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Advanced Statistical Physics II – Problem Sheet 11

Problem 1 – **Mean first passage time**

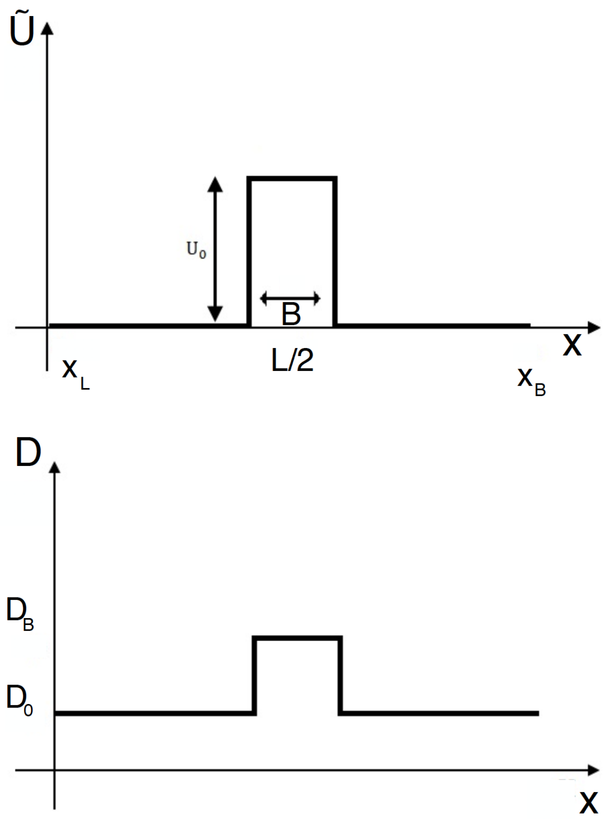

Figure 1: Potential and Diffusion shape.

The mean first passage time is the average time a particle needs to cross the barrier and reach x_B for the first time when starting at position $x_0 < x_B$. The formula for the mean first passage time for the case of a position-dependent diffusivity $D(x)$ reads

$$\tau^{MFP} = \int_{x_0}^{x_B} \mathrm{d}x' \frac{\mathrm{e}^{\tilde{U}(x')}}{D(x')} \int_{x_L}^{x'} \mathrm{d}x'' \mathrm{e}^{-\tilde{U}(x'')}, \tag{1}$$

where $x_L < x_0$ is the position of the reflective barrier.

(a) (6P) Consider the potential $\tilde{U}(x) := U(x)/k_BT$ and the diffusivity $D(x)$ as in fig.1, for $x_L = 0$ and $x_B = L$. Calculate the mean first passage time for $x_0 = 0$ as a function of all parameters.

(b) (2P) Consider τ^{MFP} for the case of no barrier ($\tilde{U}_0 = 0$) and constant diffusivity, $D_B = D_0$. Compare with the result expected from the free diffusion equation.

(c) (2P) Write down τ^{MFP} for the case of no barrier and arbitrary D_B and D_0. Discuss the limit $D_B \to \infty$.

(d) (2P) Discuss the limits $\tilde{U}_0 \to -\infty$ and $\tilde{U}_0 \to +\infty$ for the case of constant diffusivity, $D_B = D_0$.

(e) (2P) Discuss the limits $\tilde{U}_0 \to -\infty$ and $\tilde{U}_0 \to +\infty$ for arbitrary D_B and D_0.

Problem 2 – **Decomposition of mean first passage times**

(a) (2P) For the case $x_0 = x_L$, decompose the general expression (1) of the mean first passage time into three parts

$$\tau^{\text{MFP}} = \tau_I + \tau_{II} + \tau_R. \tag{2}$$

τ_I is the MFPT of a particle starting at $x_0 = x_L$ to reach x_1. τ_{II} is the MFPT of a particle starting at x_1 to reach x_B without crossing x_1 again. How can τ_R be interpreted?

(b) (4P) Consider the single barrier in fig. 1. Find τ_I, τ_{II} and τ_R for $x_1 = L/2$.