Blatt 3

1. a) $\cosh^2(x) = \frac{1}{4}(e^{2x} + 2 + e^{-2x})$

$\sinh^2(x) = \frac{1}{4}(e^{2x} - 2 + e^{-2x})$

$\Rightarrow \cosh^2(x) - \sinh^2(x) = \frac{1}{4}\left(e^{2x} - e^{2x} + e^{-2x} - e^{-2x} + 2 - (-2)\right)$

$= 1$

b) $\sinh(x+y) = \frac{1}{2}(e^{x+y} - e^{-x-y})$

Durch Bilden von $\sinh(x)\cosh(y)$ und $\sinh(y)\cosh(x)$ und Addieren, ergibt sich die gesuchte Form.

$\sinh(x)\cosh(y) = \frac{1}{4}(e^{x+y} - e^{-x-y} + e^{x-y} - e^{-x+y})$

$\sinh(y)\cosh(x) = \frac{1}{4}(e^{x+y} - e^{-x-y} - e^{x-y} + e^{x-y})$

$\Rightarrow \sinh(x)\cosh(y) + \sinh(y)\cosh(x) = \frac{1}{2}(e^{x+y} - e^{-x-y})$

$= \sinh(x+y)$

Analogie zu $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$.

c) $\cosh(x+y) = \frac{1}{2}(e^{x+y} + e^{-x-y})$

Bilden von $\cosh(x)\cosh(y)$ sowie $\sinh(x)\sinh(y)$ führt zum Ziel:

$\cosh(x)\cosh(y) = \frac{1}{4}(e^{x+y} + e^{-x-y} + e^{x-y} + e^{-x+y})$

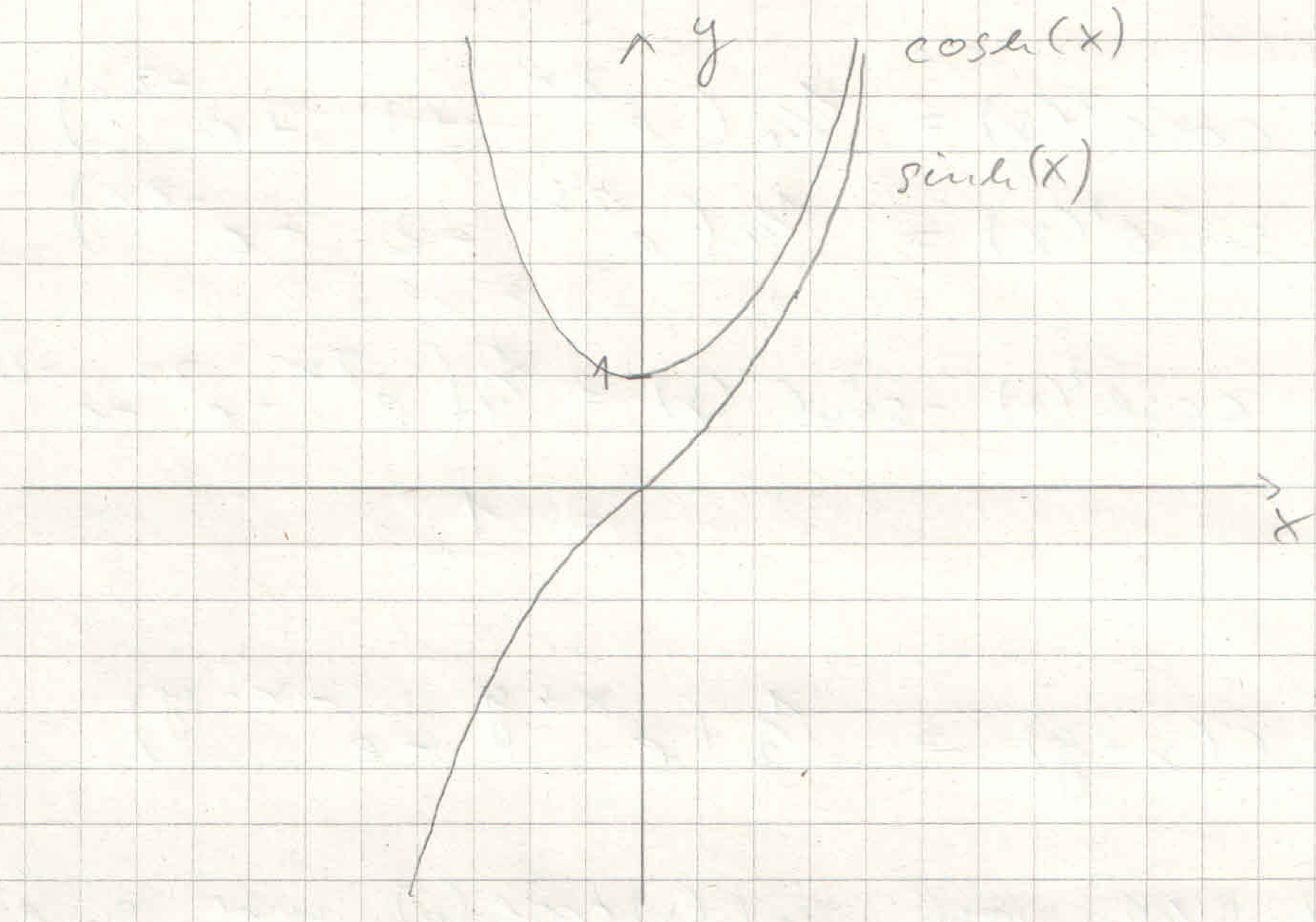
$\sinh(x)\sinh(y) = \frac{1}{4}(e^{x+y} + e^{-x-y} - e^{x-y} - e^{-x+y})$

