2 c^4} \quad (29)$$

At finite temperature, one would have to solve the integral

$$N = \frac{V}{(2\pi)^3} \int \frac{1}{e^{\frac{(\epsilon_{\vec{k}} - \mu)}{k_B T}} + 1} d^3k \quad (30)$$

as a function of μ and invert the result to obtain $\mu(N, V, T)$.

c) **(3P)** Write down the integral for the internal energy U at $T = 0$.

At $T = 0$ the internal energy is

$$U = 2 \int \epsilon_{\vec{k}} n_{\vec{k}} d^3k = 2 \frac{V}{(2\pi)^3} \int_0^{k_F} \sqrt{\hbar^2 k^2 c^2 + m^2 c^4} 4\pi k^2 dk \quad (31)$$

d) **(10P)** At low densities only the lowest momentum states will be occupied. In this limit the rest mass energy is much higher than the kinetic energy of the fermions. Assuming $\hbar^2 c^2 k^2 \ll m^2 c^4$ make a first order Taylor expansion of the integral in part b) and solve it. In the limit of low densities, only the levels with low kinetic energy will be occupied, so that $\hbar^2 k^2 c^2 \ll c^2 m^4$. Then

$$\epsilon_{\vec{k}} = \sqrt{\hbar^2 k^2 c^2 + c^2 m^4} = mc^2 \sqrt{1 + \frac{\hbar^2 k^2}{m^2 c^2}} \approx mc^2 \left(1 + \frac{\hbar^2 k^2}{2m^2 c^2}\right) = mc^2 + \frac{\hbar^2 k^2}{2m} \quad (32)$$

Now the integral can be solved easily:

$$U = \frac{2V}{(2\pi)^3} \int_0^{k_F} 4\pi k^2 mc^2 \left(1 + \frac{\hbar^2 k^2}{2m^2 c^2}\right) dk \quad (33)$$

$$= \frac{V}{\pi^2} mc^2 \left(\frac{1}{3} k_F^3 + \frac{\hbar^2}{2m^2 c^2} \frac{1}{5} k_F^5\right) \quad (34)$$

$$(35)$$

e) **(2P)** Use your result for the Fermi momentum in a) to find the internal energy as a function of N and V at $T = 0$.

Substituting $k_F = \left(3\pi^2 \frac{N}{V}\right)^{1/3}$ into the expression for the internal energy gives:

$$U = \frac{V}{\pi^2} mc^2 \left(\frac{1}{3} 3\pi^2 \frac{N}{V} + \frac{\hbar^2}{2m^2 c^2} \frac{1}{5} \left(3\pi^2 \frac{N}{V}\right)^{5/3}\right) \quad (36)$$

$$= Nmc^2 + \frac{V}{10\pi^2} \frac{\hbar^2}{m} \left(3\pi^2 \frac{N}{V}\right)^{5/3} \quad (37)$$

(Not part of the question:) This result can also be rewritten as

$$U = Nmc^2 + \frac{3}{5} N \frac{\hbar^2 k_F^2}{2m} \quad (38)$$

Problem 4 (25P)

	heat capacity kJ/(kg·K)	heat of vaporisation kJ/(kg)	molar mass/ kg/mol	density kg/dm ³
water	4	2000	0.02	1
copper	0.5	5000	0.06	9
creature	2	–	–	–
box	3	–	–	–

A thermally well-insulated closed box contains 2 kg water, both at $T_{water}^{init} = T_{box}^{init} = 300K$. Water and container are in thermal equilibrium. The empty box has a mass of 0.6 kg and a volume of 11.4l.

1. A cube of copper of mass $m_{cube} = 0.4kg$ with temperature $T_{Cu}^{init} = 1300K$ is put into the water and the box is closed again. After a while, a new equilibrium is reached.

(a) What are the new temperatures of water T_{water}^{final} , the copper cube T_{Cu}^{final} and the box T_{box}^{final} (3P)?

$$T_{all}^{final} = \frac{C_{water}T_{water} + C_{box}T_{box} + C_{copper}T_{copper}}{C_{total}}$$

$$C_{water} = 4kJ/(kg \cdot K) \cdot 2kg = 8kJ/K$$

$$C_{box} = 3kJ/(kg \cdot K) \cdot 0.6kg = 1.8kJ/K$$

$$C_{copper} = 0.5kJ/(kg \cdot K) \cdot 0.4kg = 0.2kJ/K$$

$$T_{all}^{final} = \frac{8kJ/K \cdot 300K + 1.8kJ/K \cdot 300K + 0.2kJ/K \cdot 1300K}{8 + 1.8 + 0.2kJ/K}$$

$$T_{all}^{final} = \frac{(2400 + 540 + 260)kJ}{10kJ/K}$$

$$T_{all}^{final} = \frac{3200}{10}K$$

$$T_{all}^{final} = 320K$$

(b) How much heat flows from the copper cube to water, how much to the box (2P)?

