Diode Effects in Current-Biased Josephson Junctions

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Current-biased Josephson junctions exhibit hysteretic transitions between dissipative and superconducting states as characterized by switching and retrapping currents. Here, we develop a theory for diodelike effects in the switching and retrapping currents of weakly damped Josephson junctions. We find that while the diodelike behavior of switching currents is rooted in asymmetric current-phase relations, nonreciprocal retrapping currents originate in asymmetric quasiparticle currents. These different origins also imply distinctly different symmetry requirements. We illustrate our results by a microscopic model for junctions involving a single magnetic atom. Our theory provides significant guidance in identifying the microscopic origin of nonreciprocities in Josephson junctions.

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Introduction.—The nonreciprocal behavior of diodes constitutes a central element of electronics [1,2]. Nonreciprocity is also central to microwave and radiofrequency technology [3]. It is clearly a question of both applied and fundamental concern, whether nonreciprocal behavior can be realized in superconductors. Recent experiments on superconductors [4–11] as well as related theory [12–16] indicate that the critical current can indeed depend on the current direction.

These experiments have been extended to Josephson junctions, which are particularly promising for device applications, for instance in the context of superconducting qubits. A variety of current-biased junctions have been found to exhibit nonreciprocal behavior [17–25]. Many junctions are in the weak-damping regime, where the voltage response is hysteretic [Fig. 1(a)] and the nonreciprocal behavior can occur in multiple characteristic currents. When increasing the bias current, the junction switches into the resistive state at the switching current I_{sw}. Conversely, when reducing the current bias, the junction will retrap into the supercurrent state at a smaller retrapping current I_{re}. The switching and retrapping currents are in general different from one another and from the critical current I_c of the junction, the maximal supercurrent that the junction can in principle support [26]. While the dominant nonreciprocity is typically in the switching current [17,21,23,25], it is in the retrapping current in a recent experiment [27].

Theoretical work [29–37] has largely focused on exploring scenarios in which the current-phase relation and hence the critical current are asymmetric. Here, we present a general discussion of nonreciprocities in the various characteristic currents of current-driven Josephson junctions. Focusing on the low-damping limit with well-developed hysteresis, we show that nonreciprocities in the switching and retrapping currents have different microscopic origins and require different sets of broken symmetries. While dominant nonreciprocity in the switching current results from an asymmetric current-phase relation, dominant nonreciprocity in the retrapping current originates in asymmetric quasiparticle dissipation. We illustrate our results by a microscopic calculation for junctions including a single magnetic atom. Our results give important guidance for the design and interpretation of experiments on nonreciprocal Josephson junctions.

Model.—The dynamics of Josephson junctions is conventionally described within the model of a resistively and capacitively shunted Josephson junction (RCSJ) [26]. This model assumes that the junction carries capacitive (I_C), dissipative (I_d), and noise (δI) currents in parallel to the supercurrent (I_0). Current conservation implies that these currents sum to the bias current I_b [Fig. 1(b)],

$$I_C + I_0 + I_d + \delta I = I_b. \tag{1}$$

The conventional RCSJ model assumes a sinusoidal current-phase relation $I_0 = I_c \sin\varphi$, Ohmic dissipation $I_d = V/R$ by a shunt resistance R, and a capacitive current $I_C = C\dot{V}$. Johnson-Nyquist noise associated with the shunt resistor introduces a fluctuating current with correlator $\langle \delta I(t_1)\delta I(t_2)\rangle = (2T/R)\delta(t_1 - t_2)$ at temperature T. The Josephson relation $V = \hbar\dot{\varphi}/2e$ turns Eq. (1) into a Langevin equation for the stochastic dynamics of the superconducting phase difference φ across the junction.

In its conventional form, the RCSJ model predicts reciprocal characteristic currents. Nonreciprocal behavior can in general be introduced by modified capacitive, dissipative, or supercurrent terms. Misaki and Nagaosa [32] showed that nonreciprocal behavior can originate in nonlinear contributions to the quantum capacitance. This

