Null Weak Values and Quantum State Discrimination

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We present a measurement protocol for discriminating between two different quantum states of a qubit with high fidelity. The protocol is comprised of a projective measurement performed on the system with small probability (a.k.a. weak partial-collapse), followed by a tuned postselection. We report on an optical experimental implementation of the scheme. We show that our protocol leads to an amplified signal-to-noise ratio (as compared with straightforward strong measurement) when discerning between the two quantum states.

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The notions of "state" and "measurement" were parts of the early framework of quantum mechanics. Even after several decades, these are still active and vibrant concepts for study. In fact, the study of measurements of a quantum system branched-off to many intriguing sub-topics, including: quantum state discrimination [1–5], weak values [6–15], the quantum Zeno effect [16–18], and precision measurements [19, 20], to name a few. The former is of great practical interest in quantum information processing. The ability to optimally discriminate between non-orthogonal quantum states depends on the fidelity of the measurement apparatus and on the amount of prior knowledge we have on the states between which we want to distinguish. Determining whether an unknown state is equal (or not) to a known one requires repeated measurements with a high signal-to-noise ratio (SNR). It may involve, for example, repeated strong projective measurements on replicas of this state. Here we introduce a novel procedure, related to, but differing from the protocol of *weak values* (WVs) [6]. Our protocol is employed to enhance the discrimination-fidelity between two quantum states. We then report on experimental results involving classical light, which demonstrate the practicality of our measurement protocol, denoted "null weak value." The two states to be discriminated are characterized by two different linear polarizations which are rotated with respect to each other over a large range of angles.

A WV protocol consists in weakly measuring a *system* by coupling it to a *detector*, and retaining the detector output only if the system is eventually measured to be in a chosen final state, $|\psi_f\rangle$—*postselection*. The fact that the result of a weak measurement can be anomalously large when correlated with a subsequent strong measurement (postselection) is one of the intriguing properties of this protocol [6]. Recent efforts have employed this concept for amplifying small signals both in quantum optics [10–12, 21, 22] and in solid state physics [13]. The large WV leads to a an amplification of the SNR for systems where the noise is dominated by an external (technical) component [12, 13]. When quantum fluctuations (leading to inherent statistical noise) dominate, the large WV is outweighed by the scarcity of data points, failing to amplify the signal-to-statistical-noise [13, 23]. By contrast, the method presented here leads to high fidelity discrimination between quantum states on the background of quantum fluctuations. Our approach is based on a two-step protocol: first a strong (projective) measurement is performed on the system with small probability. This leads to a partial-collapse of the system's state, and when no collapse takes place, the system experiences a weak backaction [24]. Next, a strong measurement is performed, and the result of the first measurement is weighted conditionally on the outcome of the second measurement.

In a typical state-discrimination scheme, a qubit is prepared in one of two (known a-priori) possible states. The goal is to ascertain in which of the two possible states the qubit is. Here, we consider a more general scheme. We wish to discriminate between a known qubit state $|\psi_0\rangle = \alpha_0 |0\rangle + \beta_0 |1\rangle \equiv \cos[\theta_0] |0\rangle + \sin[\theta_0] \exp[i\phi_0] |1\rangle$, and another state $|\psi_\delta\rangle = \alpha_\delta |0\rangle + \beta_\delta |1\rangle \equiv \cos[\theta_0+\delta_1] |0\rangle + \sin[\theta_0+\delta_1] \exp[i(\phi_0+\delta_2)] |1\rangle$, where δ_1, δ_2 are a-priori unknown. N replicas of each state can be measured, and the respective outcomes are compared.

Let us begin with a straightforward state discrimination protocol, achieved through a standard strong measurement of a qubit, M_s, where the occupation of the state $|1\rangle$ is measured [25]. The probabilities to detect the qubit in $|1\rangle$ in any single attempt are $P(M_{s,\delta}) = \sin^2[\theta_0+\delta_1]$, $P(M_{s,0}) = \sin^2[\theta_0]$) for the states $|\psi_\delta\rangle$, $|\psi_0\rangle$, respectively. We define the signal S to be the difference between the number of positive detections in each case,

$$S = N_{s,\delta} - N_{s,0} = N \sin^2[\theta_0 + \delta_1] - N \sin^2[\theta_0] \ , \quad (1)$$