$$\Delta Q_{cube \rightarrow all} = \Delta T_{cube} \cdot C_{cube}$$

$$\Delta Q_{cube \rightarrow all} = (1300 - 320)K \cdot 0.2kJ/K$$

$$\Delta Q_{cube \rightarrow all} = 196kJ$$

$$\Delta Q_{cube \rightarrow water} = \Delta T_{water} \cdot C_{water}$$

$$\Delta Q_{cube \rightarrow water} = 20K \cdot 8kJ/K$$

$$\Delta Q_{cube \rightarrow water} = 160kJ/K$$

$$\begin{aligned}\Delta Q_{cube \rightarrow box} &= \Delta T_{box} \cdot C_{box} \\ \Delta Q_{cube \rightarrow box} &= 20K \cdot 1.8kJ/K \\ \Delta Q_{cube \rightarrow box} &= 36kJ/K\end{aligned}$$

- (c) What is the entropy change of the copper cube, water, the box, the entire system (solve symbolically, i.e. without actual numbers) ? Which is positive, which negative (**5P**)?

$$\begin{aligned}\Delta S_{cube} &= C_{cube} \int_{T_{init}}^{T_{final}} \frac{dT}{T} \\ \Delta S_{cube} &= C_{cube} \ln \frac{T_{final}}{T_{init}} \\ \Delta S_{box} &= C_{water} \ln \frac{T_{final}}{T_{init}} \\ \Delta S_{water} &= C_{water} \ln \frac{T_{final}}{T_{init}} \\ \Delta S_{total} &= \Delta S_{cube} + \Delta S_{water} + \Delta S_{box} \\ \Delta S_{cube} &= 0.2kJ/K \ln \frac{320}{1300} \\ \Delta S_{cube} &= -0.2kJ/K \cdot 1.40 \\ \Delta S_{cube} &= -0.28kJ/K < 0 \\ \Delta S_{water} &= 8kJ/K \ln \frac{320}{300} \\ \Delta S_{water} &= 8kJ/K \cdot 0.06 \\ \Delta S_{water} &= 0.48kJ/K > 0 \\ \Delta S_{box} &= 1.8kJ/K \cdot 0.06 \\ \Delta S_{box} &= 0.108kJ/K > 0 \\ \Delta S_{total} &= (-0.28 + 0.48 + 0.108)kJ/K \\ \Delta S_{total} &= 0.308kJ/K > 0\end{aligned}$$

2. The water in the box ($T_{water}^{init} = T_{box}^{init} = 300K$) is heated and stirred by a 1000W-immersion heater.

- (a) After what time does the water start boiling (**3P**)?

$$\begin{aligned}\Delta Q &= \Delta T \cdot C_{water} \\ \Delta Q &= 73K \cdot 8kJ/K \\ \Delta Q &= 584kJ \\ time &= \frac{heat}{power} \\ t &= \frac{584kJ}{1kJ/s} \\ t &= 584s \sim 10minutes\end{aligned}$$

- (b) The box can sustain an inside pressure of 5bar. Do you expect it to be still intact after 20 minutes of further heating? Explain by an approximate calculation and treat water vapour as an ideal gas. **(6P)** NO!

$$\begin{aligned}
 Q_{further} &= 1200s \cdot 1kJ/s \\
 \Delta H_{vapour}^{water} &= 2000kJ/kg \\
 m_{water}^{vapour} &= 1200kJ / 2000kJ/kg \\
 m_{water}^{vapour} &= 0.6kg \\
 N_{water}^{vapour} &= 0.6kg / (0.02kg/mol) \\
 N_{water}^{vapour} &= 30mol \\
 p &= \frac{nRT}{V} \\
 V &= 11.4l - (2 - 0.6)l = 10l \\
 p &= \frac{30mol \cdot 8JK^{-1}mol^{-1} \cdot 373K}{0.01m^3} \\
 p &= 24000 \cdot 373J/m^3 \\
 p &\approx 24000 \cdot 400J/m^3 \\
 p &\approx 9600000J/m^3 \\
 p &\approx 9600000N/m^2 \\
 p &\approx 96bar \gg 5bar
 \end{aligned}$$

3. A creature of body mass $m_{creature} = 50$ kg eats a super power-bar. That food intake can be converted to a heat of $12 \cdot 10^3$ kJ.

- (a) Approximate how much the body temperature ($T_{body}^{init}=273K$) would rise if the body had no way to dispose the heat **(2P)**?

$$\begin{aligned}
 \Delta T_{body} &= \frac{Q}{C_{body}} \\
 \Delta T_{body} &= \frac{12000kJ}{2kJ/(kg \cdot K) \cdot 50kg} \\
 \Delta T_{body} &= \frac{12000kJ}{100kJ} K \\
 \Delta T_{body} &= 120K
 \end{aligned}$$

- (b) Assume all the heat is used to sweat out (vapourise) water. How much sweat can the creature produce that way (The creature's sweat does not contain salt or other ingredients) **(4P)**?

First the water has to heat up to the boiling temperature (373K), then it vapourises.

$$\begin{aligned}
 Q_{heatToBoil} / kg &= (373 - 273)K \cdot 4kJ/K^{-1}kg \\
 Q_{heatToBoil} / kg &= 400kJ/kg
 \end{aligned}$$

$$\begin{aligned} Q_{\text{heatToVapourise}} / \text{kg} &= 2000 \text{kJ/kg} \\ Q_{\text{sweat}} / \text{kg} &= 2400 \text{kJ/kg} \\ m_{\text{sweat}} &= \frac{12000 \text{kJ}}{2400 \text{kJ}} \text{kg} \\ m_{\text{sweat}} &= 5 \text{kg} \end{aligned}$$