VL: Mi 14-16 Uhr Prof. Dr. Kathy Lüdge
UE: Mi 16-18 Uhr ( every 2nd week)
http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-luedge/teaching/

# 5 Homework - Nonlinear dynamics

Needs to be handed in: Wed 20.1.16 (before the lecture) You can build groups of up to 3 students.

**Task 11 (10 points):** *Master stability function*

$N$ identical one-dimenional maps with local dynamics *f* are coupled together to a network

$$x^i_{k+1} = f(x^i_k) + \sigma \sum_{j=1}^{N} G_{ij}\, h(x^j_k). \tag{1}$$

$\sigma$ is the coupling strength, $G_{ij}$ the coupling matrix determining the link between $j \to i$, and $h$ is the coupling scheme. The master stability function (MSF) is a function $\mathbb{C} \to \mathbb{R}$ that maps the largest Lyapunov exponent $\lambda_{max}$ to any complex number $\alpha + i\beta$. $\lambda_{max}$ follows from the variational equation

$$\xi_{k+1} = \left[f'(x^s_k) + (\alpha + i\beta)h'(x^s_k)\right] \xi_k$$

with ${x^s}_k$ describing the synchronized dynamics. Thus $\lambda_{max}$ is given by $\lambda_{\max} = \lim_{k\to\infty} \frac{1}{k} \ln \frac{|\xi_k|}{|\xi_0|}$.

1. The local dynamics is given by the logistic map $f(x) := r\,x(1-x)$, and a coupling scheme *h(x)* $h(x) := x$. Now, chose two values for $r$ (one in the chaotic regime and one outside), plot the MSF as a function of $\alpha$ and $\beta$ and mark the contour $\lambda_{\max} = 0$. Please use your results of task 6 (homework 3).
2. For each *r*-value, please chose a network (with more than 3 elements and zero row sum) for which the synchronization is stabil and one for which it is unstable. To visualize the results, plot the eigenvalues $\sigma\gamma_k$ of the matrix $\sigma G$ into the same plot as the MSF.
3. Do a direct numeric integration of equation (1) and verify your results obtained in 2.

**Task 12 (10 points):** *Population evolution with delay*

A model for the evolution of the number $N$ of animals in a group is given by

$$\dot{N}(t) = rN(t)[1 - N(t-\tau)/K],$$

here $r$ is the growth rate and $K$ describs the adaptation rate of the surrounding.

1. Find the transformation that transforms the above equation into its non-dimensional version

$$x'(s) = x(s)[1 - x(s-a)].$$

2. Use the non-dimensional equation and show that the fixed point $x_0 = 0$ is always unstable while the fixed point $x_1 = 1$ is stable for $a = 0$.
3. Find the value for $a > 0$, where $x_1$ looses its stability in a Hopf bifurcation.