Freie Universität Berlin · Department of Physics

Basic Laboratory Course in Physics

GPI

Two Semester basic laboratory course for students of Physics, Geophysics, Meteorology and for Teacher Candidates with physics as first or second subject.

Two Semester basic laboratory course for students of Physics, Geophysics, Meteorology and for Teacher Candidates with physics as first or second major.

Aim of the Laboratory Course

Introduction to the fundamental techniques of quantitative experimental- and scientific methods in physics (measurement methods, measurement techniques, documentation, mathematical-statistical und practical evaluation methods / error calculations, critical discussion and scientific conclusion, written report and presentation). Dealing with selected topics in physics in a deeper and complementary way.

Core Rules

- Preparation based on lectures and text books according to information contained in the script.
- The experiments begin c.t. and students arriving more than 15 minutes later will be excluded from taking part.
- The two page introduction (intended as part of the experimental report) is to be presented at the beginning of the experiment.
- The tutor introduces the students to the experiment and makes sure that they are sufficiently prepared and if not, whether the work should be repeated at a later date.
- The experiment and documentation of the results is made as quick as possible under the guidance of the tutor, whereby, time for further discussions of the physical background should be taken into account.
- Evaluation of the experiment by means of tables and graphs takes place after about 3 hours with the help of the tutor. Thereafter, further work is to be done on the report (protocol).
- The 4 hours are to be fully used to complete the protocol and can then only be cut short when the tutor hands out an attestation.
- The total number of experiments (as a rule 11) must be completed within the laboratory course, whereby a maximum of 2 experiments can be repeated at the end of the course.
- Attestations for all experiments must be noted at the latest on the last day of the course, otherwise the course can not be assessed and becomes invalid.

Integration with the Physics Curricula

Two laboratory courses (GP I and II) are scheduled after the respective lecture courses (Physics I and II). Restrictions with respect to the contents of the lectures are unavoidable due to the timescale and the placement of the laboratory course. This is especially evident for students taking part in the vacation laboratory courses where subjects must be handled in advance without prior lecture material (Optics, Atomic Physics Quantum Phenomena).

Organization

Semester Course (weakly, 4 h) and *Vacation Course* (4 weeks, 12 h per week).

Laboratory course in small groups. Pairs of students performing and evaluating an experiment. A tutor assists a group of 3 pairs on the same or related experiments. Good preparation before the experiment is important. A two page introduction to the subject matter is handed out before each experiment and is intended as part of the evaluation .

Course Schedule with Experimental Work, Evaluation and (as a rule) start of the written report (protocol).

Work on the two page introduction to the subject matter (prepared beforehand), presentation of the experimental findings with summary and critical discussion of the results.

Course Material: Description of the experiment (script) containing information on the relevant physics, experimental set-up and the tasks to be performed. Report book for the written experimental protocol – to be bought by the student.

Evaluation

Experimental certificate with grades according to ECTS (European Credit Transfer System). Point system for the individual experiments. No tests or final seminar.

Experiments

Experiments with various grades of difficulty from simple experiments in GP I, to give a basic feeling for the methods involved in experimental physics, to experiments with deeper physical background, which, for a fuller understanding, require higher lecture courses in physics.

Note

A sensitive indicator for physical understanding is the application of gained knowledge. The physical principles and the connections between phenomena should be demonstrated by dealing with the problems involved and by critical observation.

As a part of scientific training, it is not the intention of the laboratory course to only impart „mechanical knowledge“ but it should lead to scientific thinking, i.e., answering questions of a physical nature or drawing conclusions from findings and laws through critical discussions in small groups and final evaluation of the observations and quantitative results.

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BASIC LABORATORY COURSE IN PHYSICS

Introduction to the fundamental techniques of quantitative experimental- and scientific methods in physics: Measurement methods, measurement techniques, documentation, mathematical-statistical und practical evaluation methods (error calculations), critical discussion and scientific conclusion, written report and presentation. Dealing with selected topics in physics in a deeper and complementary way.

Two laboratory courses (GP I and II) scheduled after the lecture courses Physics I and II, however, with reference to the complete material handled in lecture courses Physics I-IV.

Experiments and reports done in team work consisting of a group of 6 (3 pairs) under the assistance of a tutor.

Completion of introductory reports on the subject matter and physical background, presentation of the experimental findings with a summary and critical discussion of the results as an exercise in scientific writing.

Introductory text books provided the basic knowledge in a clear and connected manner, but only in passing, mention the way to the working methods of physics. Physical knowledge comes about either through quantitative observation of the natural processes, i.e., by means of experiments or by mathematical formulations of physical phenomena – theoretical work.

Laboratory courses give a feeling for the experimental methods of physics. The aim of the basic course is to introduce the students to elementary experimental and scientific working methods and critical quantitative thinking. This includes setting-up and conducting an experiment (measurement techniques and methods), documentation, evaluation (error calculations), discussion of the findings and scientific conclusions and finally presentation of the written report.

The basic course intentionally places the **scientific method** in the foreground. The physical questions presented in the course have long been answered, and the experiments are to be understood as providing classical examples for methods and techniques which recur in current research. Yet physics is always behind the work and does not differentiate between simple and difficult. It is the physicist, whether “professional” or in training, who asks the questions and thus determines the standard.

The laboratory course allows the student to tackle the work in an individual way so that the learning process is strongly self determined. Elementary and important prerequisites are curiosity and the ambition to understand.

Error Calculations

A fundamental phenomena of experimental work is the fact that the evaluation of natural processes is never absolute and all results must be considered as approximate. As a consequence, the empirical experimental data must be handled statistically in the form of error calculations.

An important aspect of the laboratory course is to introduce the student to the basic methods of error calculations. The first steps and basic exercises in error calculations are found in Annex I of this script (under the heading „ERROR CALCULATIONS”). (Practical exercises in error calculations will be given out before the laboratory course begins and must be handed in at the date of the first experiment). Learning the skills of error calculations is then the aim of the subsequent experimental work.

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Topics and Experiments

The topics of the laboratory course are coordinated with the contents of the lecture course. The experiments range from simple to demanding.

In some cases, due to organizational problems (especially in vacation courses), the topics handled in the experiments have not yet been discussed in the lectures. This requires intensive self-preparation by the student.

Preparation

Successful experimental work requires good physical- and mathematical preparation using text books and the experimental script. The laboratory course has the specific aim of deepening ones knowledge of physical processes and must be seen as complementary to the material handled in lectures and work done in tutorial exercises.

Report

The written reports serve not only as proof of experimental work but also as an exercise in the method of scientific writing. Contents and form must be such that the interested reader is introduced to the topic and the questions to be answered in an efficient and concise way and is able to follow and understand the work and conclusions. This aspect must be kept in mind and it should not be limited to a mere presentation of measured data and calculations.

Rules of the Laboratory Course

Laboratory Report Book

Laboratory regulations require that all experimental work from description to data recording and evaluation be presented in bound exercise books. Please bring suitable books (DINA4-chequered, no ring bound books) to the course. You should buy 2 – 3 books. Work done on loose or tacked paper leads to uncertainty as to its origin or loss of pages i.e. data.

Additional pages (e.g. graph paper) must be glued to a thin strip of the inside edge of a book page so that both sides of the additional page can be used. Attaching pages with paper clips is not permitted.

