

Current Correlations in a Majorana Beam Splitter

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We study the current correlation in a T -junction composed of a grounded topological superconductor and of two normal-metal leads which are biased at a voltage V . We show that the existence of an isolated Majorana zero mode in the junction dictates a universal behavior for the cross correlation of the currents through the two normal-metal leads of the junction. The cross correlation is negative and approaches zero at high bias voltages as $1/V$. This behavior survives the presence of disorder and multiple transverse channels, and persists at finite temperatures. We employ numerical transport simulations to corroborate our conclusions.

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Introduction.— Majorana Fermions in condensed matter physics are zero-energy modes which are bound to the boundaries of an otherwise gapped topological superconductor (TSC). Such an isolated Majorana bound state (MBS) is described by a self-adjoint operator and is protected against acquiring a finite energy. These properties are responsible for much of the great interest in MBSs [1, 2].

Several theoretical proposals have been raised for realizing topological superconductivity in condensed matter systems [3–10]. Promising platforms include proximity-coupled semiconductor nanowires [8, 9] and ferromagnetic atomic chains [10–17], where recent transport measurements have shown evidences consistent with MBSs [18–25].

Much emphasis has been put on investigating the differential conductance through a normal lead coupled to a MBS [26–28]. At low enough temperatures the differential conductance spectrum shows a peak at zero bias voltage which is quantized to $2e^2/h$. Various aspects of current noise in topological superconducting systems have also been studied [26, 29–33].

As a natural next step one can study the current correlations in a setup composed of multiple leads which are coupled to a MBS, namely a Majorana beam splitter. In a recent work [34] we have examined the cross correlation between currents of opposite spin species, showing that it must be negative and approach zero at high bias voltage. In the present work we generalize the conclusions of that study and show that they extend beyond the realm of spin-resolved transport [35]. An immediate experimental consequence is that this effect can be observed in a much less challenging setup, which does not require spin filters to resolve the current into its spin components.

We consider a T -junction between a topological superconductor (TSC) and two normal-metal leads as depicted in Fig. 1(a). We study the low-frequency cross correlation of the currents through the two arms of the junction,

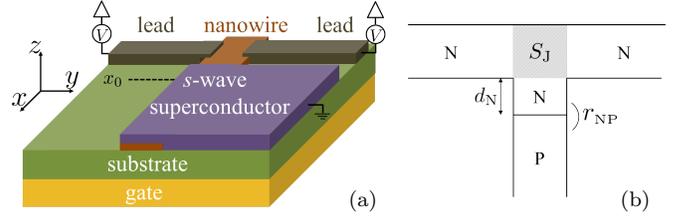


FIG. 1: (a) The proposed experimental setup is a T -junction between a topological superconductor (TSC) and two metallic leads. Here the TSC is realized by a semiconductor nanowire, proximity coupled to a conventional s -wave superconductor under an applied magnetic field. (b) We model the TSC by a spinless p -wave superconductor. It is coupled to the junction through a normal-metal section N , whose length d_N is taken to zero. Scattering at the NP interface is described by the reflection matrix r_{NP} , while scattering at the T -junction is described by the matrix S_J .

namely

$$P_{RL} = \int_{-\infty}^{\infty} dt \langle \delta \hat{I}_R(0) \delta \hat{I}_L(t) \rangle, \quad (1)$$

where $\delta \hat{I}_\eta = \hat{I}_\eta - \langle \hat{I}_\eta \rangle$, and $\hat{I}_{\eta=R,L}$ are the currents through the right and left arm of the junction respectively. The brackets stand for thermal quantum averaging. Below we show that P_{RL} has a universal behavior as a function of the bias voltage V between the two arms and the grounded superconductor. Denoting the width of the resonance due to the MBS by Γ , and the excitation gap by Δ [36], the correlation is negative for $V \ll \Gamma$ and approaches zero as $1/V$ for $\Gamma \ll V \lesssim \Delta$. For $V \gtrsim \Delta$ the behavior is nonuniversal.

This effect survives to a large extent at finite temperatures. In particular, while the temperature T should be smaller than Δ , it need not be smaller than Γ for the effect to be observed. This is in contrast to the zero-bias peak in the differential conductance spectrum which is only quantized to $2e^2/h$ for $T \ll \Gamma$.

Unlike studies which have focused on the cross correlation between currents through *two* MBSs at the two ends

of a TSC [29, 32, 37–39], here the effect is due to a *single* MBS. In these studies it was crucial that the MBSs at the two ends of the TSC were coupled [40]. Here, on the other hand, the effect is most pronounced when the two MBSs are spatially separated such that only a single MBS is coupled to the junction.

Scattering matrix approach.— The proposed experimental setup is described in Fig. 1(a). A semiconductor nanowire is proximitized to a grounded *s*-wave superconductor. When a sufficiently strong magnetic field is applied, the wire enters a topological phase [8, 9], giving rise to a MBS at each end. One of the wire’s ends is coupled to two metallic leads, biased at a voltage V .

