Freie Universität Berlin
Fachbereich Physik
Prof. Dr. Roland Netz
http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre

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Advanced Statistical Physics II – Problem Sheet 0

Problem 1 – Gaussian integrals

a) Calculate

$$I \equiv \int_{-\infty}^{\infty} dx\, e^{-ax^2}, \quad a > 0 \tag{1}$$

by expressing I^2 in polar coordinates.

b) Using the previous result, calculate

$$\int_{-\infty}^{\infty} dx\, e^{-ax^2+bx}. \tag{2}$$

c) Now we want to consider the case of n-dimensional Gaussian integrals. Calculate

$$\int d^n x\, e^{-\frac{1}{2}\vec{x}^T G \vec{x}} \tag{3}$$

where G is a positive definite, symmetric n by n matrix. *Hint:* Remember the spectral theorem.

d) Show that

$$\int d^n x\, e^{-\frac{1}{2}\vec{x}^T G \vec{x} - \vec{b}\cdot\vec{x}} = \frac{(2\pi)^{n/2}}{\sqrt{|G|}} e^{\frac{1}{2}\vec{b}^T G^{-1} \vec{b}}. \tag{4}$$

e) Solve the double integral by changing the order of integration

$$\int_0^1 dx \int_x^1 dy\; e^{y^2}. \tag{5}$$

Problem 2 – Fourier Transform

Consider the definition of the Fourier and inverse Fourier transform

$$\tilde{F}(\omega) = \int_{-\infty}^{\infty} F(t)e^{i\omega t}dt, \tag{6}$$

$$F(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{F}(\omega)e^{-i\omega t}d\omega, \tag{7}$$

and the differential equation of a driven harmonic oscillator whose amplitude $x(t)$ satisfies the following equation

$$\left[m\frac{d^2}{dt^2} + \gamma\frac{d}{dt} + \omega_0^2\right]x(t) = f(t) \tag{8}$$

where $f(t)$ is a uniform oscillating force of magnitude f_0, frequency ω_f and phase constant ϕ_f

$$f(t) = f_0 cos(\omega_f t + \phi_f) \tag{9}$$

a) Using Fourier transforms, find the solution for $\tilde{x}(\omega)$

   Hint: $\delta(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dt\ e^{i\omega t}$

b) Find the solution for $x(t)$ by taking the inverse Fourier transform of $\tilde{x}(\omega)$.

c) Calculate the Fourier transform of the following two functions

$$e^{-t/\tau}\Theta(t) \tag{10}$$

$$e^{-|t|/\tau} \tag{11}$$

where $\tau > 0$ and $\Theta(t)$ is the Heaviside step function.