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Advanced Statistical Physics II – Problem Sheet 6

Problem 1 – Non Markovian Effects and Coloured Noise
In the lecture, we considered the following set of coupled differential equations

$$m\frac{dv(t)}{dt} = -\gamma v(t) - kx(t) + F_R(t) \tag{1}$$
$$\frac{dx(t)}{dt} = v(t)$$

where $F_R(t)$ is white noise defined by $\langle F_R(t)\rangle = 0$, $\langle F_R(t)F_R(t')\rangle = 2\gamma k_B T\delta(t-t')$.
Eliminating one coupled degree of freedom gives rise to non-Markovian dynamics

$$\frac{dx(t)}{dt} = -\int_0^\infty \Gamma_x(s)\left[x(t-s) - \frac{F_R(t-s)}{k}\right] \tag{2}$$

with the new random force

$$F_x(t) = \int_0^\infty ds\Gamma_x(s)\frac{F_R(t-s)}{k}. \tag{3}$$

Here, $\Gamma_x(t) = \frac{k}{m}e^{-\gamma t/m}$ is the so-called memory kernel.

a) (1P) Calculate $\langle F_x(t)\rangle$.

b) (3P) Derive the relation between the memory kernel $\Gamma_x(t)$ and $\langle F_x(t)F_x(t')\rangle$.

c) (3P) Calculate the Fourier transform of $\langle F_x(t)F_x(t')\rangle$. Why do we call $\langle F_x(t)F_x(t')\rangle$ coloured noise? What happens in the limit $m \to 0$?

d) (2P) Calculate the real and imaginary part of the position response function $\tilde{\chi}_x(\omega) = \frac{\tilde{x}(\omega)}{\tilde{F}_R(\omega)}$ from equation (2).

e) (3P) Show that

$$\langle x^2(t)\rangle = 2\gamma k_B T\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\tilde{\chi}_x''(\omega)}{\gamma\omega}. \tag{4}$$

f) (2P) Calculate $\langle x^2(t)\rangle$.

Hint: Remember the Kramers-Kronig relation $\tilde{\chi}'(\omega) = \mathcal{P}\int_{-\infty}^{\infty}\frac{d\omega'}{\pi}\frac{\tilde{\chi}''(\omega')}{\omega'-\omega}$.

g) (1P) Now calculate the real and imaginary part of the velocity response function χ_v, defined via $\tilde{v}(\omega) = \tilde{\chi}_v(\omega)\tilde{F}_R(\omega)$.

h) (3P) Show that

$$\langle v^2(t)\rangle = 2\gamma k_B T\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\left[\frac{\tilde{\chi}_v''(\omega)}{m\omega} + \frac{k}{m}\frac{\tilde{\chi}_x''(\omega)}{\gamma\omega}\right]. \tag{5}$$

i) (2P) Calculate $\langle v^2(t)\rangle$.