Universal spectral statistics in quantum graphs

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We prove that the quantum spectrum of *individual* chaotic quantum graphs shows universal correlations, as predicted by random–matrix theory. The stability of these correlations with regard to non–universal corrections is analyzed in terms of the linear operator governing the classical dynamics on the graph.

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Fluctuations in the spectra of individual complex quantum systems (e.g. classically chaotic systems) are universal and can, typically [1], be described by the Gaussian ensembles of random–matrix theory (RMT). This statement, promoted to a conjecture by Bohigas, Giannoni and Schmit [2], has been empirically confirmed in numerous experimental and numerical analyses [3, 4]. However, never so far has it been possible to demonstrate the fidelity of spectral fluctuations of an individual chaotic system to RMT statistics analytically [5]. The aim of this work is to present such a proof for a prototypical class of chaotic quantum systems, quantum graphs.

Quantum graphs differ from generic Hamiltonian systems in two important aspects: *a.)* they are semiclassically exact (The density of states can be represented in terms of an exact semiclassical trace formula.), *b.)* the corresponding classical dynamics is not deterministic and cannot be obtained from a formal limit $\hbar \to 0$. Introduced by Kottos and Smilansky[9], graphs are attractive theorists model systems inasmuch as they display much of the behavior of generic hyperbolic quantum systems but are not quite as resistant to analytical approaches than these.

In previous work, Berkolaiko *et al.* [10] developed a perturbative diagrammatic language to analyze the semiclassical periodic–orbit representation of spectral correlation functions beyond the leading ('diagonal') approximation. However, in spite of the full knowledge of its building blocks [10, 11] a complete resummation of the perturbation series has so far been elusive (not to mention that such expansions cannot reproduce the notorious essential singularities of the spectral correlation functions). In contrast, our present approach avoids diagrammatic resummations altogether. We rather build on two alternative pieces of input, both of which have been discussed separately before: *i.)* the exact equivalence of an average over the spectrum of a large quantum graph with incommensurate bond lengths to an ensemble average over certain scattering phases [9, 12, 13], and *ii.)* the so–called color-flavor transformation [18], which is an (equally exact) mapping of the phase–averaged spectral correlation function onto a variant of the supersymmetric σ-model. A subsequent stationary phase analysis then directly leads to the RMT correlation function corresponding to the symmetry of the graph. Finally, the spectrum of the massive fluctuations around the saddle point contains quantitative information on the stability of RMT spectral statistics with regard to non–universal corrections. Although the above program can be applied for all symmetry classes we will focus on spin–rotation and time–reversal invariant graphs throughout (i.e. the symmetry class of the Circular Orthogonal Ensemble COE. The case of broken time–reversal invariance then follows as a straightforward corollary.).

Let us begin by introducing our basic setting. A quantum graph consists of V vertices j connected by B bonds b. We assume that pairs of vertices are connected by at most one bond and that no bond starts and ends at the same vertex. (While simplifying the technicalities of our analysis below, we believe both assumptions to be physically immaterial.) To account for the two different directions of wave function propagation on each bond, we introduce $2B$ double indices (b,d), where $d = 1,2$ determines the (arbitrarily defined) direction of propagation along b. Boundary conditions on the graph are set by the fixed $2B$–dimensional unitary matrix $S = \{S_{bd,b'd'}\}$ which describes the scattering of an incoming wave function on bond b to an outgoing wave function on bond b'. (Of course, $S_{bd,b'd'}$ is non–vanishing only for bonds b and b' connecting at a common vertex j.) Time–reversal invariance ($\mathcal{T}$–invariance) implies that $S^T = \sigma_1^{\rm dir} S \sigma_1^{\rm dir}$, where $\sigma_i^{\rm dir} = (\sigma_i^{\rm dir})_{dd'}$ are Pauli matrices in the space of directional indices.