Graph Paper

Graphs must be drawn on graph paper (mm paper, log-paper; available in the laboratory).

Written Preparation

A written introduction to the topic and experimental task (as part of the report) must be presented before beginning the experiment. This must be prepared by each student. Since, as a rule, one of the report books of a pair of students is in the hands of the tutor for correction, the affected student must write the introduction on loose paper and later glue it into his/her report book.

The students must be able prove that they have prepared the work through discussions with the tutor.

Insufficiently prepared students will not be permitted to take part in the experiment. The experiment is noted as failed and must be repeated at a later date. If a student is rejected because of insufficient preparation, a colloquium can be set up by the head of the course to test the student. (The rules stipulate that no more than 2 failures are allowed).

Times of the Laboratory Course

The courses begin punctually at 9.15 or 14.15 h.

$3^{3/4}$ hours (9.15-13 h and 14.15-18 h respectively) are set for the work. After the experiment is completed, the remaining time is used to evaluate important parts of the data under the direction of the tutor (e.g., graphical presentations).

Structure and Form of the Report

The report is structured in two sections: Experimental documentation (measurement protocol) and the presentation (basic theory, evaluation, conclusion and discussion). The form is such that an interested reader can follow and understand the contents, results and conclusions (and allows the tutor to make corrections in a reasonable time).

The measurement protocol must be hand written and checked by the tutor for completeness and correctness. Thereafter the tutor gives an attestation. Measurement protocols without attestation will not be recognized.

Handing Over the Report

The reports should be started during the respective experiment and must be handed over at the date of the next experiment.

Failure to hand over the report punctually leads to exclusion from the next experiment.

Missing- and Failed Experiments

If a student misses or is expelled from an experiment then her/his partner must complete the experiment alone.

The excluded partner must repeat the experiment on his/her own at a later date. (The date is set by the head of the laboratory course).

Working in Partnership

Normally students work in pairs, so that each is dependent on other. Work in conjunction with your partner and discuss each experiment so that no problems occur in completing the report and the handing out of attestations.

Attestations; Handing Out the Course Certificates

The handing out of the course certificates only takes place after presenting the complete attestations. Attestations can only be given by the responsible tutor.

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Point System

Each experiment is graded according to a point system. At the end of the course, the summed points serve to measure the total performance according to the rules of the ECTS (*European Credit Transfer System)*.

The grading is given in **% of the maximum number of points.**

[100% – 81%] = A (very good)
[80% – 61%] = B (good)
[60% – 41%] = C (satisfactory)
[40% – 27%] = D (sufficient)
[– 27%] = E (fail)

Each experiment is individually graded, whereby a maximum of 5 points can be given. The performance points for each experiment corresponds to the ETCS grades.

5 - 4.3 points = A (very good)
4 - 3.3 points = B (good)
3 - 2.3 points = C (satisfactory)
2 - 1.0 points = D (satisfactory)
< 1.0 point = E (sufficient)

(successful completion of an experiment requires, as a minimum, a grade of 1 point).

The assessment of the work done is based on the following categories:

A: Basic knowledge and understanding of the physics involved, preparing for the experiment.

B: Experimental ability (practical and methodical work and evaluation).

C: Scientific discussion and report (evaluating the experiment and the results, written report).

The points are noted on the group cards, report book, attestation certificates and the file cards by the tutor.

CONTENTS

GPI

General Information page

Experiments

Annex

REPORT GPI

The report serves as an exercise in scientific writing and presentation. It should, on the one hand, be complete and on the other concise and efficient. As an orientation, refer to the model report below.

The report consists of a *measurement protocol* and *elaboration*:

- The measurement protocol is a documentation of the experimental procedure.

  It must contain all information with respect to experimental set-up, data and observations from which one can completely understand and evaluate the experiment even after the equipment is dismantled.

- Elaboration refers to presentation and communication.

  It contains a short presentation of the basic physics involved and the question posed, evaluation, summary and critical discussion of the results and the scientific conclusions.

One of the most important aspects of a written report is its organization, i.e., how it is structured. The following describes a standard structure obligatory for the laboratory reports.

Measurement Protocol

The measurement protocol is structured as follows:

Title (Experimental Topic)

Name; Date

- Names of the students carrying out the experiment and of the tutor; date the experiment was done.

Experimental Set-Up and Equipment

- Drawing of the set-up; list of the equipment used and equipment data.

Measured Values

- Values with dimensions and units, error limits. Commentary on the error estimates. Data in the form of tables.
- Other Observations.

Elaboration

The elaboration must also be handwritten in the report book (machine written sections or formulae are glued onto the pages of the report). The elaboration is structured as follows:

Title

- (Experimental topic; name of the authors and the tutor; date of the elaboration)

Basic Physics

- A concise presentation of the basic physics with respect to the topic and the questions involved, the measurement method and the equations (copying directly from the literature is not allowed).
- The presentation must give a short but complete overview of the essential aspects of the physical quantities studied and the laws governing them. It is not required to go into details as found in text books.
- A description of the practical experimental methods is out of place here.

Evaluation

- A presentation of the evaluation in graphical form (on graph paper glued onto the appropriate page of the text), evaluated parameters, intermediate results, final results and error limits. Error discussion.
- The derivation of the results must be simple to understand and check (no scribbled notes).

Summary and Discussion of the Results

Concise Presentation:

- What was measured and how the measurements were made?
- What were the results? (clear explanation of the results).

How are the results to be assessed? (critical, qualitative and quantitative comparison of the results, with theoretical expectations, literature values. Physical assessment and conclusions, error discussion).

(1) MODEL REPORT GPI

(2) SPRING PENDULUM

Albert Ach, Paula Puh

(3) Physical Basis

(4) With an ideal spring, the restoring force is proportional to the displacement (Hook's Law):

(5) (A) $F = -D\,x$

(6) The proportionality factor D is called the spring constant. This law is examined in exercise 1.

With (A) and using Newton's Law of motion, we have, where m is the total mass displaced:

(B) $-D\,x = m\,\ddot{x}$

A solution is:

(C) $x(t) = x_0 \cos(\omega t + \Phi)$

where x_0 is the amplitude, ω the frequency and Φ is a phase constant. Substituting (C) in (B) we have for the frequency:

(D) $\omega = \sqrt{\frac{D}{m}}$ and $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{D}}$

The relationship (D) for the period T is examined in exercise 2.

Equation (B) assumes that the total mass experiences the same acceleration. This is not true for the spring itself. At the attachment point, the amplitude and the acceleration are zero. At the free end, they have the values of the attached mass.

(1) The adjacent model report serves as an example for the form and presentation of scientific writing required for the basic laboratory course.

The physical groundwork must be prepared and worked out before the experiment begins. (If the report book is not available, the work must be hand written on loose pages and latter glued in the report book).

(2) Each report begins on a new page commencing with the title of the experiment.

(3) Headings must be used to clearly structure the report.

(4) The presentation of the physical groundwork gives a short and concise introduction to the topic and the questions involved:

Which phenomena or principles are to be studied?

Which measurement methods are used?

(5) The presentation must refer to the subject matter in a short and precise way. Long textbook-type discourses and mathematical derivations of formulae for elementary facts are not required.