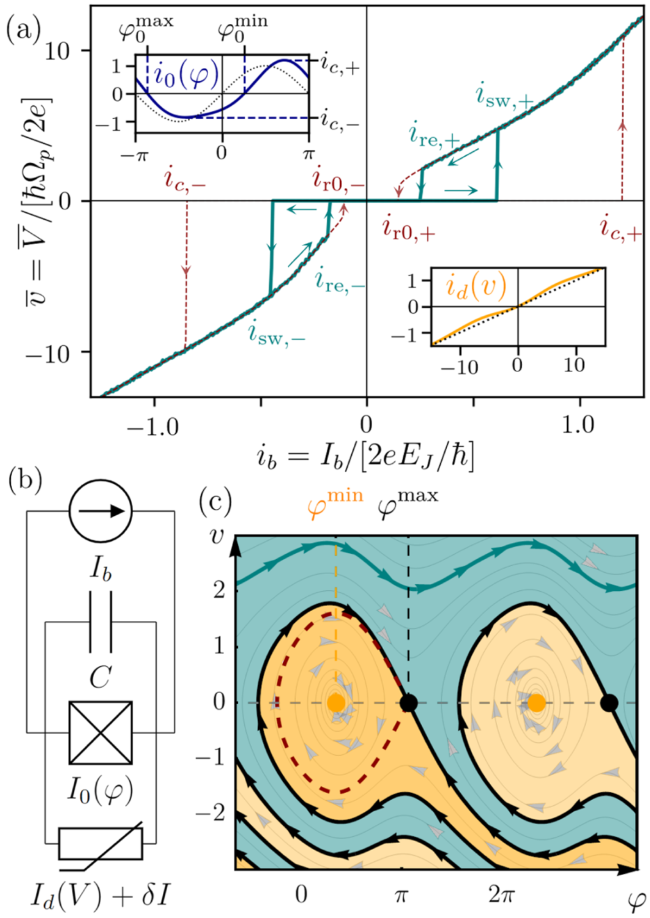

FIG. 1. RCSJ model for weakly damped Josephson junctions. (a) Hysteretic dependence of time-averaged voltage on bias current ($T = 0$: red, dashed; $T \neq 0$: green) and characteristic currents. Trace generated for asymmetric current-phase relation $I_0(\varphi)$ (left inset) and dissipative current $I_d(V)$ (right inset). Dotted traces in insets show corresponding curves used in the conventional RCSJ model. (b) Equivalent circuit of the RCSJ model. (c) Phase-space diagram of the deterministic junction dynamics with coexisting trapped (orange) and running (green) solutions. Parameters: [28].

mechanism requires different carrier densities on the two sides of the junction and applies to junctions joining two different superconducting materials.

The recent experiments [17,21,23,25,27] were performed on junctions made of a single superconductor. For this reason, we concentrate on nonreciprocities originating in the supercurrent I_0 and the dissipative current I_d. To this end, we allow for general current-phase relations $I_0(\varphi)$ and dissipative currents $I_d = I_d(V)$. Using Eq. (1) and the Josephson relation, the phase dynamics is then described by [38,39]

$$(\hbar C/2e)\ddot{\varphi} + I_d(\hbar\dot{\varphi}/2e) + I_0(\varphi) + \delta I = I_b. \qquad (2)$$

The correlator $\langle \delta I(t_1)\delta I(t_2)\rangle = K(V = \hbar\dot{\varphi}/2e)\delta(t_1 - t_2)$ of the current fluctuations is related to the dissipative

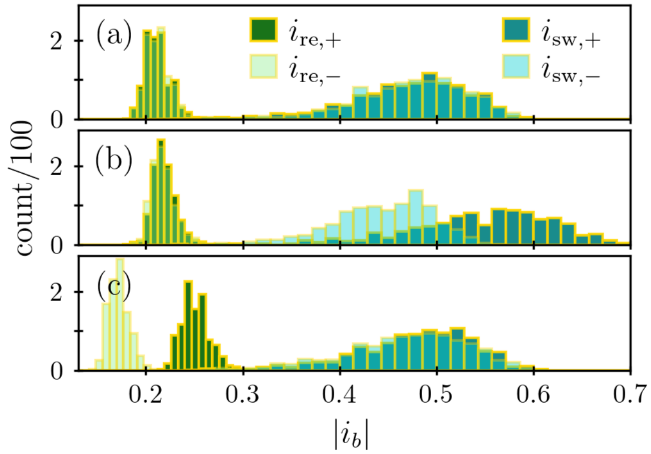

FIG. 2. Histograms of retrapping ($i_{\mathbf{re},\pm}$) and switching ($i_{\mathbf{sw},\pm}$) currents for bias currents i_b of both signs ($\pm$). (a) Conventional RCSJ model. (b) Asymmetric current-phase relation $I_0(\varphi)$ and symmetric dissipative current $I_d(\varphi)$. (c) Symmetric $I_0(\varphi)$ and asymmetric $I_d(\varphi)$. Parameters: [28].

current by the fluctuation-dissipation theorem. In the limit of low temperatures, this implies $K(V) = 2T[I_d(V)/V]$ (for a detailed discussion, see Supplemental Material [28]). Equation (2) describes the dissipative motion of a phase particle in a tilted washboard potential $U(\varphi) = U_0(\varphi) - (\hbar/2e)I_b\varphi$ with $I_0(\varphi) = (2e/\hbar)(dU_0/d\varphi)$. For definiteness, we restrict our attention to (periodic) potentials $U_0(\varphi)$ with a single minimum ($\varphi_0^{\min}$) and maximum ($\varphi_0^{\max}$) per period.