The discrete quantum time evolution in the system is described by the $2B \times 2B$ bond scattering matrix $\mathcal{S}(k) = T(k) S T(k)$. Here, the matrices $T(k)$ contain the dynamical quantum phases picked up during propagation along the bonds: $T(k) = \mathrm{diag}(e^{i\frac{kL_1}{2}}, \ldots, e^{i\frac{kL_B}{2}}, e^{i\frac{kL_1}{2}}, \ldots, e^{i\frac{kL_B}{2}})$, where L_b is the length of bond b and the two–fold replication in direction space expresses the independence of the dynamical phases on the direction of propagation. The concise formulation of this fact reads as $\mathcal{T}:\ \mathcal{S}(k)=\sigma_1^{\rm dir}\mathcal{S}^T(k)\sigma_1^{\rm dir}$. Finally, the spectrum of the graph is determined by the zeros of the spectral determinant $\xi_B(k)=\det\left(1-\mathcal{S}(k)\right)$.

Universal spectral statistics in quantum systems can be expected if the corresponding classical system is fully chaotic. What is the equivalent condition on a quantum graph? An answer has been formulated by Tanner [13] (see also [14]) in terms of the classical probability $F_{bd,b'd'}\equiv|S_{bd,b'd'}|^2=|\mathcal{S}(k)_{bd,b'd'}|^2$ to get from (b',d') to (b,d). Notice that the 'classical propagator' F has one eigenvalue $\lambda_1=1$, corresponding to a fully equilibrated distribution in bond space. The dynamics is *mixing* if, for large times, any initial probability distribution converges to this distribution, or $\lim_{n\to\infty}(F^n)_{bd,b'd'}=\frac{1}{2B}$. This condition is met if all other eigenvalues $|\lambda_{2,\dots,B}|<1$ lie inside the complex unit circle.

However, even a graph with mixing dynamics does not necessarily display universal spectral behavior[15]. Tanner rather conjectured that a stronger (and then sufficient) condition is that the spectral gap $\Delta_{\rm g}=\max_{b\in\{2,\dots,B\}}(1-|\lambda_b|)$ is constant or vanishes slow enough in the limit $B\to\infty$. Observing that the diagonal approximation to the form factor is consistent with RMT if $B\Delta_{\rm g}\stackrel{B\to\infty}{\longrightarrow}\infty$ he estimated that the gap should vanish slower than $\frac{1}{B}$.

In the sequel we will verify this conjecture, and give a stronger lower bound for the gap condition. Our analysis will be based on the assumption that all bond lengths L_b are rationally independent. Under this condition, $k\mapsto\{e^{i\frac{kL_1}{2}},\dots,e^{i\frac{kL_B}{2}}\}$ defines an ergodic flow on the B-torus implying that the average over the parameter k may be replaced by an average over B independent phases $e^{i\frac{kL_b}{2}}\mapsto e^{i\phi_b}$ [9, 12, 13]. This latter average is implemented by setting

$$\lim_{K\to\infty}\frac{1}{K}\int_0^K dk\,\mathcal{F}[T(k)]=\langle\mathcal{F}[T(\phi)]\rangle_\phi,\tag{1}$$

where $\mathcal{F}$ is a smooth function and $\langle\cdot\rangle_\phi=\frac{1}{(2\pi)^B}\int_0^{2\pi}d^B\phi\,(\cdot)$.

Throughout we will focus attention on the phase–averaged spectral determinant

$$\xi(z_{\mathbf b}^+,z_{\mathbf b}^-,z_{\mathbf f}^+,z_{\mathbf f}^-)=\left\langle\det\frac{(1-z_{\mathbf f}^+\mathcal{S}(\phi))(1-z_{\mathbf f}^-\mathcal{S}(\phi)^\dagger)}{(1-z_{\mathbf b}^+\mathcal{S}(\phi))(1-z_{\mathbf b}^-\mathcal{S}(\phi)^\dagger)}\right\rangle_\phi.\tag{2}$$

Quantities such as the two–point correlation function or the spectral form factor can then be obtained by straightforward differentiation w.r.t. the parameters $z_{\mathbf b}^+,\dots,z_{\mathbf f}^-$.

