

## Exam Solution: Advanced Statistical Physics Part I: Questions (15P)

You have 15 minutes to work on part one of the exam. Please answer the questions on this sheet and write down your name and matrikelnummer. No notes or other tools are allowed in this part. You don't have to give explanations, short answers suffice. After 15 minutes, this sheet will be collected and part two will be handed out.

Good Luck!

1. The Stefan-Boltzmann-Law is

$$\frac{U}{V} = \frac{\pi^2 (k_B T)^a}{15 (c\hbar)^3}.$$

Give the exponent  $a$  of the temperature dependence. (1P)

$$a = 4$$

2. Write down the Fermi-Dirac distribution. (1P)

$$\langle n_p \rangle = \frac{1}{1 + z^{-1} e^{\beta \epsilon_p}} = \frac{1}{1 + e^{\beta (\epsilon_p - \mu)}}$$

3. What is the efficiency  $\eta$  of an ideal, reversible heat engine operated between heat reservoirs of temperatures  $T_1 < T_2$ ? (1P)

$$\eta = 1 - \frac{T_1}{T_2}$$

4. Consider a thermodynamic cycle in the  $PV$  diagram that is operated clockwise. Is this a heat pump or a heat engine? (1P)

Heat engine.

5. What is the heat capacity of 1 mol of an ideal gas with  $f$  degrees of freedoms at constant pressure,  $C_P$ ? (1P)

$$c_p = \frac{f+2}{2} R = \frac{f+2}{2} N_A k_B$$

6. How is the order of a phase transition defined? (1P)

The order of a phase transition according to the Ehrenfest classification is the first discontinuous derivative of a thermodynamic potential with respect to one of its variables. E.g.:

$$\frac{\partial^n g_2}{\partial p^n} \neq \frac{\partial^n g_1}{\partial p^n}$$

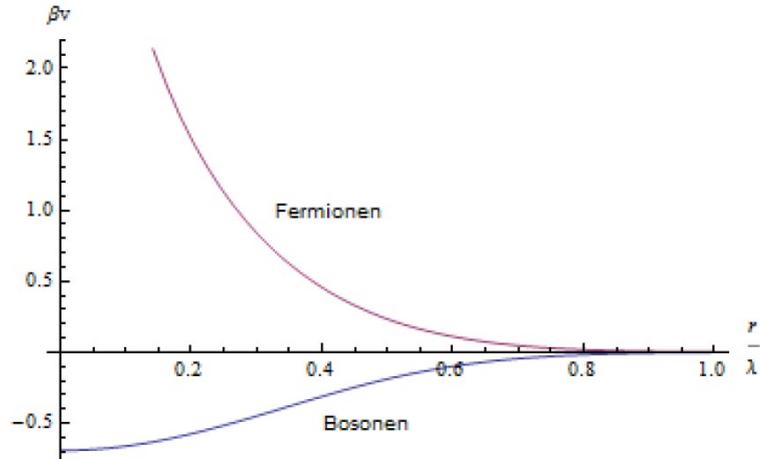
7. Consider the free energy  $F(T, V)$  as a function of volume and temperature. What is the differential of  $F$ ? (1P)

$$dF = -pdV - SdT$$

8.  $u_n(x)$  is the normalised single particle wave function for a spinless Fermion in a box in the  $n$ -th state ( $n = 1, 2, \dots$ ). Write down the normalised two particle wave function of the ground state. (1P)

$$\Psi = \frac{1}{\sqrt{2}} (u_1(x) u_2(y) - u_2(x) u_1(y))$$

9. Sketch the effective pair-interaction between Fermions and Bosons as a function of  $r/\lambda$ , where  $r$  is the particle distance and  $\lambda$  the thermal wave length. (2P)



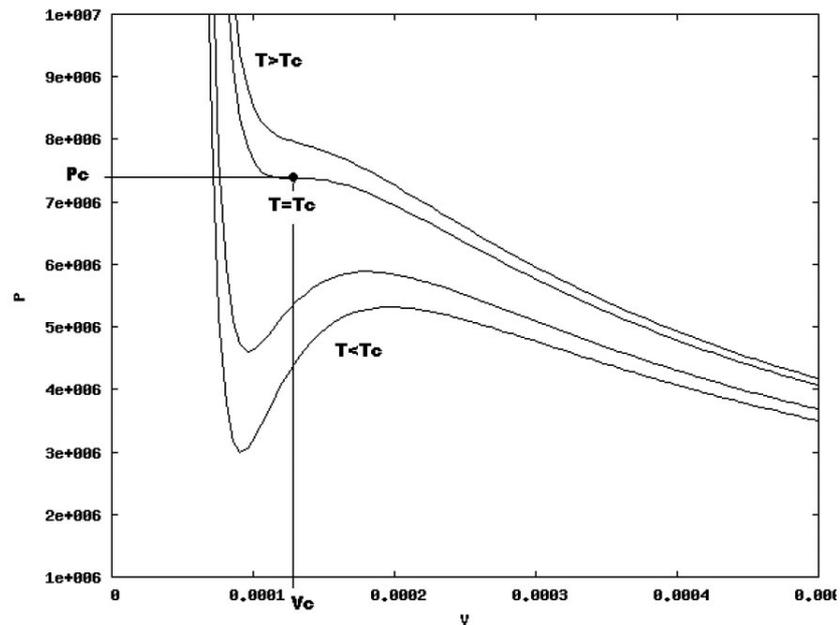
10. How is the second virial coefficient  $a_2$  defined? (1P)

$$a_2 = -\frac{1}{2} \int d^3r \left( e^{-\beta v(r)} - 1 \right)$$

11. What does the ergodic theorem state? (1P)

A system (point) in  $\Gamma$  space will reach every accessible position for long enough times.

12. Sketch the isothermals for the van der Waals equation in the P-V diagram for  $T > T_C$ ,  $T = T_C$  and  $T < T_C$ , where  $T$  is the temperature and  $T_C$  the critical temperature. (2P)



13. Give the chemical potential of an ideal gas as a function of the pressure  $p$ , the temperature  $T$  and the thermal wavelength  $\lambda_T$ . (1P)

$$\beta p = \rho = \frac{e^{\beta \mu}}{\lambda_T^3} \rightarrow \mu = \beta^{-1} \ln(\lambda_T^3 \beta p)$$