(6) The presentation must be independently written. Literature references alone or the word for word copying of text is not permitted.

(7) The mass of the spring is accounted for by an effective mass at the free end which experiences the same acceleration and thus possesses the same kinetic energy as the spring itself.

The velocity at the spring is linear:

$$\text{(E)} \qquad v(x) = v_0 \frac{x}{x_0}$$

The mass distribution along the spring is constant and for a spring element dx we have:

$$\text{(F)} \qquad dm = m_F \frac{dx}{x_0}$$

Therefore the total kinetic energy is given by:

$$E_{kin} = \frac{1}{2}\int_0^{x_0} v^2\, dm = \frac{1}{2}\frac{v_0^2}{x_0^2}\frac{m_F}{x_0}\int_0^{x_0} x^2\, dx = \frac{1}{2}\frac{v_0^2}{x_0^2}\frac{m_F}{x_0}\frac{1}{3}x_0^3$$

$$\text{(G)} \qquad = \frac{1}{3}\left[\frac{1}{2} m_F\, v_0^2\right]$$

i.e., the mass of the spring is taken as a third of the original mass.

(8) Exercises

1. Calculate the spring constant by measuring the displacement.
2. Calculate the spring constant by measuring the period of the spring pendulum.

(7) Special facts and formulas must be explained or derived respectively.

(8) Repeating the purpose of the exercises serves as an orientation and helps to make clear the aim of the experiment.

Measurment Protocol

(9) Paula Puh, Albert Ach;
Tutor: Peter Pi;
(10) 3.3.1981; Begin 10.15 am, End 12.20 pm.

(11) Equipment

Stand with mirror scale (300 mm; scale divisions 1 mm).
(12) Spring with marker and dish (Apparatus 3).
(13) Weights(5/10/20/20´/50 g).
Stop watch (accuracy 0.1 s).
Balance "Sartorius"; (accuracy 0.05 g).

Weights

~~m_5 = 4.99 g~~ (all mass errors with 0.05 g precision)
~~m_{10} = 9.92 g~~
(14) ~~m_{20} = 19.92 g~~

(Measurements discarded because of zero-point readjustment).

m_5 = 5.00 g
m_{10} = 9.90 g
(15) m_{20} = 19.90 g
$m_{20'}$ = 19.95 g
(16) m_{50} = 49.90 g

Mass of spring m_F = 15.15 g

Mass of marker and dish ms = 8.50 g

Measurement of the period (exercise 2)

Amplitude approx. 30 mm.
The period of the unloaded spring could not be measured since it did
(17) not oscillate in a regular manner.
The times were measured at the point were the displacement reverses.
Measurements at intervals of 10 T were made to reduce reaction errors.

(9) The names of the authors and tutor are important in order to know who the report belongs to and who is responsible.

(10) The date is standard information. The time can be important for subsequent discussions on disturbing influences (temperature changes, mains voltage fluctuations, ...).

(11) For the reconstruction of the experiment and the interpretation of the data (e.g., error information) a listing of all the equipment with their important nominal data must be presented (type, manufacturer; error specifications).

(12) All equipment specifications must be noted as given (measuring range, sensitivity coefficients, scale divisions, error information, ...).

(13) Information as to where the experiment was conducted and which devices or probes were used is important for later reconstruction and comparison of results.

(14) Discarded values must be recognizable (e.g. by crossing out), but readable. Do not rub out or otherwise destroy data.

(15) Zero’s are also numbers; e.g. do not write down 5 g for the measurement, but the correct value of 5.00 g. The number of digits in a value contains implicit Information on the accuracy and resolution of the value.

(16) A sketch of the experimental setup is descriptive and helps to understand the connection between the equipment and quantities to be measured. In electrical experiments this is a circuit diagram, in optical experiments the ray path with the position of the optical components as an essential prerequisite for the physical understanding of the measured data.

(17) Write down all considerations and sundry information with respect to the measurements.

(18) Displacement of spring under load conditions (exercise 1)

	Weights	Pos. Marker	M / g	x / mm
(19)	none	2.5		0
	5	17.0	5.00(5)	14.5
(20)	10	31.0	9.90(5)	28.5
	10+5	45.5	14.90(7)	43.0
	20	59.5	19.90(5)	57.0
	20+5	74.0	24.90(7)	71.5
	20+10	88.3	29.80(7)	85.5
	20+10+5	102.5	34.80(9)	100.0
	20+20'	116.0	39.85(7)	114.5
	20+20'+5	131.5	44.85(9)	129.0
	50	145.5	49.90(5)	143.0
(21)	none	2.5		

Period of spring pendulum (exercise 2)

	Weights	10 T / s	M / g	T^2 / s^2
(22)	10	5.4	9.90(5)	0.29(4)
	20	6.2	19.90(5)	0.38(4)
(23)	20+10	7.0	29.80(7)	0.49
(24)	20+20'	7.9	39.85(7)	0.62(5)
	50	8.7	49.90(5)	0.76
	50+10	9.0	59.80(7)	0.81
	50+20	9.7	69.80(7)	0.94(6)

Evaluation

Displacement as a function of load

See figure on the next page: displacement x vs. weight m. The measurement gave the expected straight-line curve with gradient:

$$D \cdot g = (0.345 \pm 0.003)\ \mathrm{kg\ m^{-1}}$$

(18) Each table must have a heading in order to see which measurements are involved.

(19) Do not confuse the units of length mm and cm. Many scale divisions (straight edges, rules, callipers) are calibrated in cm. The scale in exercise 1 has a mm-division.

(20) A scale can be read to an accuracy of better than one scale division (div.) by estimating between two divisions (estimation position; here the position after the decimal point). The error is found from the reading conditions; here the upper limit is taken as the whole scale interval between two divisions (± 0.5 div).

(21) The last measurement was made as a control of the initial value (zero point).

(22) For a better overview, integrate calculated values in the table.

(23) Note all measurements as they are made; hence do not write "15 g" or only the calculated value 14.90 g but which weight was used.

(24) Only cite error calculations and error values without further comment when they are formally calculated according to the error propagation laws and include all initial errors.

(25) To exercise 1: Displacement of a spring under load

(26)

(27)

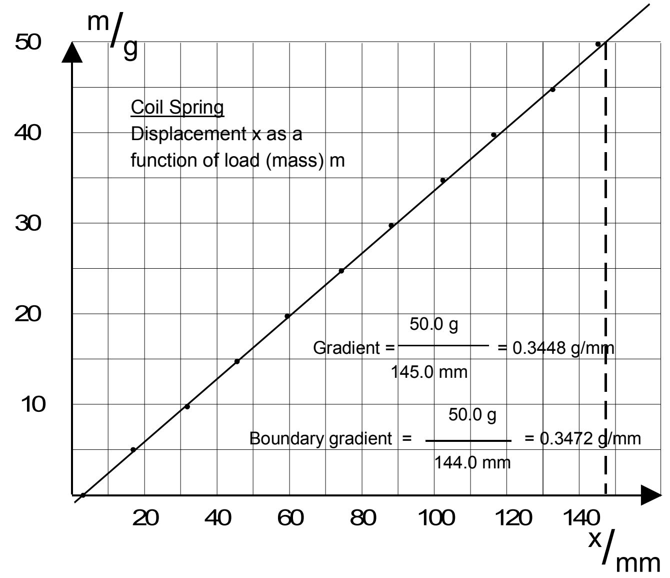
(28)