To calculate the currents through the leads and their cross correlation we use the Landauer-Büttiker formalism in which transport properties are obtained from the scattering matrix, describing both normal and Andreev scattering. We are interested in bias voltages which are smaller than the superconducting gap. An electron incident from one of the normal leads is therefore necessarily reflected from the middle (superconducting) leg. It can be reflected towards the right or the left lead, either as an electron or as a hole. Since there is no transmission into the superconductor, scattering is described solely by a reflection matrix.

Each normal lead contains $2M$ transverse channels, including both spin species. The overall reflection matrix which we wish to obtain reads

$$r_{\text{tot}} = \begin{pmatrix} r^{ee} & r^{eh} \\ r^{he} & r^{hh} \end{pmatrix}, \quad (2)$$

where each block is a $4M \times 4M$ matrix. The matrix element $r_{ij}^{\alpha\beta}$ is the amplitude for a particle of type β coming from the channel j to be reflected into the channel i as a particle of type α . Here, $i = 1, \dots, 2M$ enumerates the channels in the right lead while $i = 2M + 1, \dots, 4M$ enumerates the channels in the left lead.

We model the TSC as a spinless *p*-wave superconductor which is a valid description close to the Fermi energy [41, 42]. We artificially insert between the TSC and the junction a (spinless) normal-metal section whose length d_N is taken to zero (cf. Fig. 1b). Andreev reflection at the normal-*p*-wave superconductor interface is described by

$$r_{\text{NP}}(\varepsilon) = \begin{pmatrix} 0 & a^*(-\varepsilon) \\ a(\varepsilon) & 0 \end{pmatrix}, \quad (3)$$

where $a(\varepsilon) = \exp[-i \arccos(\varepsilon/\Delta)]$ is the Andreev reflection amplitude for $\varepsilon \leq \Delta$ [43], with ε being the energy as measured from the Fermi level. The information about the topological nature of the system is encoded in $r_{\text{NP}}(\varepsilon)$. In particular, the nontrivial topological invariant of the *p*-wave superconductor [44, 45] $\mathcal{Q} = \det[r_{\text{NP}}(0)] = -1$ dictates the existence of a MBS at each of its ends.

Scattering at the *T*-junction (which connects the added normal section to the two leads) is described by

$$S_J = \begin{pmatrix} S_e & 0 \\ 0 & S_e^* \end{pmatrix}; \quad S_e = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}, \quad (4)$$

where S_e describes scattering of electrons and S_e^* describes scattering of holes. Here, r is a $4M \times 4M$ matrix describing the reflection of electrons coming from the left and right leads (each having $2M$ transverse channels), r' is a reflection amplitude for electrons coming from the middle leg (having a single channel), t is a $1 \times 4M$ transmission matrix of electrons from the right and left leads into the middle leg, and t' is a $4M \times 1$ transmission matrix of electrons from the middle leg into the right and left leads. The matrix S_e is assumed energy-independent in the relevant energy range, but is otherwise a completely general unitary matrix.

We can now concatenate S_J with r_{NP} to obtain the overall reflection matrix r_{tot} of Eq. (2). This results in [46]

$$r^{ee} = r + \frac{a(\varepsilon)^2 r t^\dagger t}{1 + |r'|^2 a(\varepsilon)^2}; \quad r^{he} = \frac{a(\varepsilon) t'^* t}{1 + |r'|^2 a(\varepsilon)^2}. \quad (5)$$

with $r^{eh}(\varepsilon) = [r^{he}(-\varepsilon)]^*$ and $r^{hh}(\varepsilon) = [r^{ee}(-\varepsilon)]^*$ in compliance with particle-hole symmetry [47].

At zero temperature the currents in the leads and their cross correlation are given by [29, 48]

$$\begin{aligned} \langle \hat{I}_\eta \rangle &= \frac{2e}{h} \sum_{i \in \eta} \int_0^{eV} d\varepsilon |\mathcal{R}_{ii}^{he}(\varepsilon)|^2, \\ P_{\text{RL}} &= \frac{e^2}{h} \sum_{i \in \text{R}, j \in \text{L}} \int_0^{eV} d\varepsilon \mathcal{P}_{ij}(\varepsilon), \\ \mathcal{P}_{ij} &= |\mathcal{R}_{ij}^{he}|^2 + |\mathcal{R}_{ij}^{eh}|^2 - |\mathcal{R}_{ij}^{ee}|^2 - |\mathcal{R}_{ij}^{hh}|^2, \end{aligned} \quad (6)$$

where $\mathcal{R}^{\alpha\beta} = r^{\alpha e} r^{\beta e \dagger}$, $\eta = \text{R, L}$, and i, j label the channels in the right and left lead, respectively, as defined below Eq. (2).

