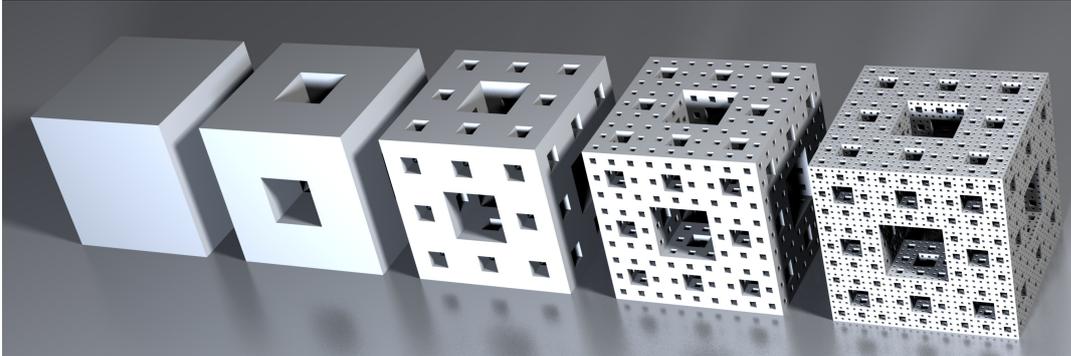


VL: Mi 14-16 Uhr Prof. Dr. Kathy Lüdge  
 UE: Mi 16-18 Uhr ( every 2nd week)  
<http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-luedge/teaching/>

### 3. Homework - Nonlinear dynamics

Need to be handed in: Wed 25.11.( before the lecture) You can build groups of up to 3 students.

#### Task 5 (8 points): Menger-sponge and fractal dimension



The Menger-sponge (named after Karl Menger) can be created by the following rule: Take a cube and look at the small cubes that are created by deviding each side into 3 segments. From these  $27 = 3 \times 3 \times 3$  equally sized cubes remove 7 (the ones in the middle of each plane as seen in the Figure). Repeat this procedure again (recursively) for each remaining small cube.

1. Calculate the surface  $F_n$  and the volume  $V_n$  as a function of the iteration step  $n$  ( $n = 0$  is the full cube at the beginning)). Which value do you get in the limit  $n \rightarrow \infty$ .
2. Calculate the Hausdorff-dimension of the Menger-sponge.

#### Task 6 (12 points): Logistic map

In this task you will investigate the route to chaos in the logistic map:

$$x_{n+1} = f(x_n) := r x_n(1 - x_n), \quad (r \in [0, 4])$$

To start your investigation please compute the bifurcation diagram and a diagram of the Lyapunov exponent as a function of  $r$ . With that you can answer the following questions:

- Which value of the Lyapunov-exponent do you find at a period doubling bifurcation? Which points in the bifurcation diagram are related to positiv Lyapunov exponents?
- Let  $R_n$  be the  $r$  values of the period doubling points in your bifurcation diagram. Find the first four ratios of

$$\delta_n = \frac{R_n - R_{n-1}}{R_{n+1} - R_n}.$$

Do they agree with the Feigenbaum constant  $\delta = 4.669 \dots$ ? (If not you might need to increase your numeric precision.)

#### Tips for the numeric:

•Use  $r$  values between  $[0, 4]$ . Iterate the logistic map with a random intitial condition for  $x_0$  for about 400 steps (to eliminate transients). •For the next 300 iterations you can collect the maxima and minima to produce the bifurcation diagram. •For the Lyapunov diagram you need appr. 2.000 iterations (then calculate the mean of all the local Lyapunov-exponents  $\ln |f'(x_n)|$  of your 2.000 iteration steps  $x_n$ ). Chose a range of  $[-1, 1]$  for plotting the Lyapunov exponent over  $r [0, 4]$ .

Use a file format with fixed resolution for plotting the results (e.g. .png) to avoid big image files.