$\Rightarrow \cosh(x)\cosh(y) + \sinh(x)\sinh(y) = \frac{1}{2}(e^{x+y} + e^{-x-y})$

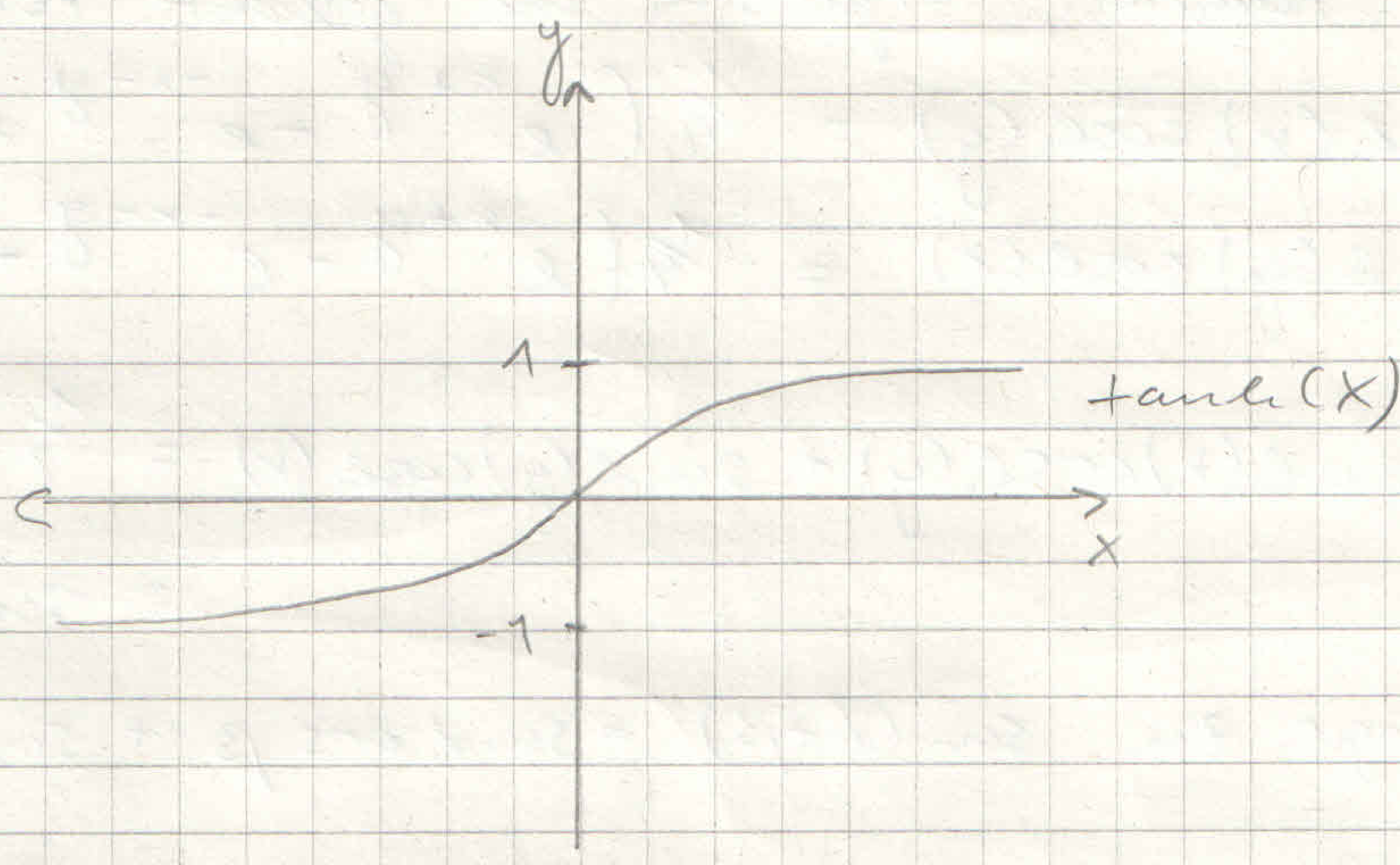
$= \cosh(x+y)$

Unterschied zu $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

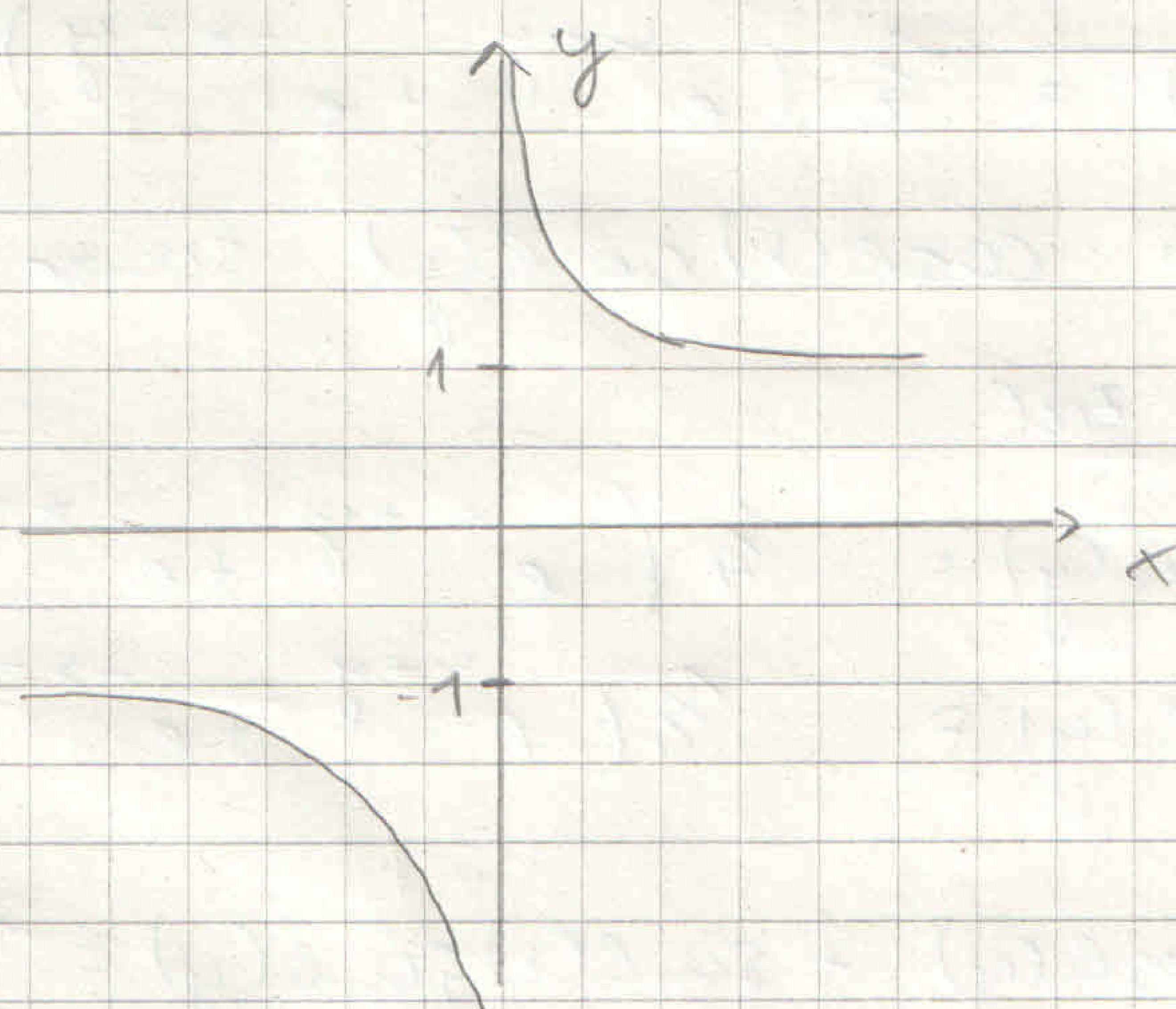
2. $\cosh(x)$, $\sinh(x)$



$\tanh(x)$



$\coth(x)$