Nonreciprocity and symmetries.—The nonreciprocity in the switching and retrapping currents have distinctly different origins. This can be seen by directly simulating the Langevin dynamics for a weakly damped junction. Resulting histograms for the switching and retrapping currents are shown in Fig. 2. The histograms are independent of the direction of the bias current I_b for the conventional RCSJ model [Fig. 2(a)]. Only the switching current is nonreciprocal, when the junction has an asymmetric current-phase relation, $I_0(\varphi) \neq -I_0(-\varphi)$, but symmetric dissipative current, $I_d(V) = -I_d(-V)$. In contrast, only the retrapping current is nonreciprocal for asymmetric $I_d(V)$, but symmetric $I_0(\varphi)$.

This difference between switching and retrapping currents reflects that switching and retrapping are due to different underlying physics. Switching is caused by escape from a minimum of the tilted washboard potential $U(\varphi)$, thus requiring asymmetry in the $U_0(\varphi)$ and hence $I_0(\varphi)$. In contrast, retrapping back into a minimum of $U(\varphi)$ is induced by frictional energy loss, which depends directly on the dissipative current $I_d(V)$ for the particular bias direction. This also implies that nonreciprocities in the switching and retrapping currents have different symmetry requirements. Asymmetries in the current-phase relation require breaking of both time-reversal and inversion symmetry [4,35]. In contrast, asymmetries of the dissipative current, $I_d(V) \neq -I_d(-V)$, require breaking of particle-hole symmetry (in the normal-metal sense) and inversion symmetry, while time reversal need not be broken (as for conventional diodes). The dissipative current has contributions from the quasiparticle current of the junction as well as the electromagnetic environment. While the latter is typically symmetric, the quasiparticle current is generically nonlinear and asymmetric in the absence of inversion and particle-hole symmetry.

Fokker-Planck description.—To develop an analytical theory, we follow standard considerations to convert Eq. (2) into the Fokker-Planck equation [40,41]

$$\frac{\partial p}{\partial \tau} = \left(-v\frac{\partial}{\partial \varphi} + \frac{\partial}{\partial v}\left[u'(\varphi) + i_d(v) + \frac{1}{2}\frac{\partial}{\partial v}k(v)\right]\right)p \qquad (3)$$

for the time evolution of the probability density $p(\varphi, v; \tau)$ as a function of the phase φ and its velocity $v = \varphi'$. Here, we have defined a dimensionless time variable $\tau = \Omega_p t$ in terms of the plasma frequency $\Omega_p = [4e^2 E_J/\hbar^2 C]^{1/2}$, where $E_J = d^2 U_0(\varphi_0^{\min})/d\varphi^2$ is the Josephson energy. [This implies $U_0(\varphi) = -E_J \cos\varphi$ for a sinusoidal current-phase relation.] Also defining dimensionless currents $i = (\hbar/2e)I/E_J$ and potentials $u = U/E_J$, the Langevin equation (2) becomes $\varphi'' + i_d(\varphi') + i_0(\varphi) + \delta i = i_b$, where primes denote derivatives with respect to dimensionless time τ. The noise correlator $\langle \delta i(\tau_1)\delta i(\tau_2)\rangle = k(\varphi')\delta(\tau_1 - \tau_2)$ involves $k(v) = 2\theta i_d(v)/v$ in terms of the reduced temperature $\theta = T/E_J$.

At zero temperature, the Johnson-Nyquist noise vanishes, $\delta i = 0$. Then, the dynamics of the phase variable becomes deterministic, with two types of solutions. For small bias currents, the phase is locked to a minimum $\varphi^{\min}$ of the washboard potential $u(\varphi)$ and the junction supports supercurrent flow, $i_b = i_0(\varphi^{\min})$. For large bias currents, there is a running solution corresponding to a resistive state of the junction. In this state, the phase variable moves in a fixed direction at all times and the energy gain due to the current bias is compensated by the friction induced by the dissipative current.

