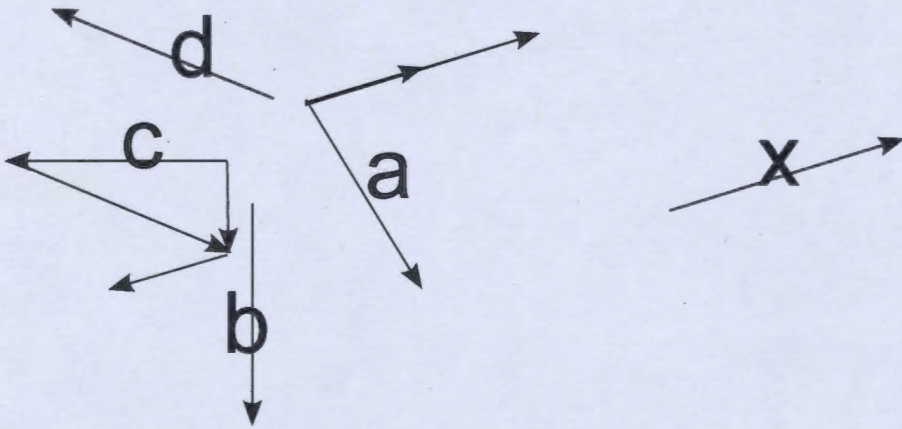
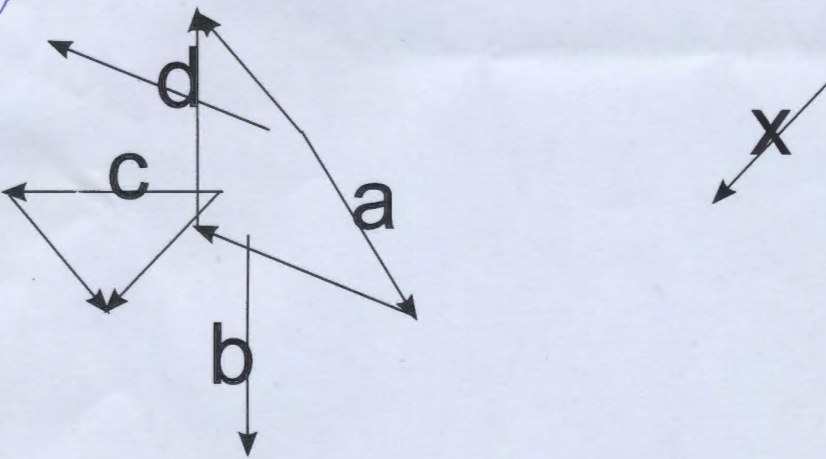


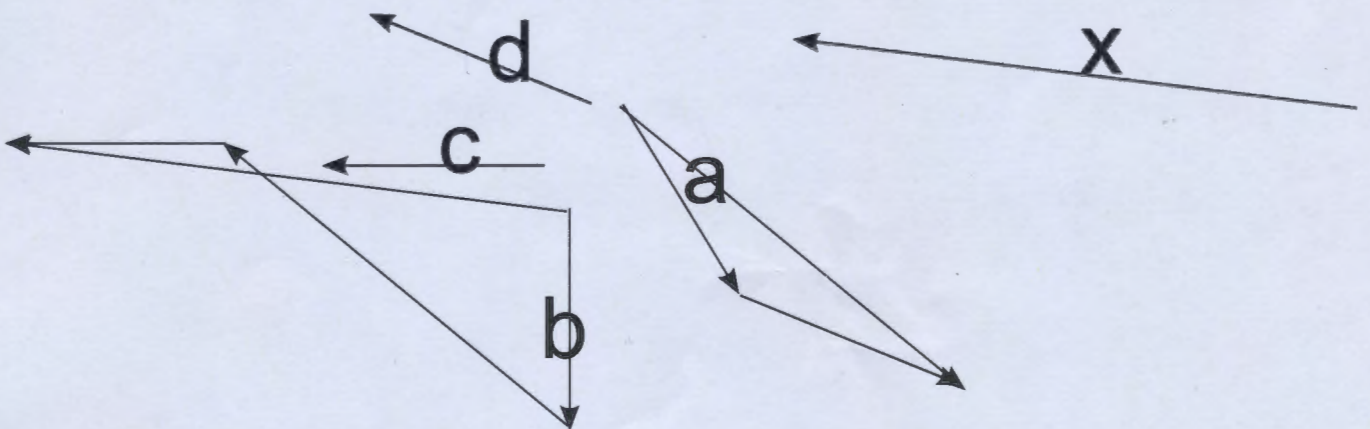
1a)



