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Advanced Statistical Physics II – Problem Sheet 1

Problem 1 – Hamiltonian Mechanics

a) (3P) Consider a bead sliding on a wire rotating at constant angular velocity ω. The wire is straight and is rotating about an axis perpendicular to itself. Furthermore, a harmonic spring with spring constant k connects the bead and the point where the wire and its axis of rotation intersect. Write down the Hamiltonian of the system in terms of its radial position and the corresponding momentum. (*Hint: Note that the rotation induces an effective potential*)

b) (5P) Discuss the solution of the resulting equations of motion and sketch the evolution in phase space with initial conditions $r(t = 0) = r_0$, $\dot{r}(t = 0) = 0$ for the cases

- $\omega^2 < k/m$
- $\omega^2 = k/m$
- $\omega^2 > k/m$

where $r(t)$ denotes the distance between the bead and the axis of rotation.

Problem 2 – Bayes' Theorem (Monty Hall Problem)

a) (3P) Imagine you are a participant in a game show where you have to choose one out of three doors. The main prize is behind one of the doors while behind the remaining two, there is just a balloon. You pick one door at random. Let's call it "A". As part of the game, the host reveals, that behind door "C" there is just a baloon. He offers you to reconsider your choice. Should you switch to door "B"? Fortunately, you have attended the "Advanced Statistical Physics II" lecture in which Bayes' Theorem was covered. Use it to calculate the probability of getting the prize for sticking with your initial choice or switching to door "B".

b) (2P) Consider the previous scenario with the difference that now the host *accidentally* opens door "C" . (He slips on a banana peel and accidentally pushes one of the doors open which *happens to be* door "C".)

Problem 3 – Liouville Equation and Correlation Functions

a) (3P) In the lecture, we showed that the Green's function of the Liouville equation can be written as

$$\rho(\tilde{\mathbf{q}}, \tilde{\mathbf{p}}, t|\tilde{\mathbf{q}}^{(0)}, \tilde{\mathbf{p}}^{(0)}) = \delta^{(3N)}(\tilde{\mathbf{p}} - \mathbf{p}(t))\, \delta^{(3N)}\,(\tilde{\mathbf{p}} - \mathbf{p}(t)) \tag{1}$$

where $\mathbf{q}(t)$ and $\mathbf{p}(t)$ solve Hamilton's equation of motion for initial conditions $\mathbf{q}(0) = \tilde{\mathbf{q}}^{(0)}$ and $\mathbf{p}(0) = \tilde{\mathbf{p}}^{(0)}$.

Show that for all times t, a solution $\rho(\tilde{\mathbf{q}}, \tilde{\mathbf{p}}, t)$ of the Liouville equation for an initial probability density distribution $\rho_0(\tilde{\mathbf{q}}, \tilde{\mathbf{p}})$ is non-negative and conserves probability, i.e $\int d\tilde{\mathbf{q}} d\tilde{\mathbf{p}} \rho(\tilde{\mathbf{q}}, \tilde{\mathbf{p}}, t) = 1$.

b) (2P) Assuming stationarity, show that

$$\frac{d^{2k}}{dt^{2k}} C_{AA}(t) = (-1)^k \langle A^{(k)}(0) A^{(k)}(t)\rangle \tag{2}$$

where $A^{(k)}(t) = d^k A(t)/dt^k$ and $C_{AA}(t) = \langle A(0)A(t)\rangle$.

c) (2P) In the lecture, we derived the formula for the time correlation function of two observables $A(q,p)$ and $B(q,p)$ for a stationary system:

$$C_{AB}(\tau) = \int \mathrm{d}q \mathrm{d}p \mathrm{d}q' \mathrm{d}p' A(q,p) B(q',p') \mathrm{e}^{-\tau L(q,p)} \delta(q-q')\delta(p-p')\rho_0(q',p') \tag{3}$$

Here, L is the Liouville operator, $\rho_0(q',p') \equiv \rho_0(\mathcal{H}(q',p'))$ is a stationary density distribution and we assume that $\mathcal{H}(q,-p) = \mathcal{H}(q,p)$. Recall from the lecture that the substitution $p \to -p, \quad p' \to -p'$ corresponds to time reversal. Find a relation between $C_{AB}(\tau)$ and $C_{AB}(-\tau)$ for the following observables:

i) $A = q$ and $B = p^2$
ii) $A = p$ and $B = H(q,p)$
iii) $A = p$ and $B = qp^3$