The determinant ξ affords the Gaussian integral representation $\xi=\operatorname{sdet}^{-1}(z^+z^-)\int d(\bar\psi,\psi)\exp(-\boldsymbol{S}[\bar\psi,\psi])$, where [16]

$$\boldsymbol{S}[\bar\psi,\psi]=\bar\psi_+\begin{bmatrix}1 & T\\ T & S^\dagger z_+^{-1}\end{bmatrix}\psi_++\bar\psi_-\begin{bmatrix}1 & T^\dagger\\ T^\dagger & z_-^{-1}S\end{bmatrix}\psi_-.\tag{3}$$

Here, $z_\pm=\operatorname{diag}(z_{\mathbf b}^\pm,z_{\mathbf f}^\pm)$ are 2×2–supermatrices and $\psi=\{\psi_{a,s,x,d,b}\}$ is a $16B$-dimensional supervector, where $a=\pm$ distinguishes between the retarded and the advanced sector of the theory (determinants involving $\mathcal{S}$ and $\mathcal{S}^\dagger$, resp.), $s=\mathbf{f},\mathbf{b}$ refers to complex commuting and anti–commuting components (determinants in the denominator and numerator, resp.), and $x=1,2$ to the internal structure of the matrix kernel appearing in (3). Using that $\det(1-z\mathcal{S})=\det(zS)\det\left(\begin{smallmatrix}1 & T\\ T & z^{-1}S^\dagger\end{smallmatrix}\right)$, one verifies that the Gaussian integration over all components of ψ yields the determinant ξ.

As a second step, we subject the phase–averaged ψ–functional to a duality transformation known as the color–flavor transformation [18]. In a variant adapted to the present context [17], the transformation states that

$$\langle\exp(-\boldsymbol{S}[\bar\psi,\psi])\rangle_\phi=\langle\exp(-\boldsymbol{S'}[\bar\Psi,\Psi])\rangle_Z,\tag{4}$$

where $\langle\cdot\rangle\equiv\int dZd\tilde Z\operatorname{sdet}(1-Z\tilde Z)(\cdot)$ and (matrix structure in advanced/retarded space)

$$\boldsymbol{S'}[\bar\Psi,\Psi]=\bar\Psi_1\begin{bmatrix}1 & Z\\ Z^{\tilde\tau} & 1\end{bmatrix}\Psi_1+\bar\Psi_2\begin{bmatrix}S^\dagger z_+^{-1} & \tilde Z^{\tilde\tau}\\ \tilde Z & z_-^{-1}S\end{bmatrix}\Psi_2.\tag{5}$$

Referring for a short discussion of the underlying technicalities to [17], we here briefly explain the notation and the physical meaning of the transformation (4). In (5), $\Psi_{1,2}=\{(\Psi_{1,2})_{a,s,t,b,d}\}$ are $16B$–dimensional independent supervectors, where the index, $t=1,2$ accounts for the time–reversal symmetry of the model. Presently, all we need to know about the variables Ψ and $\bar\Psi$ is that they contain elements of ψ and $\bar\psi$ as their components, and depend on each other through $\bar\Psi_{1,2}=\Psi_{1,2}^T\tilde\tau$. Here, the fixed supermatrix $\tilde\tau\equiv\sigma_1^{\rm dir}\otimes\tau$, where $\tau\equiv(E_{\mathbf{bb}}\sigma_1^{\rm tr}-iE_{\mathbf{ff}}\sigma_2^{\rm tr})$ ($\sigma_i^{\rm tr/dir}$ are Pauli matrices in time–reversal space (t)/direction space (d), and $E_{\mathbf{bb}/\mathbf{ff}}$ are projectors on the bosonic/fermionic sector of the theory). The newly introduced integration variables, $Z=\operatorname{bdiag}(Z_1,\dots,Z_B)$ are $8B$–dimensional block–supermatrices with 8–dimensional entries $Z_b=\{Z_{b,ss',dd',tt'}\}$. Finally, $Z^{\tilde\tau}\equiv\tilde\tau Z^T\tilde\tau^{-1}$ is in a generalized way transposed to Z, while Z and $\tilde Z$ are independent.