(29)

(30)

(31)

(32)



To exercise 2: Period of a Spring

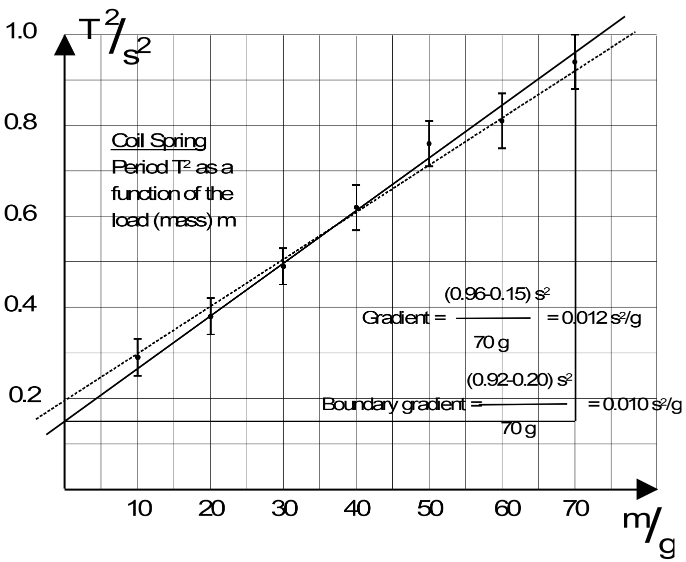
(33)

(34)

(35)

(36)

(37)



(25) See annex II *GRAFISCHE DARSTELLUNGEN UND AUSWERTUNG VON FUNKTIONEN* in this script for information concerning graphical representations.

(26) Graphical representations are to be drawn on original graph paper (mm paper, log-paper; obtained in the lab). (The adjacent graphs are shown only as examples). Graphs drawn for the lab courses are to be glued in the report protocol.

(27) For reasons of accuracy, the graphs shall not be drawn too small; The standard format in the lab course is DIN A4.

(28) For optimal use of the graph paper and for control purposes ensure that the scales selected for the axis are simple and do not contain odd- or complicated divisions (e.g., 1 unit per 30 mm or similar).

(29) Each graph must have a heading (what is represented under which conditions).

(30) The axes must be completely labelled (scale, quantities, units).

(31) All measured points must be shown. Error bars are sufficient for some representative values especially when they are constant.

(32) If the scatter of data is very small, then one must select a more sensitive representation or one must take into consideration a numerical evaluation method. In border-line cases the accuracy of the resolution and the individual values for error estimation must be taken into account.

(33) If variables are transformed, then this must be taken into consideration when labelling the axes.

(34) It is sufficient to only consider one of the two possible boundary lines for error estimation. The errors results from the differences between the lines of best fit and the boundary lines.

(35) Draw the triangles used to calculate the gradients. They should be selected as large as possible (axes intercept points), to minimize drawing and reading errors.

(36) The calculated gradients should be presented in the graph as a check.

(37) When writing down the gradients note that in general these are dimensional quantities and their units must be given.

(38) The errors Δm are negligible, the errors Δx lie at the limit of representation (point diameter).To estimate a boundary gradient an error of 1 mm in displacement was assumed taking into account drawing accuracy and scatter.

(39) With g = 9.8128(1) m/s^2 (value taken from script, error negligible) we find for the spring constant:

(40) $$D = (3.383 \pm 0.024)\ \mathrm{N\ m^{-1}}$$

Exercise 2: Displacement of a spring under load

With the mass of dish and marker m_S, the effective mass of the spring according to (G) and the variable hanging mass m we then have according to (D):

$$T^2 = \frac{4\pi^2}{D}\left(m_S + \frac{1}{3} m_F\right) + \frac{4\pi^2}{D} m$$

The transformed representation of T^2 against m (see previous page) resulted in the expected straight line with the gradient:

$$\frac{4\pi^2}{D} = (12 \pm 2)\ s^2\ kg^{-1}$$

Thus, the spring constant D is:

$$D = (3.29 \pm 0.55)\ \mathrm{N\ m^{-1}}$$

The axis intercept is

$$\frac{4\pi^2}{D}\left(\mathrm{m_S} + \frac{1}{3}\mathrm{m_F}\right) = (0.15 \pm 0.05)\ \mathrm{s}^2$$

Therefore, with the above value for the spring constant, the effective mass of the spring is:

$$m_F = (13 \pm 13)\ \mathrm{g}$$

(38) The error calculation (error propagation, error estimation of parameters) is an integral part of the evaluation. Error values and comments concerning the error calculations belong directly to the results.

(39) Writing down formal error calculations (equations for error propagation) is not required. However, error values must always be explained when individual or local aspects were taken into consideration in the estimation of errors.

(40) Error intermediate values noted during the evaluation must be written down as a two-digit number.

(41) Whereby, we have taken the D-value from exercise 1 because of the better accuracy. The large error results partly from the poor quality of the measured data (error of the axis intercept 34 %), whereby the increase to 100% results from forming the difference in the evaluation the relationship.

(42) In estimating the error, one must essentially take into consideration that the gradient and the axis intercept are correlated. This, however, was left out of consideration here since the error in D does not contribute and just like the mass of the scale was neglected.

(43) Summary and Discussion

(44) The determination of the spring constant of a coil spring by measuring
the displacement as a function of load and by measuring the period
(45) gave comparable values. For the determination of the spring constant
we used the values from the displacement exercise because of their
(46) better values:

(47) $$D = (3.38 \pm 0.03)\ \mathrm{N\ m^{-1}}$$

(48) The accuracy of the determination from the period could have been increased by increasing the number of periods.

(49) The qualitative and quantitative agreement of the measurements among each other and with the expected characteristics demonstrates the validity of Hooke's Law (A) and the law of motion (C) with (D).

In addition, from the measurement of the period as a function of the added weights one was able to determine the contribution of the mass of the spring itself. The result for the spring mass m_F of (13 ± 13) g is formally (still) in agreement with the value of (15.15 ± 0.05) g determined by weighing and at least does not contradict the approach of an effective spring mass of 1/3 m_F. However, the large error must be considered as unsatisfactory since at the other limit it allows the conclusion that the spring itself does not contribute to the mass.

(41) In special cases errors must be discussed. Which errors make the largest contribution? What influences error propagation?

(42) With correlated quantities the maximum error must at least be calculated. (It is better to investigate the correlation).

(43) The summary and discussion must present the subject, aim and results of the experiment in a clear and essential way: What was investigated? How or according to which methods were the measurements made? Which results were found? How are the results to be scientifically assessed?

(44) The results are to be compared qualitatively and quantitatively with one another or with literature values.

(45) Values are in agreement when the error intervals overlap.
Values are compatible when the threefold error intervals overlap. Values are considered significantly different when the threefold error intervals no longer overlap.

(46) A weighted mean value must be given for results with different errors. When one has results with very different errors, the final result is taken as that which is the most accurate.

(47) Results are given with the absolute error as the basis for comparison. The relative error is a measure of the accuracy.

(48) Errors of final results are rounded off and given as a single-digit.