We introduce a parameter which is the total normal transmission from the two leads into the middle leg of the *T*-junction, $D = \sum_{i=1}^{4M} |t|^2$. Inserting Eq. (5) into Eq. (6) and using the unitarity of S_e , we first obtain the differential conductance

$$\frac{dI}{dV} = \frac{2e^2}{h} \frac{\Gamma^2}{(eV)^2 + \Gamma^2}, \quad (7)$$

where $I = \langle \hat{I}_{\text{R}} \rangle + \langle \hat{I}_{\text{L}} \rangle$ is the total current through the leads, and $\Gamma = \Delta D / 2\sqrt{1 - D}$. As expected dI/dV has a peak at $V = 0$ which is quantized to $2e^2/h$. Similarly, we obtain for the cross correlation [49]

$$P_{\text{RL}}(V) = -\frac{2e^2}{h} \Gamma_{\text{R}} \Gamma_{\text{L}} \frac{eV}{(eV)^2 + \Gamma^2}, \quad (8)$$

where $\Gamma_\eta = \Delta \sum_{i \in \eta} |t'_i|^2 / 2\sqrt{1-D}$ (notice $\Gamma = \Gamma_R + \Gamma_L$). The cross correlation P_{RL} is negative for all V and approaches zero like $1/V$ for $eV \gg \Gamma$. This result is valid for $eV \leq \Delta$. It is correct even in the presence of strong disorder at the junction as we did not assume a particular form of S_e . Moreover, it does not depend on a specific realization of the TSC hosting the MBS.

The result Eq. (30) can be understood from simple considerations based on the properties of MBSs. At low bias voltage $V \ll \Gamma$ and at zero temperature the conductance through the MBS is quantized to $2e^2/h$, resulting in an overall noiseless current [50]. Upon splitting the current into the two parts I_R and I_L , the total noise P is related to the cross correlation via $P = P_R + P_L + 2P_{\text{RL}}$, where P_η is the current noise through the η lead. Since $P \rightarrow 0$ at low voltage, while P_R and P_L are positive by definition, we must have $P_{\text{RL}} \leq 0$.

At high bias voltages $V \gg \Gamma$ (i.e., off resonance) transport can be described classically by sequential tunneling events of charges. In the weak coupling limit, $\Gamma \ll \Delta$, the mechanism by which current is conducted involves a splitting of a Cooper pair, such that one electron flips the bound state (either from empty to occupied or vice versa), while the other electron is transmitted into one of the leads. Importantly, for a MBS the probabilities for being transmitted to the right and to the left do not depend on the occupation of the bound state. This is because all local properties of these two states are identical. Based on this, it can be shown [46] that $P_{\text{RL}} \propto -1/V$ in agreement with Eq. (30).

Numerical Analysis.— We now turn to check the results of the previous section using numerical simulations. We consider the system depicted in Fig. 1(a). A semiconductor nanowire of dimensions $L_x \gg W_y \gg W_z$ is proximity coupled to a conventional s -wave superconductor and is placed in an external magnetic field.

The Bogoliubov de-Gennes Hamiltonian describing the nanowire is given in Nambu representation, $\Psi^\dagger(x) = (\psi_\uparrow^\dagger, \psi_\downarrow^\dagger, \psi_\downarrow, -\psi_\uparrow)$, by

$$\mathcal{H} = \left[\frac{-\nabla^2}{2m_e} + V(x, y) \right] \tau^z + i\lambda_R (\sigma^y \partial_x - \sigma^x \partial_y) \tau^z - \frac{\mu_B g}{2} B \sigma^x + \Delta_{\text{ind}}(x) \tau^x, \quad (9)$$

where $V(x, y)$ includes both the chemical potential and an impurity potential, λ_R is the Rashba spin-orbit coupling strength, B is the magnetic field directed along the wire, μ_B is the Bohr magneton, g is the Landé g-factor, $\Delta_{\text{ind}}(x) = \Delta_0 \theta(x - x_0)$ is the proximity-induced pair potential, and σ and τ are vectors of Pauli matrices in spin and particle-hole space, respectively. Since we take W_z to be much smaller than the magnetic length, we can safely ignore the orbital effect of the magnetic field.

We approximate the continuum model of Eq. (9) by a

tight-binding Hamiltonian

$$\begin{aligned} H = & \sum_{\mathbf{r}} \sum_{s, s'} \{ [V_{\mathbf{r}} \delta_{ss'} + B \sigma_{ss'}^x] c_{\mathbf{r}, s}^\dagger c_{\mathbf{r}, s'} \\ & - \sum_{\mathbf{d}=\hat{x}, \hat{y}} [(t \delta_{ss'} + iu(\boldsymbol{\sigma}_{ss'} \times \mathbf{d}) \cdot \hat{z}) c_{\mathbf{r}, s}^\dagger c_{\mathbf{r}+a\mathbf{d}, s'} + \text{h.c.}] \} \\ & + \sum_{\mathbf{r}: \hat{x} > x_0} [\Delta_0 c_{\mathbf{r}, \uparrow}^\dagger c_{\mathbf{r}, \downarrow}^\dagger + \text{H.c.}], \end{aligned} \quad (10)$$

where \mathbf{r} runs over the sites of an N_x by N_y square lattice with spacing a . Here $t = 1/2m_e a^2$, $u = \lambda_R/2a$, $V_{\mathbf{r}} = \mu - 4t + V_{\mathbf{r}}^{\text{dis}}$, μ is the chemical potential, and $V_{\mathbf{r}}^{\text{dis}}$ is a Gaussian-distributed disorder potential with zero average and correlations $\overline{V_{\mathbf{r}}^{\text{dis}} V_{\mathbf{r}'}^{\text{dis}}} = v_{\text{dis}}^2 \delta_{\mathbf{r}\mathbf{r}'}$.