3. $\log_a x = y$

$$\Leftrightarrow \quad a^y = x$$

$$\ln(a^y) = \ln(x)$$

$$y \cdot \ln(a) = \ln(x)$$

$$\underline{\underline{y = \frac{\ln(x)}{\ln(a)} = \log_a x}}$$

4. a) $f'(x) = 12x^3 - 12x^2 + 2x - 2$

b) $f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

c) $f'(x) = \sin x + x \cos x$

d) $f'(x) = \ln x + x \cdot \frac{1}{x} - 1 = \ln(x)$

e) $f'(x) = \frac{x+2-x}{(x+2)^2} = \frac{2}{(x+2)^2}$

f) $f'(x) = -\frac{\cos(x)}{\sin^2(x)}$

5. a) $f'(x) = \cosh(x)$

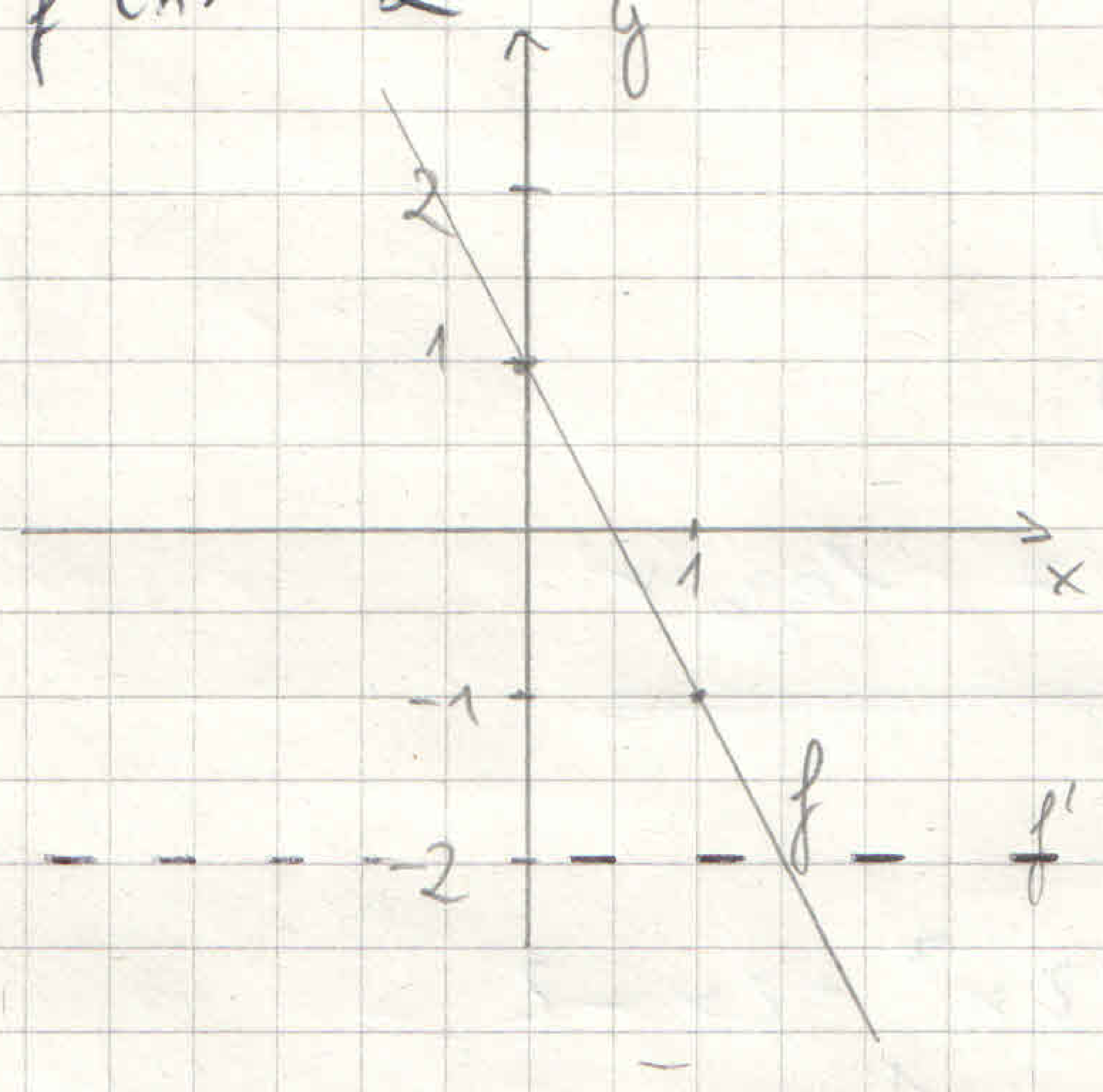
b) $f'(x) = \sinh(x)$

c) $f'(x) = \frac{\cosh^2(x) - \sinh^2(x)}{\cosh^2(x)} = 1 - \tanh^2(x) = \frac{1}{\cosh^2(x)}$