(49) Errors must also be a part of the discussion. How can the accuracy be essentially improved?

STANDARD TEXT BOOKS GPI

The following text books have been selected to provide the basic knowledge as required for the physics course and preparation for the lab work. A number of books were published in different editions and times so that information on the year of publishing was omitted. All books are available in the faculty of physics library.

Compulsory literature

[1]: Gerthsen·Kneser·Vogel;
Physik;

Springer-Verlag

[2]: Bergmann-Schaefer Band 1

[3]: Bergmann-Schaefer Band 2

[4]: Eichler Kronfeld Sahm

Das neue Physikalische Grundpraktikum

Additional literature

Alonso·Finn;
Physik;
Addison-Wesley bzw. Inter European Editions

Atkins;
Physik;
de Gruyter

Kittel·Knight·Rudermann;
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(1: Mechanik, 2: Elektrizität und Magnetismus,
3: Schwingungen und Wellen, 4: Quantenphysik,
5: Statistische Physik);
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Dransfeld·Kalvius·Kienle·Lucher·Vonach;
Physik (I: Mechanik, II: Elektrodynamik,
IV: Atome-Moleküle-Wärme);
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Feynman·Leighton·Sands;
Vorlesungen über Physik (I: Mechanik-Strahlung-Wärme, II: Elektromagnetismus und Struktur der Materie);
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Hänsel·Neumann;
Physik 1-3;
Spektrum Akademische Verlagsanstalt

Kohlrausch;
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Tipler;
Physik;
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Martienssen;
Einführung in die Physik (I: Mechanik,
II: Elektrodynamik, III: Thermodynamik,
IV: Schwingungen-Wellen-Quanten);
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Otten;
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PSSC;
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Pohl,
Einführung in die Physik (1: Mechanik-Akustik-Wärme,
2: Elektrizitätslehre, 3: Optik-Atomphysik);
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Zinth·Körner;
Physik I-III;
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Westphal;
Kleines Lehrbuch der Physik;
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Optics

Born·Wolf;
Principles of Optics;
MacMillan

Fowles;
Introduction to Modern Optics;
Dover Publication Inc.

Atomic- and Quantum Physics

Eisberg·Resnick;
Quatum Physics of Atoms, Moleculs, Solids, Nuclei and Particles;
Wiley & Sons

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Atomphysik;
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Atome, Moleküle, Festkörper;
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Error Analysis

Taylor;
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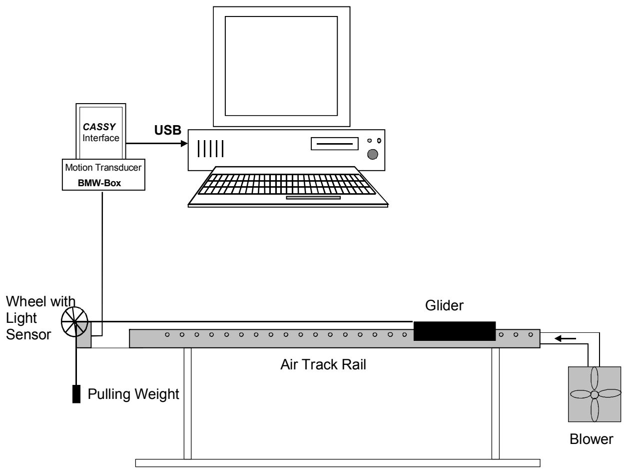
VCH Verlagsgesellschaft

LINEAR MOTION

GPI



Key Words

Newton's equation of motion

Law of motion with constant force and velocity dependent frictional force; friction, eddy current damping.

Aim of the Experiment

To discuss Newton's principle of motion and the equations of motion. Investigating the linear motion of a point mass. Deriving the law of motion (solving the law of motion) for constant and velocity proportional forces.

Observing the influence of friction. Dependence of the friction forces on the variables of motion.

Handling the computer supported experiment- and measuring system Cassy-Lab; see

lab bench script).

Literature

Gerthsen Physik (22. Edition) Chapters 1.3, 1.6, 7.4.4

Exercises

1. Adjusting the air track and the measuring system.
2. Investigating motions with constant force: Recording and plotting distance/velocity/acceleration-time measurements for different combinations of mass (glider) and traction force (pulling weight) and checking the law of motion as a function of these parameters.
3. Investigating motions under the influence of a velocity proportional damping force (magnetic eddy current damping). Calculating the damping constant from the time constant of the motion and from the limiting velocity.

Physical Principles

The glider on the air track is a rigid body, whose movement is fixed by the rail and cannot perform any rotational motion. The motion is thus one-dimensional and fully described by the application of the equation of motion on the center of gravity of the body.

Under the assumption of a constant force ***F***, the derivation of the law of motion by integrating the equation of motion is comparatively simple. Moreover, taking into account an additional frictional force δv proportional to the velocity, the equation of motion becomes:

(1) $$F - \delta v = m a = m \frac{dv}{dt}$$

Integration (after the separation of variables) results in the velocity-time law. Taking initial conditions as $v_0 = v(t = 0) = 0$ gives:

(2) $$v(t) = \frac{F}{\delta}\left(1 - e^{-\frac{\delta}{m}t}\right)$$

Be clear about the statement of this law. What happens in the limiting case for $t \rightarrow \infty$? Discuss the time constant and the limiting velocity.

Presentation of the Physical Principles in the Report

(as preparation for a part of the report): General presentation and explanation of the principles of motion and equation of motion respectively.

The essential contents of the introduction in the report shall be the above derivation of the law of motion under the influence of a constant accelerating force and the frictional forces.

Equipment

Air track with glider and additional weights. M5-nuts as pulling weights to provide a constant traction force. Permanent magnets for eddy current damping.

Motion transducer: Small pulley wheel (with spokes) between a photo sensor; *Cassy-sensor* operating unit, which converts the time dependent pulses at the output of the photo sensor into analog signals proportional to the displacement, velocity and acceleration.

Photo sensor and computer supported measuring system (*Cassy-Lab-System*) for data acquisition (velocity-time data) and evaluation.

Experiment and Evaluation

The velocity-time measurements are made on the air track and evaluated with the *Cassy-Lab.-System*. Information on handling the computer and calling the program are found in the lab bench script. Further information on measurement and evaluation methods, including data output are self-explained through the menu functions.

Exercise 1

Before beginning the experiment, ensure that the air track and motion transducer are horizontally aligned. The glider (without pulling weights) must not move on the air track when the blower is on. One cannot obtain perfect adjustment due to the high sensitivity of the equipment and internal warping of the rail but tries to get the best adjustment possible under the given conditions. Test the setup comprehensively by recording force-free motions (tipping the glider).

Apart from frictional forces, disturbances can also occur through turbulence resulting from too high blower power. The recommended blower settings are found in the lab bench script.

Exercise 2

The mass of the glider is changed by adding weights. Traction forces are implemented by attaching small nuts on a thread running over a pulley wheel; the masses of the nuts are measured on the balance in room 1.02. The experiment is evaluated from the printouts of the $\boldsymbol{s}(t^2)$, $\boldsymbol{v}(t)$ and $\boldsymbol{a}(t)$ curves and additionally, the table of values of $\boldsymbol{v}(t)$ measurements.

Verify the equation of motion by plotting the results of the measurements in a suitable form and discuss the results.