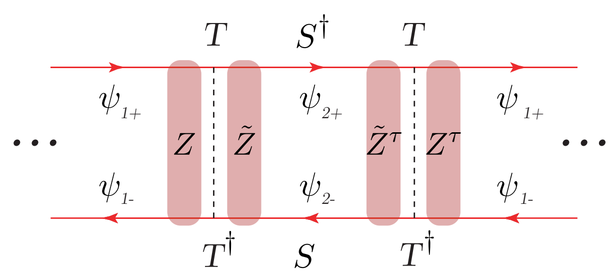

FIG. 1: On the physical interpretation of the color-flavor transformation. Explanation, see text.

What is the physical significance of the transformation (4)? Fig. 1 shows a cartoon of the retarded (upper line) and advanced (lower line) wave function dynamics in the system. During propagation, both states pick up random scattering phases T (indicated by vertical dashed lines) and suffer scattering from one bond to the other (S–matrix). The rapid succession of these events implies wild fluctuations of the wave function amplitudes. Within the field theoretical context, this translates to uncontrollable fluctuations of the bilinears $\bar\psi_{+,s,x,d,b}e^{i\phi_b}\psi_{+,s,x,d,b}$ central to the action (3). In contrast, the field Z enters the theory as $\sim \bar\Psi_{+,s,t,d,b}Z_{b,ss',tt',dd'}\Psi_{-,s',t',d',b}$, i.e. through structures that *couple* retarded and advanced field amplitudes (the 'vertical' ovals in the figure). These amplitudes generally *interfere* to form slowly fluctuating entities (the basic principle behind the formation of universal correlations.) This indicates that the Z–integral will be comparatively benign and, foreseeably, amenable to stationary phase approximation schemes.

To promote this expectation to a quantitative level, we integrate out the Ψ's, thus arriving at the *exact* representation $\xi = \int dZ d\tilde Z \exp(-\boldsymbol{S}[Z,\tilde Z])$,

$$\begin{aligned}\boldsymbol{S}[Z,\tilde Z] = &-\mathrm{str}\ln\left(1-\tilde Z Z\right)+\frac{1}{2}\mathrm{str}\ln\left(1-Z^{\tilde\tau}Z\right)\\ &+\frac{1}{2}\mathrm{str}\ln\left(1-S^\dagger z_-\tilde Z z_+ S\tilde Z^{\tilde\tau}\right).\end{aligned}\tag{6}$$

As a first step towards a better understanding of the physics of this expression, let us consider its quadratic expansion,

$$\boldsymbol{S}^{(2)}[Z,\tilde Z]=\mathrm{str}\left(\tilde Z Z-\frac{1}{2}Z^{\tilde\tau}Z-\frac{1}{2}S^\dagger z_-\tilde Z S z_+\tilde Z^{\tilde\tau}\right).\tag{7}$$

Void of non–linearities (terms of $\mathcal{O}(Z^4)$), the action $\boldsymbol{S}^{(2)}$ describes the un–interrupted propagation of two amplitudes along the *same* path in configuration space, i.e. the level of approximation underlying the diagonal approximation in semiclassical periodic–orbit theory.

This connection is made quantitative by noting that the action $\boldsymbol{S}^{(2)}$ possesses a family of approximately (up to corrections of $\mathcal{O}(B^{-1})$) 'massless' configurations, or 'zero modes' identified by $\delta_Z\boldsymbol{S}^{(2)}=\delta_{\tilde Z}\boldsymbol{S}^{(2)}=0$. Upon substitution of the ansatz $Z_{dd'}=\delta_{dd'}Z_d$ (configurations not diagonal in direction space do not qualify as solutions, see the discussion of deviations below) these equations assume the form