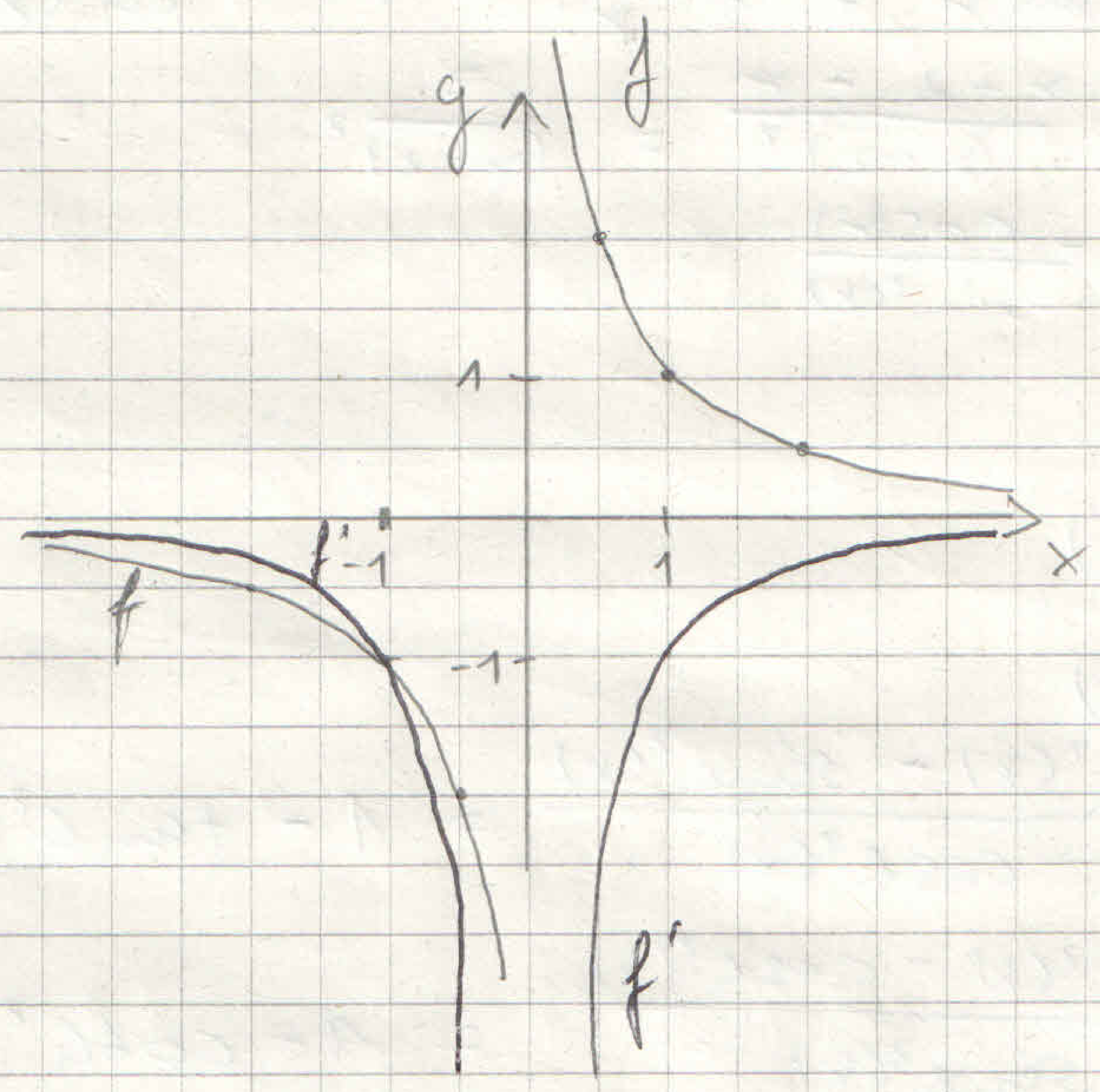
d) $f'(x) = \frac{\sinh^2(x) - \cosh^2(x)}{\sinh^2(x)} = 1 - \coth^2(x) = \frac{-1}{\sinh^2(x)}$

