Name	
Student number	

1 (20)	2 (20)	3 (20)	4 (20)	5 (20)	Σ (100)	mark

Statistical Physics and Thermodynamics (SS 2016)
Practice exam 14 July 2016

Total available time: 90 minutes

1 Open questions

a) In a phase diagram, what is meant by "triple point"?

b) In equilibrium, which quantities are extremized in the (i) microcanonical, (ii) canonical and (iii) grand canonical ensemble?

c) How does the heat capacity of a classical monoatomic gas depend on the temperature?

d) Write down the equation of state of an ideal gas.

e) Which of the following quantities are extensive: volume, temperature, particle number, pressure, chemical potential, entropy, density and mass?

f) How do the vapor pressure, boiling temperature and melting temperature of a mixture compare to the same quantities of a pure liquid?

g) How does the entropy change in a reversible process?

h) Sketch the Maxwell-Boltzmann distribution $P(|v|)$ as a function of the velocity v for an ideal gas. Which value is higher: the average velocity or the most probable velocity?

2 Ideal monoatomic gas

Consider an ideal monoatomic gas with a mas of $m = 28$ g/mol and a heat capacity of $C_V = 4$ J/(mol K) (a mol consists of $N_A = 6 \cdot 10^{23}$ particles). An amount of this gas with a total mass of 1 kg is heated from 210 K to 420 K at constant pressure using a heat pump. For this exercise you can use the gas constant $R = N_A k_B = 8$ J/(mol K).

a) How much heat is required by the process?

b) How much does the internal energy U increase?

c) How much external work W is being been done?

d) What is the efficiency η of the heat pump?

3 Liquid-gas phase transition

Consider a substance with the enthalpy of the liquid phase given by

$$H_{liq}(p,S) = 2(apSN)^{1/2}, \tag{1}$$

and the enthalpy of the gas phase by

$$H_{gas} = 3(pS/2)^{2/3}(bN)^{1/3}, \tag{2}$$

with a and b being positive constants, S is the entropy, p is the pressure, and N is the total number of particles.

a) Calculate the liquid-gas coexistence temperature T_g as a function of pressure.

b) Calculate the densities of the liquid and the gas phase at the phase transition line.

c) Calculate the entropy change per volume $\Delta S/\Delta V$ at the phase transtion line.

4 Particle adsorption

An ideal gas at temperature T and chemical potential μ is in contact with a surface with N adsorption sites. Each adsorption site may be occupied by 0, 1 or 2 gas particles. The energy of a vacant site is zero, the energy of a site with one adsorbed particle is ϵ and the energy with two adsorbed molecules is $(3/2)\epsilon$. The energy ϵ can be positive or negative. There are no additional interactions between adsorbed particles.

a) Calculate the grand canonical partition function for a fixed number of adsorption sites.

b) Use the grand canonical partition function to derive the mean number of adsorbed particles per site $\langle n \rangle$ and the mean internal energy per site $\langle u \rangle$ as a function of T, μ and ϵ.

c) Sketch $\langle n \rangle$ for $T = 0$ and constant μ as a function of ϵ.

d) Calculate $\langle n \rangle$ for large temperatures.

5 Lenoir cycle

Consider 1 mol of an ideal gas, which initially has a volume V_1, temperature T_1 and pressure p_1. The gas undergoes the following cyclic process:
$1 \to 2$: isochoric (constant volume) heating to T_2
$2 \to 3$: isentropic expansion to V_3
$3 \to 1$: isobaric cooling

a) Sketch the P–V and the T–S diagrams for this process.

b) For each leg, calculate the performed work W and the transferred head Q in terms of p_1, V_1 and V_3.

c) Calculate the efficiency η in terms of $\alpha = V_3/V_1$.