Exercise 3

Eddy current damping of the motion is achieved by placing one or two permanent magnets in a recess in the glider. As a preliminary to the experiment, note the following parameters (mass of the glider, traction force or mass of pulling weight respectively, number of damping magnets).

According to equation (2) the damping constant δ is determined from the limiting velocity and from the time constant of the exponential term. The measurements are evaluated from the printouts of the $\boldsymbol{v}(t)$-diagram and the logarithmic evaluation requires the printouts of the $\boldsymbol{v}(t)$-table.

For the logarithmic presentation and evaluation, the limiting velocity $\boldsymbol{v}_\infty$ must be subtracted from the measured values. A first estimate follows from the data for large time values. Furthermore, the criterion for a consistent value of $\boldsymbol{v}_\infty$ is the linearity of the logarithmic curve. If the values for large times lie too high then too much was subtracted (the estimate for $\boldsymbol{v}_\infty$ too high), if the values lie too low then the converse is true (the estimate for $\boldsymbol{v}_\infty$ too low).

This test is very sensitive and may, under the circumstances, require an iterative procedure (finding the best fit with pencil and rule).

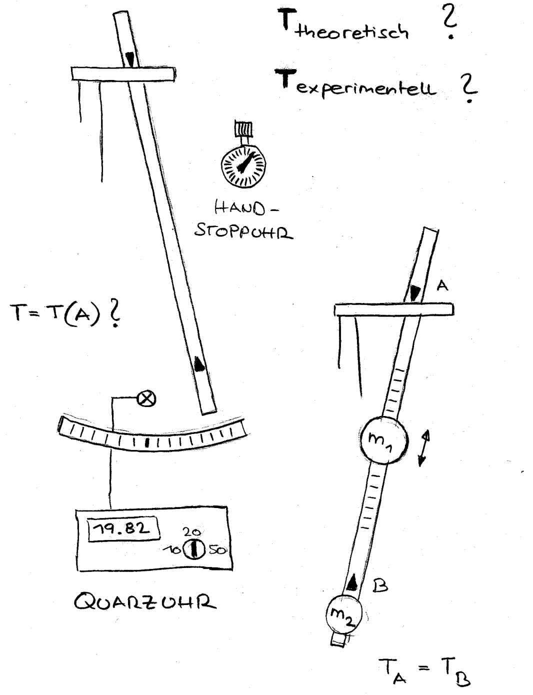
Supplementary Questions

1. To a first approximation, the accelerated mass is that of the glider. How do the following influence the acceleration:

- the pulling weights?
- the pulley wheel?
- the mass and stiffness of the thread?

In the approach to the equation of motion with velocity proportional damping, the traction force is taken as constant. Is this true for the given experimental procedure?

GRAVITY PENDULUM GPI



Key Words

Rotation of rigid bodies; torque;
moment of inertia, **Stein's Theorem**.

Free, harmonic oscillations (rotating pendulum),
Law of motion.

Reduced pendulum length; Reversing pendulum.

Aim of the Experiment

Investigation of the oscillations of a gravity pendulum. Deeper insight into error calculations and error discussions in the framework of the comparatively high accuracy of the apparatus.

Literature

Standard text books (see list).

[1]: Chapter 1.4, 2.2, 2.3

[2]: Chapter 22.1, 22.3

Exercises

1. Measuring the period of a gravity pendulum (peddulum without weights) as a function of the amplitude.
2. Measuring the acceleration due to gravity by the reversing pendulum method (pendulum with additional weights).

Physical Principles

The equation of motion for the rotation of a rigid body about a fixed axis is:

$$M = I\,\frac{d^2\phi}{dt^2} \tag{1}$$

where M is the torque, I the moment of inertia and ϕ angle of rotation. For a (physical) pendulum with a restoring torque $M = -\,s\,m\,g\,\sin\phi$ (s distance from fulcrum to center of gravity, m mass), and under the approximation $\sin\phi = \phi$ one has from (1) a harmonic oscillation with a frequency or period found from the eigenvalue equation of the trial solution:

$$T_0 = 2\,\pi\sqrt{\frac{I}{m\,s\,g}} \tag{2}$$

By introducing the **reduced pendulum length** $L = I/(m\,s)$ one establishes an analogy to the mathematical pendulum:

$$T_0 = 2\,\pi\sqrt{\frac{L}{g}} \tag{3}$$

The solution of the equation of motion with the correct torque $-\,s\,m\,g\,\sin\phi$ leads to a series expansion:

(4)

$$T = T_0\left[1+\frac{1}{4}\sin^2\frac{\phi_0}{2}+\left(\frac{1\cdot 3}{2\cdot 4}\right)^2\sin^4\frac{\phi_0}{2}+\ldots\right]$$

i.e. to an amplitude dependence of ***T***. If only the first term is taken into account and again one approximates $\sin\phi$ by ϕ we have:

$$T = T_0\left[1+\frac{{\phi_0}^2}{16}\right] \tag{5}$$

Reversing Pendulum

For a given pendulum the calculation of the moment of inertia I and the center of gravity distance s can be difficult. One characterizes the **reversing pendulum** as a measurement method or arrangement where the expression $I/(m\cdot s)$ leads back to a simple to measure length.

If one expresses the moment of inertia, according to **Stein's Theorem,** as being composed of a „form component“ I_s (moment of inertia to center of gravity axis) and a „point mass component“ $m\cdot s^2$, then we have from (2):

$$T = 2\,\pi\sqrt{\frac{I_s + m\,s^2}{m\,s\,g}} \tag{6}$$

Equating this relationship for two values s_1 and s_2 allows one to search for two center of gravity distances (fulcrums) for which the pendulum has the same period. One gets a quadratic equation with the solutions:

(7a) $$s_2 = s_1$$

(7b) $$s_2 = \frac{I_s}{m\,s_1}$$

The first solution is trivial and brings no further useful information. If one forms the sum of the distances for the second case one gets the reduced pendulum length L:

(8) $$s_1 + s_2 = \frac{I_s}{m\,s_1} + s_1 = \frac{I_s + m\,s_1^2}{m\,s_1} = \frac{I}{m\,s} = L$$

If one arranges two fulcrums on a line through the center of gravity of the pendulum, each with a different distance to the center of gravity, so that both fulcrums give the same period, then the distance between the fulcrums is equal to the reduced pendulum length ***L***.

Presentation of the Physical Principles

(As preparation for part of the report): Presentation of the solution of the equation of motion in the approximate form for a linear restoring moment.

Calculation of the moment of inertia of a linear mass distribution and the proof for exercise 2.

Apparatus and Equipment

Pendulum (steel bar, ca. 1.70 m long) with two symmetrically arranged fulcrums (edge blades) and additional weights (see title page).

Hand stopwatch (resolution 1/100 s). photo-sensor controlled quartz timer (accuracy 10^{-5}, resolution 1/100 s).

Experiment and Evaluation

To Exercise 1

For comparison purposes, the measurements must be carried out with a hand stopwatch as well as with the quartz timer. In both cases, the accuracy and the reproducibility must be investigated and consideration given to the extent of the measurements (number of periods). The maximum swing amplitude (at the end of the pendulum is) approx. 30 cm.