6. a) $f(x) = 1-2x$

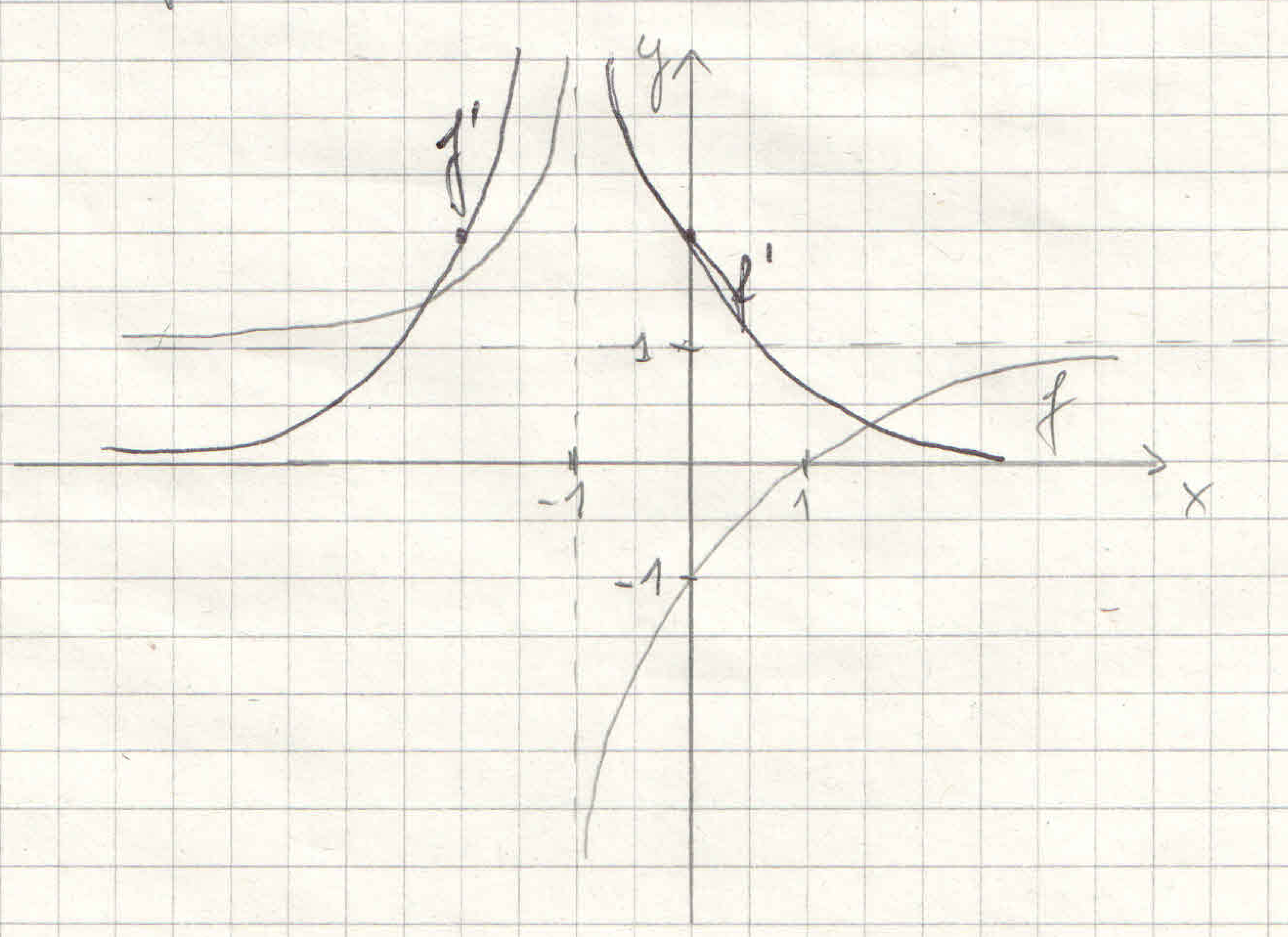
$f'(x) = -2$



b) $f(x) = \frac{1}{x}$, $f'(x) = \frac{-1}{x^2}$



c) $f(x) = \frac{x-1}{x+1}$, $f'(x) = \frac{2}{(x+1)^2}$



7. a) $f(x) = \frac{1}{x^2+a^2}$; $f'(x) = \frac{-2x}{(x^2+a^2)^2}$; $f''(x) = \frac{6x^2-2a^2}{(x^2+a^2)^3}$

