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Advanced Statistical Physics (WS11/12)
Problem sheet 9

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Problem 1: Diesel Cyclic Process

Consider an engine with a compression ratio of $\epsilon = V_1/V_2$, a cut-off ratio of $\phi = V_3/V_2$ and filled with an ideal gas of temperature T_1 and pressure P_1 with an adiabatic index of $\gamma = c_P/c_V$. This engine undergoes a reversible Diesel cyclic process, which is defined as follows:

- $1 \rightarrow 2:$ isentropic compression from V_1 to V_2,
- $2 \rightarrow 3:$ isobaric heating from T_2 to T_3 at P_2,
- $3 \rightarrow 4:$ isentropic expansion from V_3 to V_1,
- $4 \rightarrow 1:$ isochoric cooling from T_4 to T_1 at V_1.

Draw the corresponding P-V- and S-T-diagram. Calculate the efficiency η as a function of γ, ϵ and ϕ.

Problem 2: Solid-Liquid Phase-Transition

We want to consider a substance with the free energy of the liquid phase

$$F_l(V,T) = \frac{aN^2}{2TV}$$

and the free energy of the solid phase

$$F_s(V,T) = \frac{bN^3}{3TV^2},$$

where a and b are constants, V is the total volume of the system, T is the temperature and N is the total number of particles.

a) Calculate the pressure P_m, at which the substance melts.

b) Calculate the densities of the liquid and the solid phase at the phase transition point.

c) Calculate the entropy change per particle at the phase transition point.

d) Use the Clausius-Clapeyron equation and your results in parts b) and c) to calculate $dP_m(T)/dT$. Compare the result with the result in part a).

Problem 3: Interacting Gas

A gas of interacting atoms has an equation of state given by

$$p(T,V) = aT^{1/2} + bT^3 + cV^{-2}$$

and the heat capacity at constant volume given by

$$C_V(T,V) = dT^{1/2}V + eT^2V + fT^{1/2},$$

where a through f are constants which are independent of T and V.

a) Find the differential of the internal energy $\mathrm{d}U(T,V)$ in terms of $\mathrm{d}T$ and $\mathrm{d}V$.

b) Find the relationships among a through f due to the fact that $U(T,V)$ is a state variable.

c) Find $U(T,V)$ as a function of T and V.

d) Use the equations of state to derive a simple relation between P and U for an ideal monoatomic gas. If the gas in the previous parts was to be made ideal, what would be the restrictions on the constants a through f?

Problem 4: Curie Magnet

The susceptibility χ of a certain paramagnetic salt obeys Curie's law, i.e. $\chi = b/T$, which holds true for small magnetic fields and temperatures T well above the Curie temperature T_C. The salt's heat capacity per unit volume (at constant magnetic field) is assumed to be inversely proportional to the square of the absolute temperature, i.e. $C_H = \alpha V/T^2$, where $\alpha = b + aH^2$, a and b being constants and V is the volume.

A sample of this salt at temperature T_i is placed in a magnetic field of strength H. The sample is adiabatically demagnetised by slowly reducing the strength of the field to zero. What is the final temperature, T_f, of the salt for $H = 0$? Assume thereby, that V is constant during this process.

Hint: The solution of the differential equation $\frac{\mathrm{d}f(x)}{\mathrm{d}x} = \frac{\lambda x}{1+\kappa x^2} f(x)$ is

$f(x) = (1 + \kappa x^2)^{\lambda/2\kappa} f(x = 0)$.