b)



c)



$$2a) (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$

$$0 = (\vec{a} + \vec{b}) \cdot \vec{a} - (\vec{a} + \vec{b}) \cdot \vec{b} = |\vec{a}|^2 + \vec{b} \cdot \vec{a} - \vec{a} \cdot \vec{b} - |\vec{b}|^2$$
$$\Rightarrow |\vec{a}|^2 = |\vec{b}|^2 \Rightarrow |\vec{a}| = |\vec{b}|$$

Die Länge der Vektoren muss gleich (ug) sein.

$$b) |(\vec{a} \times \vec{b})| \cdot |(\vec{a} \times \vec{b})|$$

$$= (\vec{a} \cdot \vec{a}) \cdot (\vec{b} \cdot \vec{b}) - (\vec{b} \cdot \vec{a}) \cdot (\vec{a} \cdot \vec{b})$$

~~≠~~

≠

$$\Rightarrow |(\vec{a} \times \vec{b})|^2 = \sqrt{(\vec{a} \cdot \vec{a})(\vec{b} \cdot \vec{b}) - (\vec{a} \cdot \vec{b})^2}$$

3a)

$$\vec{a} + \vec{b} = \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix} \quad \vec{a} - \vec{c} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

$$\vec{a} \cdot \vec{b} = 2 + 2 + 9 = 13$$

$$\vec{b} \cdot \vec{c} = 4 + 0 + 3 = 7$$

$$3b) \quad \vec{a} \cdot \vec{c} = 2 + 3 = 5 \quad |\vec{a}| = \sqrt{14} \quad |\vec{b}| = \sqrt{14} \quad |\vec{c}| = \sqrt{5}$$

$$\Delta(a, b) = \arccos \left( \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \right)$$

$$\Delta(a, b) = \arccos \left( \frac{13}{14} \right) = 0,38$$

$$\Delta(b, c) = \arccos \left( \frac{7}{\sqrt{14} \sqrt{5}} \right) = 0,58$$

$$\Delta(a, c) = \arccos \left( \frac{5}{\sqrt{14} \sqrt{5}} \right) = 0,93$$



4)

Vector senkrecht geg. durch

$$\vec{a} \times \vec{b} = \begin{pmatrix} 1 \cdot (-1) - 1 \cdot (0) \\ 1 \cdot 1 - 1 \cdot (-1) \\ 1 \cdot 0 - 1 \cdot (1) \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

5) a)  $3\vec{a}$

b)  $-\vec{a}$

c)  $2\vec{a} + \vec{b}$

d)  $-\vec{a} + \vec{b}$

e)  $\vec{y} = \vec{c} - \vec{a} - \vec{d} + \vec{b} = -\vec{a} + \vec{b} + \vec{c} - \vec{d}$

g)  $\lambda(\vec{a} + 2\vec{b})$

h)  $-\vec{a} - 2\vec{b} = -2(\vec{a} + \vec{b})$

6 a)  $\vec{a} \cdot \vec{b} = -2 - 1 = -3$

$\vec{a} \cdot \vec{c} = 1 - 1 = 0$

$\vec{b} \cdot \vec{c} = -2 + 1 + 1 = 0$

b)  $|\vec{a}| = \sqrt{2}$   $|\vec{b}| = \sqrt{6}$   $|\vec{c}| = \sqrt{3}$

$\angle(\vec{a}, \vec{b}) = \arccos\left(\frac{-3}{\sqrt{2}\sqrt{6}}\right) = 2,62$