where $N_{s,\delta} = NP(M_{s,\delta})$, $N_{s,0} = NP(M_{s,0})$. The signal

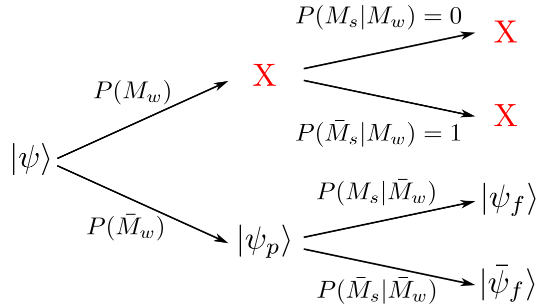

FIG. 1. A tree diagram of the qubit state evolution under subsequent partial-collapse measurements; the respective probabilities are indicated: $P(M_w)$ [$P(\bar{M}_w)$] is the probability that the detector "clicks" [no "click"] upon the first measurement. If it does "click", the system is destroyed, hence there are no clicks upon further measurements [this is marked by a (red) X]. Note that following $P(\bar{M}_w)$ (null detection of the qubit), the weak backaction rotates $|\psi\rangle$ into $|\psi_p\rangle$.

is a function of two variables $S(N_{s,\delta}, N_{s,0})$. The uncertainty in the signal is then given by

$$\Delta S = \sqrt{\left(\frac{\partial S}{\partial N_{s,\delta}}\right)^2 \Delta N_{s,\delta}^2 + \left(\frac{\partial S}{\partial N_{s,0}}\right)^2 \Delta N_{s,0}^2} = \sqrt{N \sin^2[\theta_0 + \delta_1] + N \sin^2[\theta_0]}\,, \quad (2)$$

where for the second equality we assumed Poissonian noise (which is dominant for coherent light experiments discussed below), $\Delta N_{s,\delta}^2 = N_{s,\delta}$ and $\Delta N_{s,0}^2 = N_{s,0}$. The obtained SNR is

$$\mathrm{SNR}_{\mathrm{std}} = \frac{S}{\Delta S} \approx \sqrt{2}\cos[\theta_0]\delta_1\sqrt{N}\,, \quad (3)$$

where the approximation is for $\delta_1 \ll 1$.

We now turn to describe our new measurement protocol (null-WV) (cf. Fig.1). The qubit state is measured twice. The first measurement M_w is a strong (projective) measurement which is performed on the system with small probability. Here the states $\{|0\rangle, |1\rangle\}$ are measured with probabilities $\{p_0, p_1\}$, respectively. For simplicity, hereafter, we assume that only the state $|1\rangle$ is measured with probability $p_1 = p$, $p_0 = 0$. If the detector "clicks" (the measurement outcome is positive), the qubit state is destroyed. Very importantly, having a "null outcome" (no click) still results in a weak backaction on the system. We refer to this stage of the measurement process as "weak partial-collapse". Subsequently the qubit state is (strongly) measured a second time (postselected), M_s, to be in the state $|\psi_f\rangle$ (click) or $|\bar{\psi}_f\rangle$ (no click), where $|\psi_f\rangle, |\bar{\psi}_f\rangle$ form a different qubit basis than $|0\rangle, |1\rangle$. We propose to discriminate between the two possible initial qubit states via repeating the protocol for $|\psi_0\rangle$ and $|\psi_\delta\rangle$ and comparing the respective conditional outcomes of $P(M_{w,0}|\bar{M}_{s,0})$ and $P(M_{w,\delta}|\bar{M}_{s,\delta})$, i.e. [having a click the first time conditional to *not* having a click the second time]. Events in which the qubit is measured strongly (in the second measurement), M_s, are discarded. In other words, we define our signal to be $\tilde{S} \equiv P(M_{w,\delta}|\bar{M}_{s,\delta}) - P(M_{w,0}|\bar{M}_{s,0})$.