We express H in first quantization using a $4N_x N_y \times 4N_x N_y$ matrix \mathcal{H}_{TB} [46], from which one extracts the retarded Green function

$$G^R(\varepsilon) = (\varepsilon - \mathcal{H}_{\text{TB}} + i\pi W W^\dagger)^{-1}, \quad (11)$$

and subsequently the reflection matrix [51, 52]

$$r_{\text{tot}}(\varepsilon) = 1 - 2\pi i W^\dagger G^R(\varepsilon) W. \quad (12)$$

Here, W is a matrix describing the coupling of the eigenmodes in the leads to the end of the nanowire as depicted in Fig. 1(a) and specified in the Supplemental Material (SM) [46]. The metallic leads are described in the wide band limit by W which is energy independent. With the help of Eqs. (5) and (6) we then obtain the currents through the leads and their cross correlation (see SM [46] for a formula valid at finite temperatures).

In the present work we use parameters consistent with an InAs nanowire, namely $E_{\text{so}} = m_e \lambda_R^2 / 2 = 75 \mu\text{eV}$, $l_{\text{so}} = 1/(m_e \lambda_R) = 130 \text{nm}$, and $g = 20$ [20]. The induced pair potential is taken to be $\Delta_0 = 150 \mu\text{eV}$. The length of the wire is $L_x = 2 \mu\text{m}$, with the section not covered by the superconductor being $x_0 = 200 \text{nm}$ in length, and the width of the wire is $W_y = 130 \text{nm}$.

In Fig. 2 we present the cross correlation $P_{\text{RL}}(V)$ and the differential conductance dI/dV at various temperatures for $\mu = 0$ and $B = 520 \text{mT}$. For these values of μ and B the system is in the topological phase [8, 9, 53]. P_{RL} is negative and approaches zero in agreement with the analytic expression of Eq. (30). Interestingly, this behavior persists even at nonzero temperatures. The main effect of temperature is to increase the voltage above which P_{RL} starts approaching zero. Since the gap in the system is about $100 \mu\text{eV}$, the effect can be seen even at the relatively high temperature of $T = 100 \text{mK}$, a temperature for which the ZBCP is much lower than $2e^2/h$.

Next we study the effect of disorder on P_{RL} . Fig. 3(a) presents P_{RL} for 10 different realizations of random disorder with $v_{\text{dis}} = 75 \mu\text{eV}$. As expected, the behavior of P_{RL} does not change significantly. We can compare this

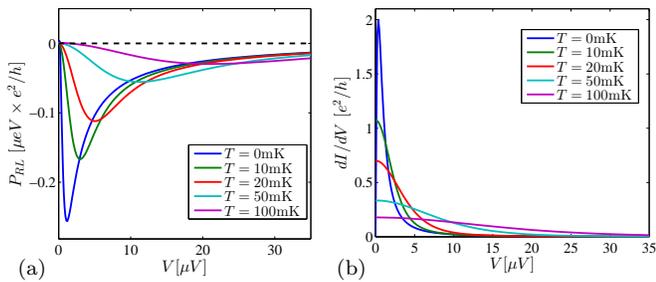


FIG. 2: (a) Zero-frequency cross correlations P_{RL} (defined in Eq. (1)) of the currents through the left and right leads as a function of bias voltage V at various temperatures. P_{RL} is negative for all V and approaches zero at voltages which are larger than both the resonance width and the temperature. (b) Total differential conductance, dI/dV , where $I = I_R + I_L$. At zero temperature dI/dV exhibits a zero-bias conductance peak quantized to $2e^2/h$. A nonzero temperature widens the peak and reduces its height to a nonuniversal value.

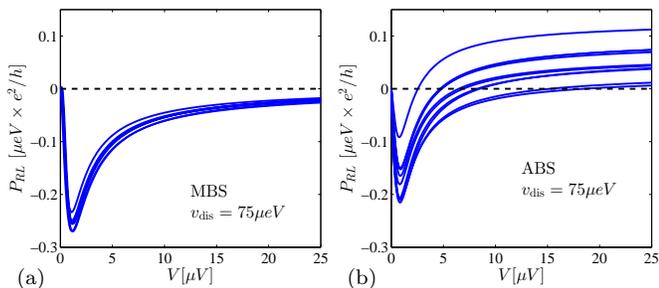


FIG. 3: Current cross correlation P_{RL} vs. bias voltage V at $\mu = 0$ for different realization of short-range Gaussian disorder. (a) $B = 520\text{mT} > B_c$, the system is in the topological phase with a Majorana zero mode (MBS) at each end of the wire. The behavior of $P_{RL}(V)$ is not affected by the presence of disorder. (b) For each realization of disorder the magnetic field is tuned to have an Andreev bound state (ABS) at the end of the wire, while keeping the system in the topologically-trivial phase, $B = 170 - 200\text{mT} < B_c$. The behavior of $P_{RL}(V)$ varies significantly for different realizations of disorder. In all cases $P_{RL} > 0$ for large V in contrast to the topological case where it goes to zero.

to the case of an ordinary Andreev state which is tuned to zero energy. The end of the wire which is not covered by a superconductor ($x < 0$ in Fig. 1(a)) hosts Andreev bound states which are coupled to the leads. For each realization of disorder, we tune the magnetic field to bring one of them to zero energy, and calculate P_{RL} . In all the realizations, the resulting tuned magnetic field was below the critical field $B_c = 260\text{mT}$, i.e., the system is in the trivial phase. As shown in Fig. 3(b), the behavior of P_{RL} is nonuniversal and varies significantly from one realization of disorder to another. Importantly, in all cases P_{RL} is positive at large V .