For weak damping, the two types of solutions coexist at intermediate bias currents, see the phase-space diagram in Fig. 1(c). Then, the junction transitions between the two types of solutions due to Johnson-Nyquist noise and exhibits hysteresis. The nonreciprocal behavior of Josephson junctions is controlled by the transition rates between the trapped and running states, which we now derive for general current-phase relations and dissipative currents.

Switching rate.—We first consider the switching rate out of the trapped into the running state. For weak damping, the energy—and consequently the action—of the undamped motion are slowly varying variables. Then, the Fokker-Planck equation can be reduced to a drift-diffusion equation for the distribution function $p(J)$ of the action $J = \oint d\varphi v$ (here, $\oint$ denotes an integral over one period of the trapped motion) [42,43]. Using the general drift-diffusion equation $\partial_\tau p = \partial_J[-v_D p + D\partial_J p]$ and deducing the drift velocity $v_D = -\oint d\varphi i_d(v)$ as well as the diffusion coefficient $D = [2\pi\theta/\omega(J)]\oint d\varphi i_d(v)$ from the Langevin equation, we obtain (see Ref. [28] for details)

$$\frac{\partial p}{\partial \tau} = \frac{\partial}{\partial J}\left\{\varepsilon_d(J)\left[1 + \frac{2\pi\theta}{\omega(J)}\frac{\partial}{\partial J}\right]\right\}p. \qquad (4)$$

Here, we introduced the (dimensionless) energy $\varepsilon_d = \oint d\varphi i_d(v)$, which is dissipated per period. The current-phase relation and the bias current also enter via the angular frequency $\omega(J) = 2\pi dh(J)/dJ$ of the trapped motion, where $h = \frac{1}{2}v^2 + u(\varphi)$ is the Hamiltonian of the undamped junction.

Deriving the transition rate from the trapped into the running state is now an escape problem out of a metastable well [41]. In the low-temperature limit, we find the (dimensionless) activation rate (see Ref. [28])

$$\gamma_{\text{sw}} = \frac{\varepsilon_d(J_B)\omega_0}{2\pi\theta}\exp\left\{-\frac{\varepsilon_B}{\theta}\right\}. \qquad (5)$$

The transition rate depends exponentially on the activation barrier $\varepsilon_B = h(J_B) - h(J = 0)$ out of the metastable well. [Here, J_B is the action of the undamped separatrix motion beginning and ending at the unstable maximum $\varphi^{\max}$, see red dashed contour in Fig. 1(c), and $J = 0$ corresponds to the phase particle at rest in the stable minimum $\varphi^{\min}$.] Dissipation only affects the preexponential attempt frequency through $\varepsilon_d(J_B)$, the limit of the dissipated energy per period as the separatrix between running and trapped motion is approached. ω_0 is the oscillation frequency in the minimum of the tilted washboard potential $u(\varphi)$.

Retrapping rate.—We now consider the retrapping rate from the running into the trapped state. For weak damping, the retrapping current is parametrically smaller than the critical current and proportional to the strength of dissipation. We can thus restrict attention to small bias currents implying a weakly tilted washboard potential. Under these conditions, we focus on the action $J = \int_0^{2\pi \mathrm{sgn}(v)} d\varphi v$ evaluated for the Hamiltonian $h_0 = \frac{1}{2}v^2 + u_0(\varphi)$ of the unbiased junction. Note that the action J is now defined as an integral over all φ, as appropriate for the running state. The action is again slowly varying with time and satisfies a drift-diffusion equation [44]. The drift involves a contribution from the bias current i_b in addition to the dissipative term, $v_d = -\int_0^{2\pi \mathrm{sgn}(v)} d\varphi\, i_d(v) + 2\pi|i_b|$. The diffusion constant becomes $D = [2\pi\theta/\omega(J)]\int_0^{2\pi \mathrm{sgn}(v)} d\varphi\, i_d(v)$. This gives [28]

$$\frac{\partial p}{\partial \tau} = \frac{\partial}{\partial J}\left\{\varepsilon_d(J) - 2\pi|i_b| + \varepsilon_d(J)\frac{2\pi\theta}{\omega(J)}\frac{\partial}{\partial J}\right\}p \qquad (6)$$

for $p = p(J)$. Here, $\varepsilon_d(J) = \int_0^{2\pi \mathrm{sgn}(v)} d\varphi\, i_d(v)$ is the dissipated energy per period in the running state.