When investigating large amplitudes please displace the pendulum carefully to avoid collisions with other parts of the apparatus!

Evaluation is made by plotting the period against the square of the amplitude.

The calculation of the period follows from the geometric dimensions and the acceleration due to gravity g found in the lab bench script with the application of **Stein's Theorem**. The pendulum is idealized as a linear body. In addition, the given data can be used to take into account the notches in the bar and the edge blades. Compare and discuss the theoretical- and experimental results.

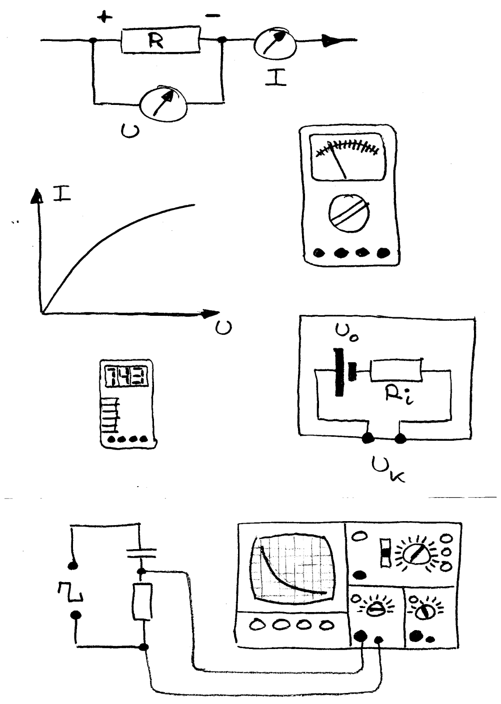
To Exercise 2

The measurement requires that the weights be attached to the pendulum bar. The larger weight must be positioned between the edge blades, the smaller weight outside the edge blades so that the red arrow mark points to the notch mark A. In this form of the pendulum, the reduced length is given by the distance between the edge blades (L = 0.9941 m ± 0.0002 m), however, the position of the center of gravity can be changed, especially by moving the larger mass.

An asymmetric search for a position with equal period for both fulcrums is unsatisfactory for two reasons. Firstly, a search for the correct position is a matter of chance and successive approximation. Secondly, all measurements needed to reach this position would no longer contribute to the result and its accuracy. It is far more advantageous to measure the period as a function of the position of the larger weight successively for both fulcrums and to plot the results in a common diagram. One should get an intercept point $T_1 = T_2$, whose position and accuracy is determined by the totality of all measurements.

ELECTRICAL MEASUREMENT TECHNIQUES

GPI



Key Words

Current conduction; Current (charge current); Electric voltage. *Kirchhoff's* laws. Resistance; *Ohm's* law.

Electrical work and power.

Wheatstone Bridge; Compensation measurement.

Internal resistance of measuring equipment and current sources.

Alternating current; RMS-values. Impedance (alternating current resistance).

Phase shift and alternating current power.

Capacitor and capacitance; Coil and self-induction.

R-C circuit (time constant) and *L-C* circuit (natural frequency, damping constant).

Aim of the Experiment

Electric circuits and measuring equipment and the use of electric quantities are elementary aids in all fields of physical experiments and the associated fundamental principles of measuring techniques can not be restricted to the theory of electricity. The main aim of the experiment is to provide an introduction to measuring techniques with the basic quantities, voltage, current and resistance and to get acquainted with the respective equipment. Furthermore, the aim is to communicate, in a logical manner, simple models of the current conduction mechanism.

The concept of the experiment is mainly to get practice in working with experimental- and measuring equipment. A deeper understanding of the basics of some of the electrical circuits is provided in later experiments.

Literature

Standard literature (see list of the standard text books).

Exercises

The exercises are ordered according to subject matter. The requirement is to work on 4 exercises from different subject areas; two from A-D and two from E-G.

Subject area A (Current, voltage and resistance; Kirchhoff's laws. Internal resistance of measuring equipment. Voltage- and current divider; Voltage- and current sources. Internal resistance. Power matching):

A1. Setting up, measuring and calculating the value of the resistance of a "cube-circuit" comprising 100 Ω resistors with "diagonal" current flow.

A2. Plotting the characteristic curve of a voltage divider under load conditions.

Subject area B (Voltage and current sources. Internal resistance. Power matching):

B1. Measuring the internal resistance of a battery and the apparent internal resistance of a constant voltage source (power supply unit).

Subject area C (Bridge methods):

C1. Measuring a very small resistance (copper wire) with a bridge circuit and determining the specific resistance of copper.

Subject area D (Current conduction mechanisms. Characteristic curves):

D1. Plotting the characteristic curve of a metal conductor (lamp bulb) or a semiconductor (NTC-resistor).

D2. Measuring the temperature dependence of the resistance of a metal or semiconductor. Calculating the temperature coefficient (in the case of a semiconductor, from the logarithmic plot of the measurement). Determining the mean energy difference between valence- and conduction electrons in semiconductors.

Subject area E (Frequency generator, Oscilloscopes; Characteristics of alternating currents):

E1. Observing an alternating-, a triangular- and a square-wave voltage with the Oscilloscope and at the same time making root-mean-square (rms) measurements of the voltages with a multimeter. Compare the different results of the voltage values (amplitude, rms values) and the frequencies with each other.

Subject area F (Capacitor and Coil):

F1. Setting up an *R-C* circuit. Periodic excitation with a square-wave voltage and recording the charge-/discharge curve. Checking the expected functional dependence and calculating the time constant. Compare with the expected value from the values of the capacitance and resistance. (Additional exercise: Observing the discharge curve of the R-C circuit with very small and very large resistance and interpreting the results).

F2. Setting up an *L-C* circuit. Periodic excitation with a square.wave voltage and recording the oscillation curve. Checking the expected functional dependence and calculating the natural frequency and damping constant. Compare with the expected value from the values of the inductance and resistance of the coil and the capacitance of the capacitor.

Subject area G (Alternating current resistance and alternating current power):

G1. Determining the alternating current resistance of a capacitor as a function of the frequency. Checking the functional dependence of the frequency and determining the capacitance of the capacitor.

G2. Determining the alternating current resistance of a coil as a function of the frequency. Checking the functional dependence of the frequency and determining the inductance (self-induction coefficient) of the coil.

G3. Setting up an *R-C* voltage divider (high-pass/low-pass). Measuring the divided voltages as a function of the frequency and determining the capacitance of the capacitor from the transfer frequency.

Physical Principles

Detailed presentations on the subject matter are found in all basic text books on physics. In the following we will present a short discussion on the relationships required for the calculations in the exercises.

Resistance *R*

The (direct current) *resistance R* is defined as the ratio of the voltage *U* to the charge current strength *I* in a conductor:

$$R = \frac{U}{I} \quad \left(= \frac{|U|}{|I|} \right) \tag{1}$$

Relationship (1) defines and is used for calculating the positive definite quantity "resistance". It is not a law giving statements on the quantities involved or the functional dependence of resistors as given by *Ohm's law*.

For a homogeneous conductor, the resistance is proportional to the length ℓ and inverse proportional to the cross-sectional area *A*:

$$R = \rho \cdot \frac{\ell}{A} \tag{2}$$

The (temperature dependent) material constant ρ is called the *specific resistance*.