It is interesting to examine the case when more than a single transverse channel is occupied. At weak pair-

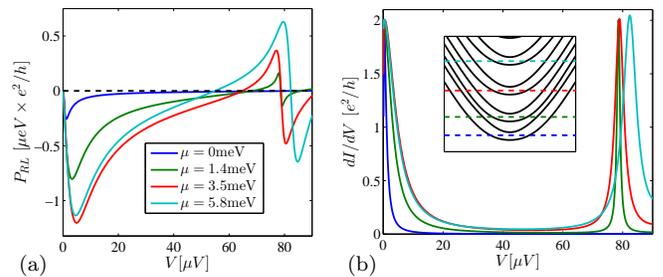


FIG. 4: (a) Cross correlation and (b) differential conductance at various chemical potentials μ , corresponding to a different odd number of occupied transverse channels. The calculations are performed at $T = 0$, $v_{\text{dis}} = 0$, and $B = 520\text{mT}$. The addition of occupied channels introduces extra subgap states which coexists with the Majorana zero mode. These appear as peaks in the differential conductance spectra at finite V (see (b) at $V \simeq 80\mu\text{eV}$). Above this voltage the behavior of P_{RL} is no longer universal.

ing [54] the system is in the topological phase whenever an odd number of channels is occupied. Figure 4 presents P_{RL} and dI/dV for various values of μ , each corresponding to a different odd number of occupied channels between 1 and 7. When more than a single channel is occupied we can have subgap Andreev bound states which coexist with the MBS. One such state can be seen in Fig. 4(b) as a peak at $V \simeq 80\mu\text{eV}$. It is only below this voltage that the behavior of $P_{RL}(V)$ remains qualitatively the same as for a single occupied channel. In this respect, the existence of subgap states reduces the effective energy gap below which $P_{RL}(V)$ exhibits its universal features. Another effect of introducing higher transverse channels is the stronger coupling of the middle leg of the T-junction to the two leads [55].

Conclusions.— When current from a topological superconductor is split into two metallic leads, the current cross correlation P_{RL} exhibits universal behavior as a function of bias voltage V . The cross correlation is negative for all V and approaches zero at high voltage. This behavior does not rely on a specific realization of the topological superconductor hosting the Majorana, or on a specific form of coupling to the leads. It can be observed even in disordered multichannel systems at finite temperature. For the effect to be observed the width of the Majorana resonance Γ has to be smaller than the energy of the first subgap state. Importantly, the temperature T does not have to be smaller than Γ .

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Note added.— While preparing for submission we be-

came aware of a work by Valentini *et al.* [56]. The results are consistent where they overlap.

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SUPPLEMENTARY MATERIAL

CLASSICAL PICTURE OF TRANSPORT

The behavior of P_{RL} at high voltages can be derived based on simple classical considerations. More specifically we look at the limit $eV \gg \Gamma$, where Γ is the resonance width, such that transport can be described by sequential tunneling events. Consider Cooper pairs (CPs) of electrons which arrive from the superconductor (SC) towards the junction in the presence of a zero energy bound state (either an Andreev or a Majorana bound state). The bound state defines two degenerate many-body states $|0\rangle$ and $|1\rangle$. The mechanism by which current is conducted involves one electron of the CP being transmitted into one of the leads, while the other electron flips the bound state from $|0\rangle$ to $|1\rangle$ or vice versa. We note that in the weak coupling limit ($\Gamma \ll \Delta$) the process in which two electrons are transmitted is highly suppressed.

The possible outcomes of each arrival of a CP at the junction are (i) The CP is reflected back towards the SC, (ii) one electron is transmitted into the right leads, and (iii) one electron is transmitted into the left lead. In general, the probability for each process depends on the state of the system. We denote by p_{ref}^0 , p_{R}^0 and p_{L}^0 the probabilities of processes (i), (ii) and (iii) respectively when the system is in the state $|0\rangle$, and p_{ref}^1 , p_{R}^1 and p_{L}^1 when the system is in the $|1\rangle$ state. Each time an electron is transmitted, the probabilities for the next event therefore change.