Our protocol takes advantage of the correlation between the two measurements. To shed some light on its outcome we calculate explicitly the conditional probabilities following the measurement procedure sketched in Fig. 1. For example, if the first measurement results in a "click" the system's state is destroyed and any subsequent measurement on the system results in a null-result, implying $P(M_s|M_w) = 0$, and $P(\bar{M}_s|M_w) = 1$. This represents a classical correlation between two measurements. By contrast, $P(\bar{M}_s|\bar{M}_w)$ embeds non-trivial quantum correlations. The first partial-collapse measurement of a given preselected state $|\psi_\delta\rangle$ results in the detector clicking with probability $P(M_{w,\delta}) = p\sin^2[\theta_0 + \delta_1]$. If no click occurs [with probability $P(\bar{M}_{w,\delta}) = 1 - P(M_{w,\delta})$], the qubit's state is modified by the measurement backaction into $|\psi_{\delta,p}\rangle = \big[\cos[\theta_0+\delta_1]\,|0\rangle + \sqrt{1-p}\sin[\theta_0+\delta_1]\,e^{i(\phi_0+\delta_2+\phi_M)}\,|1\rangle\big]/\sqrt{P(\bar{M}_{w,\delta})}$, with ϕ_M being the phase accumulated due to the measurement procedure. A second strong measurement, M_s, yields a click [no click] with probability $P(M_{s,\delta}|\bar{M}_{w,\delta}) = |\langle\psi_f|\psi_{\delta,p}\rangle|^2$ $\big[P(\bar{M}_{s,\delta}|\bar{M}_{w,\delta}) = |\langle\bar{\psi}_f|\psi_{\delta,p}\rangle|^2\big]$. Finally, using Bayes theorem, we can write $P(M_{w,\delta}|\bar{M}_{s,\delta}) = P(M_{w,\delta})/[P(M_{w,\delta}) + P(\bar{M}_{w,\delta})P(\bar{M}_{s,\delta}|\bar{M}_{w,\delta})] = N_{w,\delta}/(N_{w,\delta} + N_{p,\delta})$, where the last equality is obtained by taking the measured estimator for the conditional probability [Theoretically $N_{w,\delta} = Np\sin^2[\theta_0 + \delta_1]$ is the number of clicks in the first measurement and $N_{p,\delta} = (1 - p\sin^2[\theta_0+\delta_1])|\langle\bar{\psi}_f|\psi_{\delta,p}\rangle|^2$ is the number of no-clicks in the (second) postselection]. Note that if the detector clicks in the first measurement, the protocol is truncated, and no second step is to be carried out. This finally leads to the signal

$$\tilde{S} = \frac{N_{w,\delta}}{N_{w,\delta} + N_{p,\delta}} - \frac{N_{w,0}}{N_{w,0} + N_{p,0}}\,. \quad (4)$$

In complete analogy with the case of a single strong measurement, we define the uncertainty in the signal

$$\Delta\tilde{S} = \sqrt{\sum_{i=w,p}\sum_{j=0,\delta}\left(\frac{\partial\tilde{S}}{\partial N_{i,j}}\right)^2 \Delta N_{i,j}^2}\,, \quad (5)$$

where $(\partial\tilde{S}/\partial N_w)^2\Delta N_w^2 = [1/(N_w + N_p) - N_w/(N_w + N_p)^2]^2 N_w$ and $(\partial\tilde{S}/\partial N_p)^2\Delta N_p^2 = N_w^2 N_p/(N_w + N_p)^4$.

We focus on obtaining a large

$$\mathrm{SNR}_{\mathrm{NWV}} = \tilde{S}/\Delta\tilde{S}\,, \quad (6)$$

for discriminating between the two states. The $\mathrm{SNR}_{\mathrm{NWV}}$ depends on the choice of reference state $|\psi_0\rangle$ and the postselection basis $|\psi_f\rangle, |\bar{\psi}_f\rangle$. For the purpose of the present theoretical and experimental analysis it is sufficient to discuss the case of states in a plane, i.e. $\delta_2 = 0$,

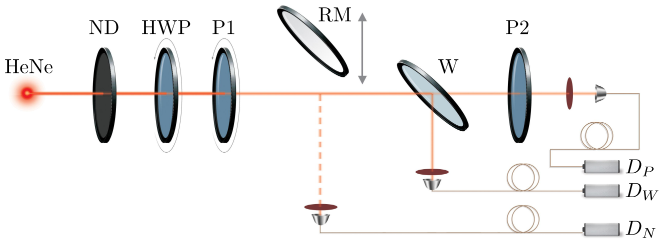

FIG. 2. A sketch of the experimental apparatus. Single spatial mode light from a helium-neon laser (HeNe) passes through a neutral density filter (ND) followed by a half-wave plate (HWP) and polarizer (P1) to prepare the initial state. During data acquisition, the HWP is used to maintain a constant photon flux which is measured using a removable mirror (RM). A glass window (W) weakly reflects vertically polarized light. Photons that pass through the window are then projected onto a linear polarization state with a second polarizer (P2). The photons in each spatial mode are passed through colored glass filters to block background, collected via multi-mode fiber and sent to single photon counting modules (D_N, D_W and D_P).