$$\tilde Z = Z^{\tilde\tau},\qquad (\mathbf{1}-\hat F)Z=0,\tag{8}$$

where we have set the external parameters $z^\pm_{\mathbf{b},\mathbf{f}}=1$[19]. Owing to the fact that on a chaotic quantum graph, the 'classical propagator' $\hat F$ has only one eigenvalue 1, Eq. (8) possesses the unique solution $Z_{bd,b'd'}=B^{-1/2}\delta_{bd,b'd'}Y$, proportional to the invariant equidistribution. Notice that $Y^{\tilde\tau}=Y^\tau\equiv\tau Y^T\tau^{-1}$, where the matrix τ differs from $\tilde\tau$ by the absence of the (now redundant) matrix σ_1^{dir}. Technically, the relation $\tilde Y = Y^\tau$ identifies $(Y,\tilde Y)$ as generators of the orthosymplectic algebra osp(4|4). Having identified the zero mode, let us explore the significance of other field configurations. A glance at Eq. (7) shows that deviations from the first of the two equations in (8) are penalized by a large action $\boldsymbol{S}^{(2)}=\mathcal{O}(1)$. Upon integration, these modes produce no more than a factor unity to the spectral determinant. (Similarly, modes that are off–diagonal in direction space can be integrated out to give a factor unity — due to our technical assumptions on the connectivity of graphs the term $\mathrm{str}S^\dagger z_-\tilde Z z_+ S\tilde Z^{\tilde\tau}$ exactly vanishes for such modes.) To explore the more interesting role played by deviations from the equation $(\mathbf{1}-\hat F)Z=0$, let us expand a general configuration $Z_{bd}=\sum_{m=1}^{2B}Y_m\xi_{m,bd}$ in the basis of eigenfunctions ξ_m of the operator $\hat F$. Here, Y_m are four–dimensional supermatrices obeying the symmetry $\tilde Y_m = Y_m^\tau$ and the identification $Y_1\equiv Y$ is understood. Substituting this expansion into the quadratic action we obtain

$$\boldsymbol{S}^{(2)}[Y,\tilde Y]=\frac{1}{2}\sum_1^{2B}\mathrm{str}\,(Y_m\tilde Y_m-\lambda_m Y_m z_-\tilde Y_m z_+),$$

and, upon quadratic integration over the Y's,

$$\xi^{(2)}=\prod_{m=1}^{2B}\frac{(1-\lambda_m z^+_{\mathbf{f}}z^-_{\mathbf{b}})^2(1-\lambda_m z^+_{\mathbf{b}}z^-_{\mathbf{f}})^2}{(1-\lambda_m z^+_{\mathbf{f}}z^-_{\mathbf{f}})^2(1-\lambda_m z^+_{\mathbf{b}}z^-_{\mathbf{b}})^2}.\tag{9}$$

Intending to probe correlations on the scale of the mean level spacing, we set $z^\pm_{\mathbf{b},\mathbf{f}}=\exp(i\pi\epsilon^\pm_{\mathbf{b},\mathbf{f}}/B)$ ($\epsilon^\pm_{\mathbf{b},\mathbf{f}}=\mathcal{O}(1)$) and expand Eq. (9) in powers of B^{-1}. While the zero–mode factor corresponding to $\lambda_1=1$ evaluates to $\frac{(\epsilon^+_{\mathbf{f}}+\epsilon^-_{\mathbf{b}})^2(\epsilon^+_{\mathbf{b}}+\epsilon^-_{\mathbf{f}})^2}{(\epsilon^+_{\mathbf{f}}+\epsilon^-_{\mathbf{f}})^2(\epsilon^+_{\mathbf{b}}+\epsilon^-_{\mathbf{b}})^2}+\mathcal{O}(B^{-2})$ all other factors equal unity up to a correction of order $\frac{\lambda_m\epsilon^2}{B^2(1-\lambda_m)^2}$. (In the two-point correlation function each of these corrections leads to an additive constant of order $\frac{\lambda_m}{B^2(1-\lambda_m)^2}$.) There being $2B-1$ such factors, we conclude that the cumulative contribution of the massive modes can be neglected provided that $B\Delta_g^2\overset{B\to\infty}{\to}\infty$, which is a slightly stronger condition than Tanner's. For higher order correlation functions the same condition applies (unless the order of the correlation function is of order B).