After N arrivals of CPs there are a total of Q_{R} (Q_{L}) electrons transmitted into the right (left) lead. The average current through each of the leads is given by

$$\langle I_{\text{R}} \rangle = \frac{e\langle Q_{\text{R}} \rangle}{N\Delta t} ; \quad \langle I_{\text{L}} \rangle = \frac{e\langle Q_{\text{L}} \rangle}{N\Delta t}, \quad (13)$$

and the current cross correlation is given by

$$\begin{aligned} P_{\text{RL}} &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt_1 \int_{\infty}^{\infty} dt_2 \langle \delta I_{\text{R}}(t_1) \delta I_{\text{L}}(t_2) \rangle = \\ &= \frac{e^2}{N\Delta t} (\langle Q_{\text{R}} Q_{\text{L}} \rangle - \langle Q_{\text{R}} \rangle \langle Q_{\text{L}} \rangle) \end{aligned} \quad (14)$$

where Δt is the time interval between arrival events.

In the case of a Majorana bound state all the local properties of the states $|0\rangle$ and $|1\rangle$ are identical. This is usually stated as the fact that one cannot make a local measurement which would reveal in which of the two ground states the system is in. In particular, it means that $p_{\text{ref}}^0 = p_{\text{ref}}^1 \equiv p_{\text{ref}}$, $p_{\text{R}}^0 = p_{\text{R}}^1 \equiv p_{\text{R}}$, and $p_{\text{L}}^0 = p_{\text{L}}^1 \equiv p_{\text{L}}$. Transport is then described by a trinomial distribution

and accordingly one obtains [57]

$$\begin{aligned} \langle Q_{\text{R}} \rangle &= Np_{\text{R}}, \\ \langle Q_{\text{L}} \rangle &= Np_{\text{L}}, \\ \langle Q_{\text{R}} Q_{\text{L}} \rangle - \langle Q_{\text{R}} \rangle \langle Q_{\text{L}} \rangle &= -Np_{\text{R}}p_{\text{L}}. \end{aligned} \quad (15)$$

Before setting Eq. (15) in Eq. (13) and Eq. (14) we make use of the fact that the probability for an electron to be transmitted (either to the left or the right lead) is related to the voltage and the resonance width by

$$1 - p_{\text{ref}} \simeq \Gamma/eV. \quad (16)$$

Given that an electron was transmitted, we can define the conditional probabilities of being transmitted into the right or left lead s_{R} and s_{L} ,

$$p_{\text{R}} = (1 - p_{\text{ref}})s_{\text{R}} ; \quad p_{\text{L}} = (1 - p_{\text{ref}})s_{\text{L}}, \quad (17)$$

where s_{R} and s_{L} are determined by the details of the setup, and they obey $s_{\text{R}} + s_{\text{L}} = 1$. Finally we can replace the time interval between consecutive arrivals of CPs by $\Delta t \simeq \hbar/eV$ which is the time-domain width of the electronic wave packet. One then obtains for the case of a Majorana bound state

$$\langle I_{\text{R}} \rangle = \frac{e}{\hbar} \Gamma_{\text{R}} ; \quad \langle I_{\text{L}} \rangle = \frac{e}{\hbar} \Gamma_{\text{L}}, \quad (18)$$

and

$$P_{\text{RL}} = -\frac{e}{\hbar} \frac{\Gamma_{\text{R}} \Gamma_{\text{L}}}{V}, \quad (19)$$

where we have defined $\Gamma_{\text{R}} = s_{\text{R}}\Gamma$ and $\Gamma_{\text{L}} = s_{\text{L}}\Gamma$. P_{RL} is negative and approaches zero as $1/V$. We have therefore manage to reproduce the high-voltage limit of Eq.(8) of the main text.

Unlike the case of a Majorana bound state, for a general Andreev bound state, the probabilities can depend on the state of the system, namely $|0\rangle$ or $|1\rangle$. We concentrate on the extreme case

$$p_{\text{L}}^0 = 0 ; \quad p_{\text{R}}^1 = 0 \quad (20)$$

where the electron can only go right if the system is in $|0\rangle$, and it can only go left if the system is in $|1\rangle$ [58]. Because each time an electron is transmitted the state of the system changes (either from $|0\rangle$ to $|1\rangle$ or vice versa), it is obvious that $Q_{\text{R}} = Q_{\text{L}}$ is given by half the number of times an electron has been transmitted. For simplicity we assume $p_{\text{ref}}^0 = p_{\text{ref}}^1 \equiv p_{\text{ref}}$. Based on a binomial distribution with success probability $1 - p_{\text{ref}}$ one then obtains

$$\langle Q_{\text{R}} Q_{\text{L}} \rangle - \langle Q_{\text{R}} \rangle \langle Q_{\text{L}} \rangle = \frac{1}{4} N p_{\text{ref}} (1 - p_{\text{ref}}). \quad (21)$$

and the cross correlation is therefore given by

$$P_{\text{RL}} = \frac{1}{4} \frac{e^2}{\hbar} \Gamma \left(1 - \frac{\Gamma}{eV}\right). \quad (22)$$

P_{RL} is monotonically increasing, asymptotically approaching a positive constant. This is in agreement with Fig. 3(b) of the main text and with the results of Ref. [34].