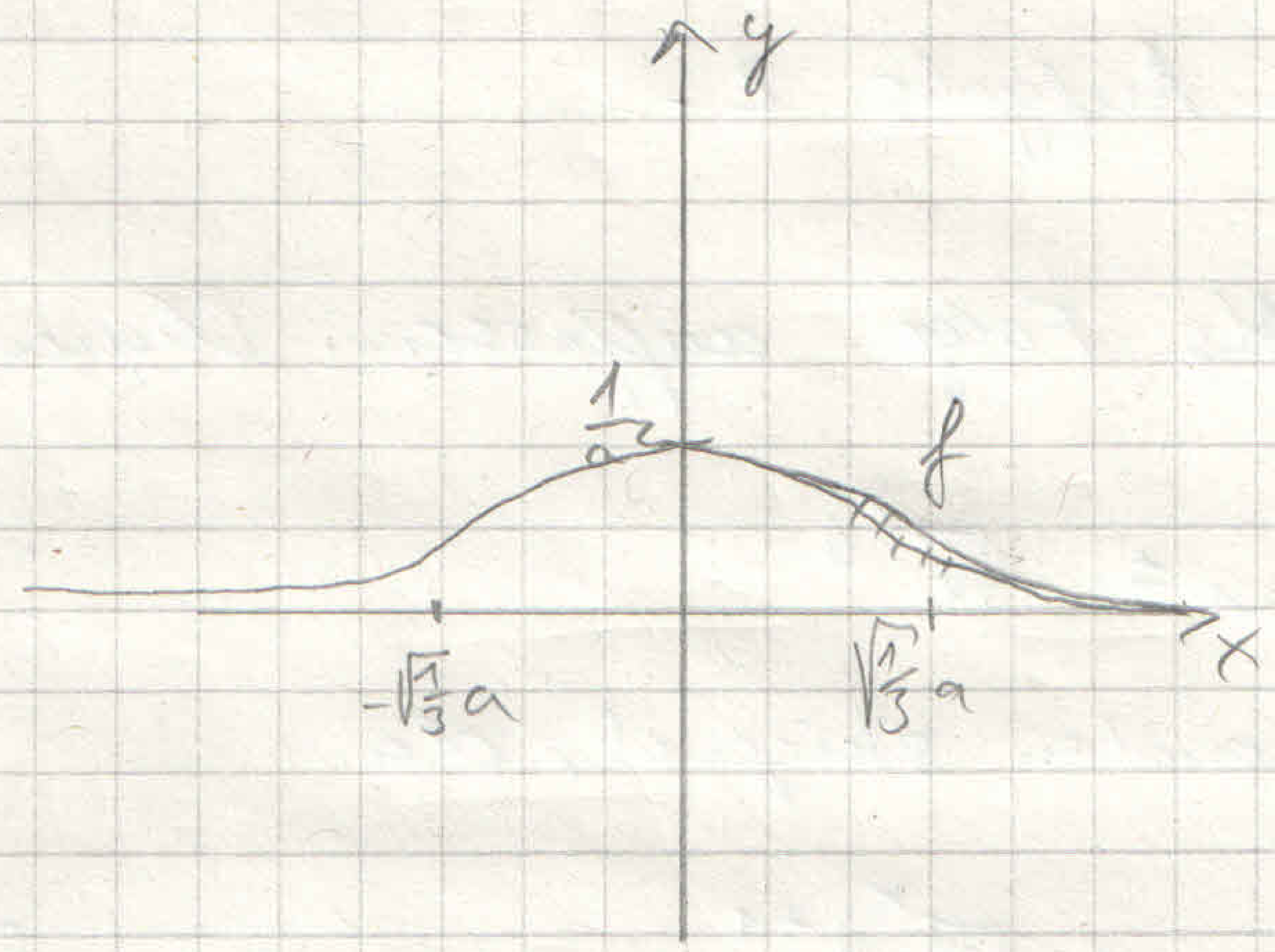
Wendepunkte:

