Freie Universität Berlin
Fachbereich Physik
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Statistical Physics and Thermodynamics (SS 2018)

Problem sheet 9

Hand in: Friday, June 29th during the lecture

http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre/

1 Expressions for TdS for different independent variables (8 points)

The differential first law of thermodynamics for a system with a constant number of particles, $TdS = dU + PdV$, can be expressed as a function of either dV and dP, dP and dT or dT and dV. In the lecture, the relation $TdS = C_V dT + T(\alpha/\kappa_T)dV$ has been derived.

a) Using that U can be written as a function of P and V, derive the expression for TdS in terms of dP and dV and the corresponding partial derivatives. **(1 point)**

b) Derive that $(\partial U/\partial V)_P + P = C_P/(\alpha V)$ with $C_P = (\partial(U + PV)/\partial T)_P$ and $\alpha = (1/V)(\partial V/\partial T)_P$. **(1 point)**

c) Derive that $(\partial U/\partial P)_V = C_V \kappa_T/\alpha$ with $C_V = (\partial U/\partial T)_V$ and $\kappa_T = -(1/V)(\partial V/\partial P)_T$. **(1 point)**

d) Using your results from b) and c), rewrite the expression for TdS of part a) in terms of C_V, C_P, α, κ_T and V. **(1 point)**

e) Using that V can be written as a function of P and T, and therefore U can also be written as a function of P and T, derive the expression for TdS in terms of dP and dT and the corresponding partial derivatives. **(1 point)**

f) Show that $(\partial U/\partial T)_P + P(\partial V/\partial T)_P = C_P$. **(1 point)**

g) Derive that $(\partial U/\partial P)_T + P(\partial V/\partial P)_T = -\alpha TV$. **(1 point)**

h) Using your results from f) and g), rewrite the expression for TdS of part e) in terms of α, T, V and C_P. **(1 point)**

2 The relation between C_P and C_V (5 points)

a) Using your results from problem 1, show that

$$\left(\frac{C_P - C_V}{\alpha V} - \frac{\alpha T}{\kappa_T}\right) dV + \left((C_P - C_V)\,\frac{\kappa_T}{\alpha} - \alpha TV\right) dP = 0.$$

(1 point)

b) From a), using that P and V are independent variables, derive an expression for $C_P - C_V$. Explain your reasoning. **(3 points)**

c) What is the sign of $C_P - C_V$? Explain the reason in terms of mechanical stability. **(1 point)**

3 Thermodynamic efficiency of a jet engine (7 points)

A jet engine, pictured in Fig. 1, is operated according to the following cycle:

1. A-B: Adiabatic, quasi-static compression in the inlet and compressor
2. B-C: Constant-pressure expansion by fuel combustion
3. C-D: Adiabatic, quasi-static expansion in the turbine and exhaust nozzle
4. D-A: Constant-pressure cool down back to the initial condition.

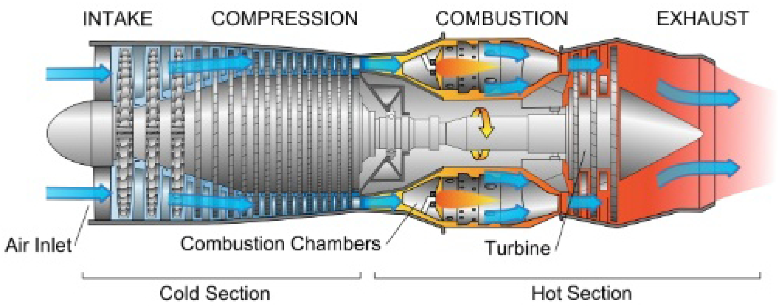

Figure 1: Cross section of a jet engine

a) Draw the thermodynamic cycle in a PV-diagram. Indicate the parts of the cycle where heat is removed from and added to the system by ΔQ_1 and ΔQ_2, respectively. **(2 points)**

b) Use the variation of the internal energy U to derive the work done by the system in terms of the heat ΔQ_1 and ΔQ_2. **(1 point)**

c) Calculate ΔQ_1 and ΔQ_2 in terms of the heat capacity C_P (assumed constant as a function of the temperature) and the temperatures $T_{A...D}$. **(1 point)**

d) Using that $P^{1-\gamma}T^{\gamma}$ is constant for adiabatic processes, write down the relation between the temperatures T_A, T_B, T_C and T_D. **(1 point)**

e) Calculate the efficiency η in terms of the temperatures $T_{A...D}$, and rewrite it using your result of part d) as a function of the temperatures at the entrance of the combustion chamber T_B and the atmospheric temperature T_A only. **(2 points)**