Going beyond the level of the quadratic approximation, we note that the saddle point equations $\delta_Z\boldsymbol{S}=\delta_{\tilde Z}\boldsymbol{S}=0$ of the full action (6) are still solved by the zero mode configurations (8). While deviations from the zero modes continue to be negligible (as long as $B^2\Delta_g\to\infty$), the action of the latter now reads

$$\boldsymbol{S}[Y,\tilde Y]=\frac{B}{2}\mathrm{str}\Big(\ln(1-z_-\tilde Y z_+ Y)-\ln(1-\tilde Y Y)\Big),\tag{10}$$

where we have rescaled $Y\to(2B)^{1/2}Y$. Eq. (10) defines an exact representation of the COE spectral determinant[18]. To represent this result in a perhaps more widely recognizable form, let us define $z^\pm = 1+i\delta\frac{\epsilon^\pm}{2}$ ($\delta=\pi B^{-1}$ is the level spacing), and the 8×8–matrix

$$Q=\begin{pmatrix}1 & Y\\ \tilde Y & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1 & Y\\ \tilde Y & 1\end{pmatrix}^{-1}.$$

It is then straightforward to verify that the leading order expansion of (10) in powers of B^{-1} obtains Efetov's[8] action for the GOE correlation function

$$\boldsymbol{S}[Q] = \frac{i}{2}\,\mathrm{str}\,(Q\hat{\epsilon}), \tag{11}$$

where $\hat{\epsilon} = \mathrm{diag}(\epsilon^+_{\mathbf{b}}, \epsilon^+_{\mathbf{f}}, \epsilon^-_{\mathbf{b}}, \epsilon^-_{\mathbf{f}})$.

Finally, let us discuss the crossover to the case of unitary symmetry. Time–reversal invariance can be broken either by lifting the symmetry $S = \sigma_1^{\mathrm{dir}} S^T \sigma_1^{\mathrm{dir}}$, or by perturbing the phase balance between the left and right moving states on the bonds. Either way, the symmetry $\mathcal{T} : \mathcal{S} = \sigma_1^{\mathrm{dir}} \mathcal{S}^T \sigma_1^{\mathrm{dir}}$ of the full scattering operator gets lost. Within our field theoretical setting, a lack of $\mathcal{T}$–invariance implies the formation of mass terms for the components of the generators Y off–diagonal in time–reversal space. Referring for a more detailed discussion of the crossover physics to [20], we note that even a miniscule violation $\mathcal{S} - \sigma_1^{\mathrm{dir}} \mathcal{S}^T \sigma_1^{\mathrm{dir}} \sim \mathcal{O}(B^{-1})$ induces a crossover to CUE spectral statistics. (The exact CUE correlation function is obtained upon neglecting the off–diagonal components — the Cooperon modes in the jargon of mesoscopic physics — altogether.)

Summarizing, we have proven Tanner's conjecture on universal spectral statistics (on the scale of the mean level spacing) in large chaotic quantum graphs. The corrections to universality turn out to be of order $\frac{1}{B\Delta_g^2}$ (or smaller) which gives a more conservative lower bound $\Delta_g > \mathrm{const}B^{-\frac{1}{2}}$ on the spectral gap Δ_g in the limit $B \to \infty$ than Tanner's estimate $\Delta_g > \mathrm{const}B^{-1}$. An interesting direction of future research may be the analysis of correlations at moderately high energies $\epsilon > B\Delta_g$ where aspects of non–ergodic dynamics begin to play a role.

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To each of the B phase averages over $e^{i\phi_b} \in \mathrm{U}(N_c = 1)$ we now apply the color–flavor transformation in a variant $(N_c = 1, N_f = 4)$, i.e. for one color and four 'flavors' (the latter labeled by $t = 1, 2$ and $d = 1, 2$). Which in practice means [18] that the phase action gets replaced by $\boldsymbol{S} \to \boldsymbol{S}' = 2\bar{\Psi}_{+1} Z \Psi_{-1} + 2\bar{\Psi}_{-2} \tilde{Z} \Psi_{+2} + \boldsymbol{S}_0$ where Z and $\tilde{Z}$ are $8B$–dimensional block matrices containing 8×8–supermatrices Z^b and $\tilde{Z}^b$ as independent sub–blocks. We finally use relations such as $\bar{\Psi}_{-1} Z \Psi_{+1} = \bar{\Psi}_{+1} Z^\tau \Psi_{-1}$ to symmetrize the action and arrive at (5).

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