and $\phi_0 + \phi_M = 0$. We propose two possible measurement schemes for obtaining a large $\mathrm{SNR_{NWV}}$. In the first scheme we choose the postselection such that the reference state satisfies $|\langle\bar{\psi}_f|\psi_0\rangle|^2 = 0$. This means that the reference state $|\psi_0\rangle$ would have always clicked in the second measurement had it not been first weakly measured. This implies $P(M_{w,0}|\bar{M}_{s,0}) \approx 1$ ($N_{p,0} \approx 0$). We call this *scheme* ***A***. Alternatively, in *scheme* ***B***, we choose the postselection such that $|\langle\bar{\psi}_f|\psi_p\rangle|^2 = 0$: the weakly measured and, thus, rotated state $|\psi_0\rangle \to |\psi_p\rangle$ always clicks in the second measurement, $P(M_{w,0}|\bar{M}_{s,0}) \equiv 1$ ($N_{p,0} \equiv 0$). Note that in both schemes $P(M_{w,\delta_1}|\bar{M}_{s,\delta_1}) \sim p\sin^2[\theta_0+\delta_1]/\sin^2[\delta_1]$ for $p \ll \sin^2[\delta_1]$. Thus, $\mathrm{SNR_{NWV}}(N) \sim \sin[\delta_1]/(\sin[\theta_0+\delta_1]\sqrt{p})\sqrt{N}$ which becomes large for $p \to 0$ (weak partial-collapse). This is because the condition $N_{w,0} \gg N_{p,0}$ is satisfied vis-a-vis the null-WV of the reference state. Varying δ_1 such that $|\langle\psi_\delta|0\rangle|^2$ is increased leads to a decrease of $N_{w,0}$ and an increase of $N_{p,0}$. A large $\mathrm{SNR_{NWV}}$ [cf. Eq. (6)] is obtained when $P(M_{w,\delta_1}|\bar{M}_{s,\delta_1})$ crosses to a regime where $N_{w,\delta_1} \le N_{p,\delta_1}$.This happens first with scheme ***A***. Hence, scheme ***A*** produces a larger $\mathrm{SNR_{NWV}}$ for smaller δ_1; scheme ***B*** leads to far larger $\mathrm{SNR_{NWV}}$ for larger δ_1.

We measure the null-WV protocol and its amplified SNR using an optical technique, where the qubits are replaced by photons from a dramatically attenuated coherent beam with measurements performed by single-photon detectors. The experimental setup is sketched in Fig. 2. A linearly polarized, 633 nm helium-neon laser is attenuated to the picowatt level before the preparation of the initial state of the photons. We encode this state in the polarization degree of freedom; this is done by passing the beam through polarizer (P1), giving $|\psi_\delta\rangle = \cos[\delta_1-\theta_0]\,|0\rangle + \sin[\delta_1-\theta_0]\,|1\rangle$ where $\{|0\rangle, |1\rangle\}$ correspond to the horizontal and vertical polarization states, respectively. We perform a partial-collapse (weak) measurement by sending the photons through a glass window (W) set at Brewster angle. The window therefore weakly reflects vertically polarized light, with probability [26] $p = 0.15$, and passes horizontal light with near unit probability. We set the second polarizer (P2) in the transmitted arm to strongly project the photon into the state $|\bar{\psi}_f\rangle$ which is represented by scheme ***A*** or ***B***, as desired. The photons are then detected with single photon counting modules in each port and their arrival times are recorded.

From the recorded arrival times, we can separate photon detection events into time bins and determine the average number of photons N_w and N_p and their variances as we vary the input and post-selection states. For the data included here, we count photons for approximately 5 s, with time bins of 150 μs and 25 ms for the null-WV and standard schemes, respectively. These times are chosen to ensure that each method uses an equal number of prepared photons per measurement. We subtract dark counts, which constitute much less than 1% of the total counts on average.

We consider each scheme mentioned above for $\theta_0 = 0.1$ rad and plot the results in Fig. 3. We find that, for scheme ***A***, we can discriminate between the two states with a higher SNR than the standard scheme over the whole range of angles considered. Similarly, while the SNR of the standard technique nearly coincides with that of scheme ***B*** for small angles, we see that the sensitivity of the two schemes diverge quickly for larger angles; in this regime ($\delta_1 \approx \theta_0$), the null-WV scheme ***B*** is significantly better. The curves represent the expected SNR from the theory above assuming modest errors in the calibration of polarizer angles and reflection/transmission probabilities. The reflection and transmission probabilities for 633 nm light were calculated (and verified experimentally) using the Fresnel equations; the total number of photons per measurement was measured using the removable mirror, including effects from detection efficiencies. The deviation from the data is due to the technical noise present in the detectors (e.g., dark current) which was not included in the theory.