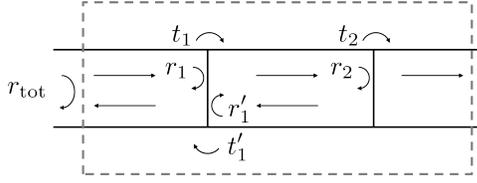


FIG. 5: Concatenating scattering matrices to obtain the total reflection matrix of modes incident from the left. In our case the first scattering matrix (from the left) is S_J , while the second reflection matrix is r_{NP} .

CONCATENATING SCATTERING MATRICES

To obtain the blocks of the reflection matrix (see Eq. (5) of the main text) we concatenate the scattering matrices r_{NP} and S_J of Eqs. (3) and (4) of the main text. We use the formula for concatenating two matrices in a row (see Fig. (5)), relating their blocks to the overall reflection matrix

$$r_{\text{tot}} = r_1 + t'_1(1 - r_2 r'_1)^{-1} r_2 t_1. \quad (23)$$

In our case one has (in the proper basis)

$$r_1 = \begin{pmatrix} r & 0 \\ 0 & r^* \end{pmatrix}, \quad r'_1 = \begin{pmatrix} r' & 0 \\ 0 & r'^* \end{pmatrix}, \quad r_2 = \begin{pmatrix} 0 & a^*(-\varepsilon) \\ a(\varepsilon) & 0 \end{pmatrix} \\ t_1 = \begin{pmatrix} t & 0 \\ 0 & t^* \end{pmatrix}, \quad t'_1 = \begin{pmatrix} t' & 0 \\ 0 & t'^* \end{pmatrix}. \quad (24)$$

Inserting Eq. (24) into Eq. (23) results in Eq. (5) of the main text, where one has to use the relation $a^*(-\varepsilon) = -a(\varepsilon)$, and the unitarity of S_J , from which it follows that $t' r'^* = -r t^\dagger$.

HAMILTONIAN APPROACH

The results presented in Eqs. (7) and (8) of the main text can be derived from a Hamiltonian approach of transport. We start from an effective low-energy Hamiltonian describing a multiple number of conducting channels which are coupled to a single Majorana bound state. Each of the channels belongs either to the left lead or to the right lead (although the calculation proceeds similarly in the case of a different number of leads). The Hamiltonian reads

$$H = H_L + H_T, \\ H_L = \sum_{ik} \epsilon_{ik} \eta_{ik}^\dagger \eta_{ik}; \quad H_T = i\gamma \sum_{ik} (\lambda_i \eta_{ik} + \text{h.c.}), \quad (25)$$

where γ describes the Majorana bound state, η_{ik}^\dagger creates an electron with momentum k and energy ϵ_{ik} in the i^{th} channel, and λ_i is the coupling constant of the i^{th} channel to the Majorana.

In the wide-band limit the reflection matrix can be obtained by [51, 52]

$$r_{\text{tot}}(\varepsilon) = 1 - 2\pi i W_M^\dagger \left(\varepsilon + i\pi W_M W_M^\dagger \right)^{-1} W_M, \quad (26)$$

with W_M being a vector of coupling constants given by

$$(W_M)_i = \sqrt{\nu_i} \begin{cases} \lambda_i & , i = 1, \dots, 4M \\ \lambda_i^* & , i = 4M + 1, \dots, 8M \end{cases}, \quad (27)$$

where ν_i is the density of states of the i^{th} channel at the Fermi energy, and M is the number of spinful channels in each lead (all together there are $4M$ electronic channels). One obtains for the blocks of r_{tot} (see also Eq. (2) of the main text)

$$r_{ij}^{ee} = \delta_{ij} + \frac{2\pi \lambda_i^* \lambda_j}{i\varepsilon - \Gamma}, \quad r_{ij}^{he} = \frac{2\pi \lambda_i \lambda_j}{i\varepsilon - \Gamma}, \quad (28)$$

with $r^{hh}(\varepsilon) = [r^{ee}(-\varepsilon)]^*$ and $r^{eh}(\varepsilon) = [r^{he}(-\varepsilon)]^*$, and where we have defined $\Gamma = 2\pi \sum_{i=1}^{4M} \nu_i |\lambda_i|^2$.

Inserting Eq. (28) into Eq.(6) of the main text results in

$$\frac{dI}{dV} = \frac{2e^2}{h} \frac{\Gamma^2}{(eV)^2 + \Gamma^2}, \quad (29)$$

and

$$P_{\text{RL}}(V) = -\frac{2e^2}{h} \Gamma_{\text{R}} \Gamma_{\text{L}} \frac{eV}{(eV)^2 + \Gamma^2}, \quad (30)$$

where $\Gamma_\eta = 2\pi \sum_{i \in \eta} \nu_i |\lambda_i|^2$. We have therefore managed to rederive Eqs. (7) and (8) of the main text. We note that the definition of Γ here is in terms of the coupling constant, while in the main text it is given in terms of transmission amplitudes. In both cases, however, it equals the width of the Majorana-induced resonance.