$f''(x) = 0$

$0 = 6x^2 - 2a^2$

$\Leftrightarrow \quad x^2 = \frac{1}{3}a^2$

$x_{1,2} = \pm\sqrt{\frac{1}{3}}\,a$



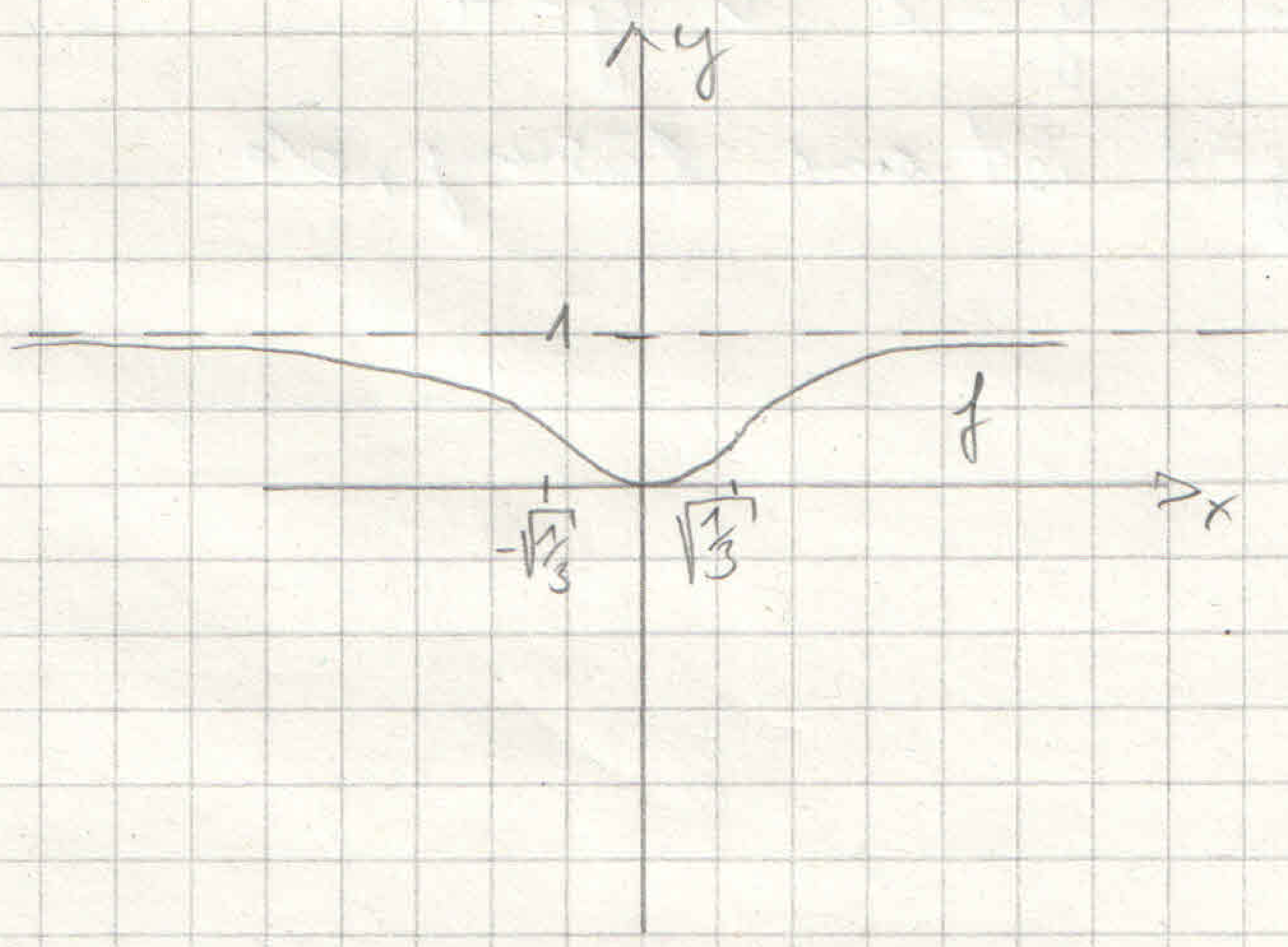
b) $f(x) = \frac{x^2}{x^2+1}$; $f'(x) = \frac{2x}{(x^2+1)^2}$; $f''(x) = \frac{2-6x^2}{(x^2+1)^3}$

$f''(x) = 0$

$0 = 2 - 6x^2$

$x^2 = \frac{1}{3}$

$x_{1,2} = \pm\sqrt{\frac{1}{3}}$



7. c) $f(x) = \tanh(x)$; $f'(x) = 1 - \tanh^2(x)$

$f''(x) = -2\tanh(x)\,(1 - \tanh^2(x)$

Wendepunkte:

$f''(x) = 0$

$0 = -2\tanh(x) \quad \vee \quad 0 = 1 - \tanh^2(x)$

$\Rightarrow \quad x = 0 \qquad\qquad \tanh(x) \neq 1$

Skizze: siehe Aufgabe 2.

8. Ansatz: Als Folge auffassen. Elemente x_j mit

$$x_{j+1} = \sqrt{2}^{x_j}$$

Streng monoton steigend, da

$$x_{j+2} = \sqrt{2}^{x_{j+1}} > x_{j+1} = \sqrt{2}^{x_j}$$

Folge konvergiert gegen den Wert 2

$$\Rightarrow \quad y = \sqrt{2}^{y}$$

$$\log y = y \log \sqrt{2}$$

Lässt sich graphisch lösen.

Lösungen: $y = 2 \vee y = 4$

Nur $y = 2$ ist eine Lösung, da