In conclusion we have presented here a new protocol based on a weak partial-collapse measurement followed by a tuned postselection. Our protocol enables one to discern between quantum states with better accuracy than a standard measurement would allow. We demonstrate the feasibility and effectiveness of our protocol by discriminating between different polarization states of light. By contrast to earlier protocols [5] tuned to discriminate between two prescribed states, the present one consists of a two-step correlated outcome measurement. It facilitates the study of an amplified SNR for a wide range

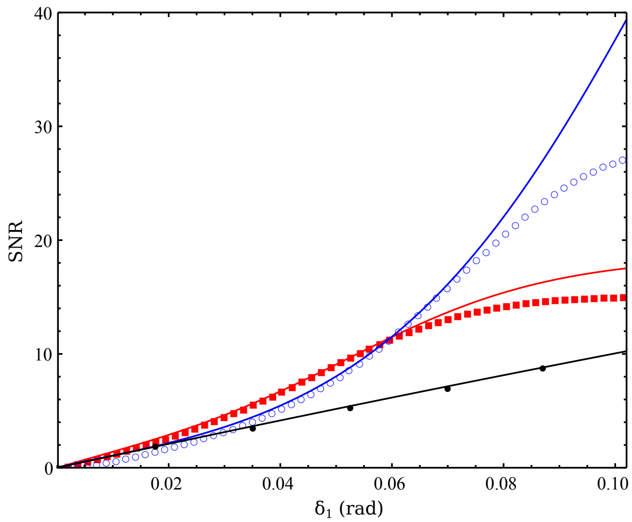

FIG. 3. A graph of the theoretical and experimental SNR obtained for different measurement schemes. Scheme $\boldsymbol{A}$ (red) and $\boldsymbol{B}$ (blue) correspond to the null-WV technique ($\mathrm{SNR}_{\mathrm{NWV}}$). The parameter δ_1 denotes the distance between the measured and the reference state; it is varied by changing the angles for the input polarizer P1. For a given P2 and W (cf. Fig. 2) the reference state is determined by finding P1 for which $|\langle\bar{\psi}_f|\psi_0\rangle|^2$, $|\langle\bar{\psi}_f|\psi_p\rangle|^2$ is minimal for schemes $\boldsymbol{A}$, $\boldsymbol{B}$, respectively. The standard scheme (black) is that defined by Eq. (3), and is represented by a single polarizer with no weak measurement. Dots correspond to calculations from data and lines correspond to the theoretical predictions. Each scheme used approximately the same number of photons, with $N \approx 11250$ per measurement.

of possible polarizations of one of the states, which is not a-priori known. Three features distinguish our protocol from earlier implementations of WVs [10–13]: (i) Here we make use of a partial-collapse measurement, in which the system experiences a weak backaction only for a subset of all possible measurement outcomes; (ii) In conventional WV-amplification procedures one needs to employ two entangled degrees-of-freedom: the "system" which serves as a WV-amplifier and is subsequently post-selected and the "detector". In the present procedure there is a single degree-of-freedom employed; the detector is classical, hence no explicit use of quantum entanglement is required to achieve amplification; (iii) We obtain a SNR amplification versus inherent quantum and statistical fluctuations, and not only against external detector noise.

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[25] For simplicity, we consider here, as reference, a standard strong measurement applied to $|\psi_0\rangle$ and $|\psi_\delta\rangle$ which differ from each other in the oocupation of the state $|1\rangle$. The null-WV protocol, defined below, is useful in discriminating between two states that differ from each other also in the phase ϕ_0 only, i.e. , for which the occupation of $|1\rangle$ is identical.
[26] Photons which are reflected from the front face of the window are collected and counted as N_w. However, photons are also reflected from the back face of the window (with $p = 0.067$), causing an additional backaction on the light which passes through the window. We thus change the definition of our conditional $P(M_{w,\delta_1}|\bar{M}_{s,\delta_1}$ to include the probability of being weakly measured at

the first protocol step (being reflected from the window's front face), conditional on [not being reflected off the back face] *and* [not being being absorbed by P2], i.e. $P(M_{w,\delta_1}|\bar{M}_{s,\delta_1} \wedge \bar{M}2_{w,\delta_1}) = N_w/(N_w + N_p)$.