NUMERICAL SIMULATIONS

To obtain the scattering matrix using Eqs. (10-12) of the main text we express the Hamiltonian H in first quantized form using a $4N_x N_y \times 4N_x N_y$ matrix \mathcal{H}_{TB} defined by

$$H = \sum_{mn} \Psi_m^\dagger \mathcal{H}_{\text{TB}} \Psi_n; \quad \Psi^\dagger = (\Phi^\dagger, \Phi), \quad (31)$$

where $\Phi_{m=2N_y(n_x-1)+2(n_y-1)+s}^\dagger = c_{\mathbf{r}=(n_x, n_y, a), s}^\dagger$ creates an electron with spin s on site (n_x, n_y) of an $N_x \times N_y$ square lattice. Here, $s = 1$ for spin \uparrow and $s = 2$ for spin \downarrow . In our simulations we used $N_x = 90$, and $N_y = 6$.

The matrix W in Eq. (11) of the main text describes the coupling between the extended modes of the leads and the sites of the lattice. In each lead there are M spinful transverse channels. In our simulations $M = 4$ (see Fig. 6(b)). Including both leads, both spin species,

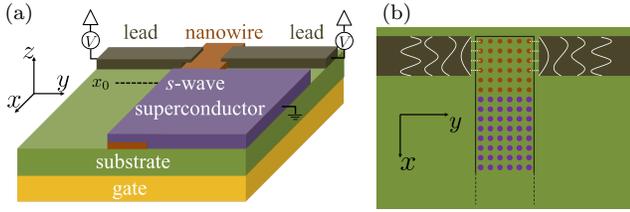


FIG. 6: (a) The proposed experimental setup, and (b) the corresponding tight-binding model. Each lead is tunnel-coupled to the sites adjacent to it. The purple sites are ones in which there is a non-vanishing induced pair potential (cf. Eq. (10) of the main text).

and the particle-hole degree of freedom, W is a $4N_x N_y \times 8M$ matrix of the following form

$$W = \begin{pmatrix} W_e & \mathbf{0} \\ \mathbf{0} & -W_e^* \end{pmatrix}; \quad W^e = (W_L \quad W_R), \quad (32)$$

where W_L and W_R described the coupling to the left and right lead, respectively. As depicted in Fig. 6(b)), each lead is coupled only to those lattice sites which are adjacent to it. Moreover, the coupling to each site is modulated according to the transverse profile of the particular channel. This is described by

$$W_L = W^0 \otimes \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{N_y} \otimes \sigma^0; \quad W_R = W^0 \otimes \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}_{N_y} \otimes \sigma^0,$$

$$W_{nm}^0 = \begin{cases} w_m \sin \frac{\pi n m}{M+1}, & 1 \leq n \leq M \\ 0, & M < n \leq N_x \end{cases}, \quad m = 1, \dots, M, \quad (33)$$

where σ^0 is a 2×2 identity matrix in spin space, and w_m is

a set of coupling constants for each transverse channel of the leads. In this work we have used $w_m^2 = 0.03\Delta_0$, $\forall m \in \{1, 2, 3, 4\}$.

Given the coupling matrix W and the first-quantized Hamiltonian \mathcal{H}_{TB} the reflection matrix is calculated by [51, 52]

$$r_{\text{tot}}(\varepsilon) = 1 - 2\pi i W^\dagger (\varepsilon - \mathcal{H}_{\text{TB}} + i\pi W W^\dagger)^{-1} W. \quad (34)$$

Finite Temperature

Figure (4) of the main text presents the current and the cross correlation at finite temperatures. This is calculated using the formula [48]

$$I = \frac{e}{h} \sum_{\substack{i,j=1,\dots,4M \\ \alpha,\beta \in \{e,h\}}} \text{sgn}(\alpha) \int_0^\infty d\varepsilon A_{jj}^{\beta\beta}(i, \alpha; \varepsilon) f_\beta(\varepsilon),$$

$$P_{\text{RL}} = \frac{e^2}{h} \sum_{i \in \text{R}, j \in \text{L}} \sum_{\substack{k,l=1,\dots,4M \\ \alpha,\beta,\gamma,\delta \in \{e,h\}}} \text{sgn}(\alpha) \text{sgn}(\beta) \int_0^\infty d\varepsilon \quad (35)$$

$$\times A_{kl}^{\gamma\delta}(i, \alpha; \varepsilon) A_{lk}^{\delta\gamma}(j, \beta; \varepsilon) f_\gamma(\varepsilon) [1 - f_\delta(\varepsilon)],$$

$$A_{kl}^{\gamma\delta}(i, \alpha; \varepsilon) = \delta_{ik} \delta_{il} \delta_{\alpha\gamma} \delta_{\alpha\delta} - (r_{\text{tot}})_{ik}^{\alpha\gamma*} (r_{\text{tot}})_{il}^{\alpha\delta},$$

with $f_e(\varepsilon) = 1 - f_h(-\varepsilon) = 1/(1 + \exp\{[\varepsilon - eV]/k_B T\})$ being the distribution of incoming electrons in the leads, and where $I = \langle \hat{I}_{\text{R}} \rangle + \langle \hat{I}_{\text{L}} \rangle$ is the total current in the leads. Equation (6) of the main text is obtained upon setting $T = 0$ in Eq. (35).