$\angle(\vec{a}, \vec{c}) = \arccos(0) = \frac{\pi}{2}$

$\angle(\vec{b}, \vec{c}) = \arccos(0) = \frac{\pi}{2}$

c)  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \varphi$

$\varphi = 60^\circ$   $|\vec{a}| = |\vec{b}| = 2$

$= 4 \cdot \cos 60^\circ = 2$

FACHBEREICH PHYSIK

FREIE UNIVERSITÄT BERGHA

FREIE UNIVERSITÄT BERGHA

FÜ

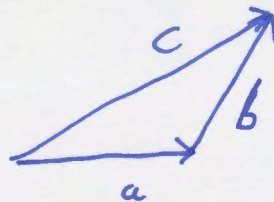
BERGHA

$$\begin{aligned}
 7a) \quad (\vec{a} + \vec{b})(\vec{a} - \vec{b}) &= (\vec{a} + \vec{b})(\vec{a} + (-\vec{b})) \\
 &= (\vec{a} + \vec{b})\vec{a} + (\vec{a} + \vec{b})(-\vec{b}) \\
 &= \vec{a}\vec{a} + \vec{b}\vec{a} + \vec{a}(-\vec{b}) + \vec{b}(-\vec{b}) \\
 &= |\vec{a}|^2 + \vec{b}\vec{a} - \vec{a}\vec{b} - |\vec{b}|^2 \quad \vec{a}\vec{b} = \vec{b}\vec{a} \\
 &= \cancel{\vec{b}\vec{a} - \vec{a}\vec{b}} \\
 &= a^2 - b^2
 \end{aligned}$$

Wenn  $a=b$  schließen  $(\vec{a} + \vec{b}), (\vec{a} - \vec{b})$  rechten Winkel ein

$$\begin{aligned}
 b) \quad |(\vec{a} - \vec{b})|^2 &= (\vec{a} - \vec{b})(\vec{a} - \vec{b}) \\
 &= \vec{a}\vec{a} - \vec{b}\vec{a} - \vec{b}\vec{a} + \vec{b}\vec{b} \\
 &= a^2 - 2\vec{a}\vec{b} + b^2 \quad \text{Wenn } \vec{a} \perp \vec{b} \rightarrow \vec{a}\vec{b} = 0 \\
 &= a^2 + b^2
 \end{aligned}$$

$$8a) \quad \vec{a} = \begin{pmatrix} 7 \\ 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -6 \\ 4/2 \end{pmatrix}$$



$$\Rightarrow \vec{c} = \vec{a} + \vec{b} = \begin{pmatrix} 1 \\ 5,5 \end{pmatrix} = \begin{pmatrix} 1 \\ 11/2 \end{pmatrix}$$

$$|\vec{a}| = \sqrt{50} \quad |\vec{b}| = \sqrt{\frac{225}{4}} = \frac{15}{2} \quad |\vec{c}| = \sqrt{\frac{225}{4}} = \frac{15}{2}$$

$$b) \quad \vec{a} \cdot \vec{b} = -\frac{75}{2} \quad \vec{a} \cdot \vec{c} = \frac{25}{2}$$

$$\angle(\vec{a}, \vec{b}) = \arccos\left(\frac{-\frac{75}{2}}{\sqrt{50} \cdot \frac{15}{2}}\right) = \frac{3}{4}\pi \rightarrow \frac{\pi}{4} = 0,79$$

$$\angle(\vec{a}, \vec{c}) = \arccos\left(\frac{\frac{25}{2}}{\sqrt{50} \cdot \frac{5\sqrt{5}}{2}}\right) = 1,25$$

$$\rightarrow \angle(\vec{b}, \vec{c}) = \pi - 0,79 - 1,25 = 1,10$$



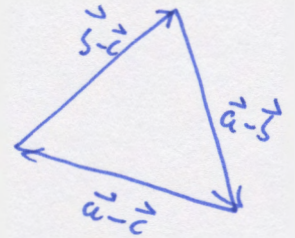
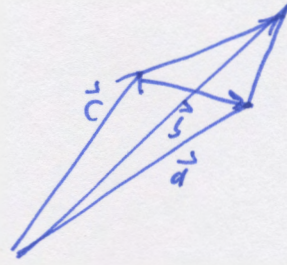
g)

~~a, b, c~~

$$\vec{a} - \vec{b} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

$$\vec{a} - \vec{c} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$$

$$\vec{b} - \vec{c} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$



$$(\vec{a} - \vec{b})(\vec{a} - \vec{c}) = 2 + 6 = 8$$

$$(\vec{a} - \vec{b})(\vec{b} - \vec{c}) = -2 + (2) = 0 \Rightarrow \text{rechtwinklig}$$