Following analogous steps as for the switching rate, we obtain the (dimensionless) retrapping rate [28]

$$\gamma_{\mathrm{re}} = \sqrt{\frac{i_d'(\bar{v})}{i_d(\bar{v})/\bar{v}} \frac{[|i_b| - i_{r0,\pm}]^2}{2\pi\theta}} \times \exp\left\{-\frac{1}{2\theta}\frac{1}{i_d'(\bar{v})\frac{i_d(\bar{v})}{\bar{v}}}[|i_b| - i_{r0,\pm}]^2\right\}. \quad (7)$$

Here, we define the retrapping currents $i_{r0,\mathrm{sgn}v} = (1/2\pi)\int_0^{2\pi \mathrm{sgn}(v)} d\varphi\, i_d(v(J_B))$ in the absence of fluctuations, where J_B is the action of the separatrix beginning and ending at neighboring unstable maxima $\varphi_0^{\max}$. The average phase velocity $\bar{v}$ is the solution of $i_b = i_d(\bar{v})$ and the upper (lower) sign applies for $i_b > 0$ ($i_b < 0$). The expression for the retrapping rate is valid at low temperatures and for bias currents sufficiently far from $i_{r0,\pm}$. For Ohmic friction and sinusoidal current-phase relation, our result reduces to the classic expression of Ben-Jacob *et al.* [45].

Nonreciprocity of switching and retrapping currents.— With these preparations, we are in a position to discuss nonreciprocity in weakly damped Josephson junctions in rather general terms. First, consider the nonreciprocity properties of the switching rate. If $u_0(\varphi)$ is symmetric about $\varphi = 0$ [and thus $i_0(\varphi) = -i_0(-\varphi)$], Eq. (4) governing the switching rate is explicitly symmetric under sign changes of the bias current i_b. This follows since the dissipation parameter ε_d and the frequency $\omega(J)$ are expressed as integrals over a full period of the trapped motion, in which the phase velocity v changes sign [28]. This conclusion remains true even if the dissipative current $i_d(v)$ is asymmetric in v. We thus find that as for the critical current, nonreciprocal switching rates require breaking of time reversal symmetry, such that $u_0(\varphi)$ is no longer symmetric under $\varphi \to -\varphi$ and the barrier ε_B becomes dependent on the sign of the bias current i_b.

Equation (6) governing the retrapping rate is explicitly symmetric under sign changes of i_b as long as the dissipative current $i_d(v)$ is symmetric. This is true for any potential $u_0(\varphi)$. A nonreciprocal retrapping rate originates in asymmetry of $i_d(v)$, which leads to $i_{r0,+} \neq i_{r0,-}$. Thus, a nonreciprocal retrapping rate requires breaking of particle-hole symmetry (in the normal-metal sense), so that the contribution of the quasiparticle current to $i_d(v)$ can be asymmetric. In contrast, breaking of time-reversal symmetry [asymmetric $u_0(\varphi)$] is not required.

The switching and retrapping rates allow one to derive expressions for the average switching and retrapping currents. We assume a bias-current ramp $i_b(\tau) = a\tau$ with rate a. Then the switching current i_{sw} can be defined through $P_t(i_{\mathrm{sw}}) = \frac{1}{2}$, where P_t is the probability to be in the trapped state. Using the switching rate (5), one finds that the shift in the switching current $\Delta i_{\mathrm{sw},\pm} = i_{\mathrm{sw},\pm} - i_{c,\pm}$ relative to the critical current is equal to [28]

$$\frac{\Delta i_{\mathrm{sw},\pm}}{i_{c,\pm}} \approx \left\{\frac{\theta}{\varepsilon_{B0}} \ln\left[\frac{\varepsilon_d(J_{B0})}{2\pi a \ln 2 |\varphi_0^{\max} - \varphi_0^{\min}|_\pm}\right]\right\}^{1/\mu_\pm}, \quad (8)$$

where J_{B0} is the action of the separatrix in the absence of damping and bias current and $\mu_\pm = |\varphi_0^{\max} - \varphi_0^{\min}|_\pm i_{c,\pm}/\varepsilon_{B0}$. Similarly, we find that fluctuations shift the retrapping rate away from $i_{r0,\pm}$ by [28]

$$\Delta i_{\mathrm{re},\pm} \approx \left\{\theta i_d'(\bar{v}) \frac{i_d(\bar{v})}{\bar{v}} \ln\left[\frac{\theta [i_d'(\bar{v})]^3}{2\pi (a \ln 2)^2} \frac{i_d(\bar{v})}{\bar{v}}\right]\right\}^{1/2}, \quad (9)$$

where the right hand side is evaluated for $i_b = \pm i_{r0,\pm}$. These expressions make the nonreciprocities of the average switching and retrapping currents explicit. While Eqs. (8) and (9) assume sufficiently high barriers as well as smooth drift and diffusion of the action [28], our qualitative results are valid more widely as indicated by the numerical results in Fig. 3. We also note that while our analytical results focus on the regime of thermally activated switching and retrapping, quantum tunneling may become relevant at sufficiently low temperatures. This will affect explicit

