

## Statistical Physics and Thermodynamics (SS 2018)

### Problem sheet 11

Hand in: Friday, July 13th during the lecture

<http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre/>

#### 1 Joule-Thomson process (10 points)

Consider a pipe with thermally insulated walls. A porous plug divides the pipe into two parts. The porous plug in the pipe provides a constriction to the flow of the gas (alternatively, a valve which is only slightly opened may provide such a constriction). We consider a situation where gas flows from left to right. At the left side the gas has a pressure  $P_1$  and a volume  $V_1$ . After the constriction the gas expands to a volume  $V_2$  greater than the volume  $V_1$ . The presence of the constriction results in a constant pressure difference being maintained across this constriction. Thus the gas pressure  $P_1$  to the left of the constriction is greater than the gas pressure  $P_2$  to the right of the constriction. Let  $T_1$  denote the temperature of the gas on the left-hand side of the constriction. In this exercise, we derive what the gas temperature  $T_2$  on the right-hand side will be.

- Write down the change of the internal energy,  $\Delta U$ , and the net work  $W$  done by the floating gas. **(1 point)**
- Use the fact that no heat is exchanged during the process and your result from a) to show that  $H_2 = H_1$ , where  $H = U + PV$  is the enthalpy. **(1 point)**
- Use the first law of thermodynamics to show that  $dH(S, P) = TdS + VdP$  **(1 point)**
- In our case, where  $H$  is constant,  $dH(S, P) = 0$ . Use this to show that

$$C_P dT + V(1 - T\alpha) dP = 0, \quad (1)$$

where  $C_P$  is the heat capacity at constant pressure and  $\alpha$  is the coefficient of expansion. **(4 points)**

*Hint: To obtain Eq. (1) calculate the total differential  $dS(T, P)$  and find a suitable Maxwell relation for  $(\partial S/\partial p)_T$ .*

- Use your previous result to calculate the so called Joule-Thomson coefficient  $\mu_{JT} := (\partial T/\partial P)_H$ . What does this coefficient describe physically? **(1 point)**
- Calculate  $\mu_{JT}$  for the ideal gas. Argue what the temperature  $T_2$  at the right-hand side of the constriction will be. **(1 point)**
- Calculate  $\mu_{JT}$  for the van der Waals gas. What does this result mean for the temperature  $T_2$  at the right-hand side of the constriction? **(1 point)**

## 2 Two interacting dipoles (10 points)

Two identical ideal dipoles are located in a two-dimensional plane with a distance  $r$  between them. The dipoles are positioned at angles  $\theta_1$  and  $\theta_2$  relative to the connecting line. The interaction energy between the dipoles is given by

$$V(r, \theta_1, \theta_2) = -\frac{d^2}{4\pi\epsilon r^3} \left[ \frac{3}{2} \cos(\theta_1 + \theta_2) + \frac{1}{2} \cos(\theta_1 - \theta_2) \right],$$

with  $d$  being the dipole moment and  $\epsilon$  being the dielectric constant of the medium between the dipoles.

- a) Calculate the average interaction potential between two dipoles,  $\bar{V}(r)$ , from a Boltzmann-weighted integral over all possible orientations

$$\bar{V}(r) = -\ln \int_0^{2\pi} \frac{d\theta_1}{2\pi} \int_0^{2\pi} \frac{d\theta_2}{2\pi} \exp \left[ -\frac{V(r, \theta_1, \theta_2)}{k_B T} \right]. \quad (4 \text{ points})$$

*Hint: The integral*

$$\int_0^{2\pi} d\theta \exp [A \cos \theta] = 2\pi I_0(A),$$

*can be written as*

$$I_0(A) = \sum_{k=0}^{\infty} \frac{\frac{1}{4} (A^2)^k}{(k!)^2},$$

*where  $I_0(A)$  is the modified Bessel function of the first kind.*

- b) Expand  $\bar{V}(r)$  to leading order for large distances  $r$ , which means  $r^3 > d^2/(4\pi\epsilon k_B T)$ . How does the average interaction potential depend on the distance  $r$ ? **(3 points)**
- c) For large arguments  $A$ , the modified Bessel function scales like

$$I_0(A) \sim \frac{\exp A}{\sqrt{2\pi A}}.$$

Approximate  $\bar{V}(r)$  to leading order for small distances  $r$ . How does the average interaction potential depend on the distance  $r$ ? **(3 points)**