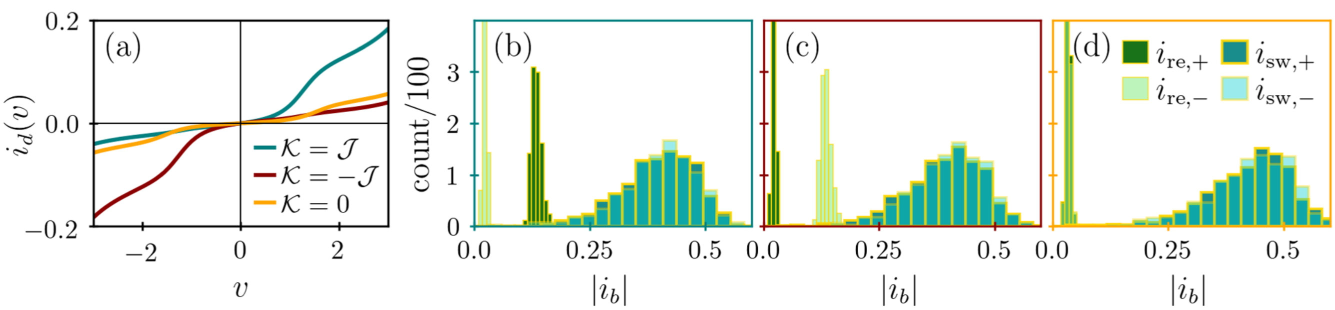

FIG. 3. Microscopic model of Josephson junction with magnetic impurity. (a) *I-V* characteristics of quasiparticle current due to YSR state associated with magnetic impurity for different potential scatterings $\mathcal{K}$. (b)–(d) Histograms of switching and retrapping currents corresponding to *I-V* characteristics in (a) (note color-coded box), emphasizing the importance of particle-hole symmetry. Parameters: [28].

temperature dependences, but leaves our qualitative results unaffected.

Yu-Shiba-Rusinov junctions.—We illustrate the importance of particle-hole symmetry for the retrapping current by microscopic results for a Josephson junction hosting a magnetic adatom coupled to one of the electrodes (for a recent experiment, see Ref. [27]). The spin of the adatom couples to the electrode electrons via both exchange scattering $\mathcal{J}$ and potential scattering $\mathcal{K}$, with the latter being nonzero only when particle-hole symmetry is broken [46]. These couplings induce Yu-Shiba-Rusinov (YSR) resonances within the superconducting gap, which are symmetric in energy, but in general asymmetric in intensity for nonzero potential scattering. This provides a microscopic model for an asymmetric quasiparticle current [Fig. 3(a)] accompanied by a symmetric current-phase relation. Consistent with our general theoretical analysis, a simulation of the junction dynamics based on a standard model for YSR states [47–50] (see Ref. [28] for details) exhibits asymmetric retrapping currents for nonzero potential scattering $\mathcal{K}$, with the direction of the asymmetry dependent on the sign of $\mathcal{K}$ [Figs. 3(b) and 3(c)]. Symmetric retrapping currents are observed for $\mathcal{K} = 0$, when particle-hole symmetry is preserved [Fig. 3(d)].

Conclusions.—We have developed a general theory of nonreciprocity in current-biased Josephson junctions, focusing on the hysteretic behavior for weak dissipation (high quality factor). We have shown that a nonreciprocal switching current originates from nonreciprocity of the supercurrent. In contrast, a nonreciprocal retrapping current originates from quasiparticle dissipation which is asymmetric under a sign change of i_b. Moreover, these different sources of nonreciprocity have different symmetry requirements. While nonreciprocal switching currents require breaking of time reversal symmetry, nonreciprocity of the retrapping current requires breaking of particle-hole symmetry, but not of time reversal symmetry. Recent experiments on weakly damped Josephson junctions revealed dominant nonreciprocities in both, the switching and the retrapping current. Our theory implies that these nonreciprocities have fundamentally different microscopic origins.

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