Ohm's Law

Ohm's law specifies the idea of a constant resistance:

$$R = const \tag{3}$$

In general the resistance depends in a complicated way on the voltage, the current, the temperature or other parameters and only as a limiting case do metals at (not too low) and constant temperature follow Ohm's law.

Kirchhoff's Laws

Currents and voltages in branched circuits are described by the *Kirchhoff's laws* for the elementary components of *nodes* and *closed loops* making up a network:

In each *node*, the sum of all currents vanishes:

$$\sum I_k = 0 \tag{3a}$$

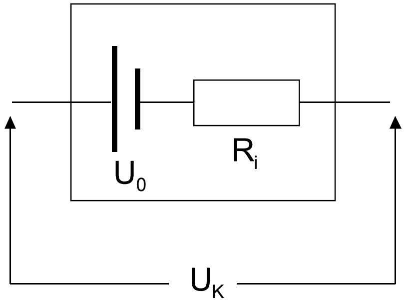
In each *closed loop* in a circuit the sum of all voltages is zero:

$$\sum U_k = 0 \tag{3b}$$

From these laws one can derive the rules for the series- and parallel connection of resistors.

Internal Resistance of Voltage- and Current Sources

When doing calculations on circuits one must take into account that real voltage- and current sources possess a finite *internal resistance* across which a part of the source voltage U_0 (*primary voltage, electromotive force*) drops under load conditions, so that the output voltage U_K (*terminal voltage*) drops depending on current. The actual behavior of real circuit elements or components must, in the model picture, be substituted by an *equivalent circuit* diagram:



Internal Resistance of Measuring Instruments

Measuring instruments for voltage and current also have their own (finite or non-vanishing) internal resistance and in some cases must be taken into account during measurements.

In classical moving-coil measuring instruments (analog multimeters) the internal resistance (resistance of the moving coil plus the series resistance when measuring voltages or shunts when measuring currents) depends on the respective measuring range.

Digital multimeters have, in general, a constant internal resistance when measuring voltages (input resistance of the measuring circuit; typical 10 MΩ), but different shunt resistances when measuring current.

Bridge Methods

Bridge circuits can be used to increase the accuracy and sensitivity of measurements or to lessen the effect of the measurement on the test object (Wheatstone bridge, compensation bridge). In the present case, a very small resistance is to be measured (whereby the current-voltage measurement is difficult in practice) with the aid of a bridge circuit containing suitable reference resistors (see block schematic of the circuit under *Experiment and Evaluation (Subject Area C)*).

Relationships between Current and Voltage for *R*, *C* and *L*

From the definitions of *R*, *C* and *L*, the general relationships between voltage and current are:

$$U_R = -R \cdot I \tag{1}$$

$$I_C = -C\frac{dU}{dt} \tag{2}$$

$$U_L = -L \cdot \frac{dI}{dt} \tag{3}$$

The signs arise under the additional consideration of the orientations of the quantities, whereby "cause" und "effect" are in the opposite direction (*Lenz's rule*). With respect to the sign convention of a positive current from plus to minus "flowing out of a source" a negative current flows through the resistor from minus to plus "into the resistor" and is connected with a proportional (counter-) voltage (the voltage drop). At the capacitor a (negative) current "into" the capacitor leads to a build up of (counter-) voltage and vice versa. An increase in current in a coil leads to an induced (counter-) voltage and vice versa.

The resistance *R*, capacity *C* and the self induction coefficient *L* as formulated above characterize ideal resistors, capacitors and coils. Real circuit elements only behavior approximately as *R*, *C* and *L* and in particular with coils one must, even with the simplest consideration of coil resistance and other effective resistances using an *equivalent circuit diagram*, take into account an additional *R*.

Impedance (Alternating Current Resistance)

The *Impedance Z* (ac resistance) for sine-wave voltages and currents is defined as the ratio of voltage- to current amplitude:

$$Z \equiv \frac{U_0}{I_0} \tag{4}$$

Alternating Current Power

The average time-value of the alternating current power is given by:

$$P = U_0 \cdot I_0 \cdot \cos(U, I) \tag{5}$$

Presentation of the Physical Principles

(as preparation for part of the report): A short description and discussion of the selected circuits with details of the required basic relationships and derivation of the equations for the measurements.

Apparatus and Equipment

Plug-in circuit board (experimental board). Cables.

Voltages sources (power supply unit, function generator).

Various resistors and circuit components in plugable modules, on *plug-in boards* or in a temperature bath.

Fluorescent lamp ("energy saving lamp") with terminals for voltage- and current measurement.

(Digital-) multimeter to measure various electrical quantities. Oscilloscope.

Experiment and Evaluation

With respect to the experiment and to the measuring equipment note the information in the lab bench script! In particular, the error data of the individual pieces of equipment are noted in the script.

The limit values for Current and Voltage of the Components must be kept to!

It is essential that the power supply units labeled with "Strombegrenzung 200 mA" (limiting current 200 mA) are used. On the lab benches other similar power supply units are available which are not limited to this current and can deliver 2.5 A. Here the danger exists that components may become overloaded and get damaged due to an unintentional circuit fault.

Subject Area A

To Exercise A1

Resistors are connected together to form a „cube“. When calculating the expected total resistance, make use of the symmetry properties of the circuit.

To Exercise A2

A 10-step rotary potentiometer is used as a voltage divider. The load resistor is present on the plug-in circuit board with the *unknown resisto*r (≈ 200 Ω; the exact value is to be measured with a multimeter).

Subject Area B

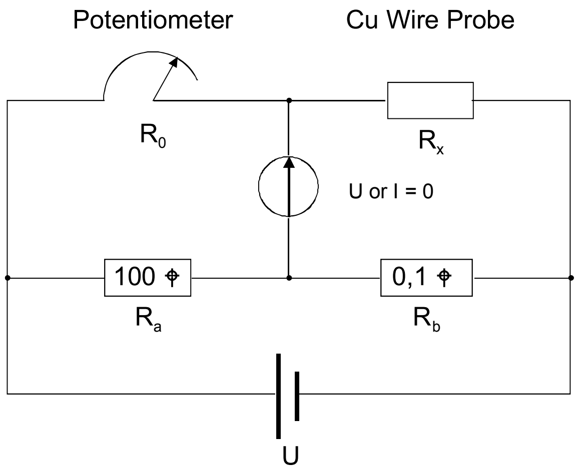
To Exercises B1 and B2

Plot the voltage current characteristic curve of a 1.5 V battery (monocell) and the power supply unit. The load is provided by five *resistors* (3.3 - 56 Ω) on a small circuit board. To measure the characteristic curve of the power supply unit it is advantageous to use an open circuit voltage of just under 2 V (for optimal utilization of the resolution of the multimeter).

Subject Area C

To Exercise C1:

The measuring bridge is formed by a fixed voltage divider in one branch of the circuit and the copper wire and the 10-step rotary potentiometer (1 kΩ) in the other branch. A digital multimeter is used as the zero instrument.



Which currents or voltages are to be expected when the bridge is not compensated? Take note that because of the low-ohm arrangement, high currents can destroy the potentiometer!

Determine on hand the result the specific resistance of copper and compare this value with the literature value!

Subject Area D

To Exercise D1

The components consist of a 24 V lamp and a NTC-resistor (negative temperature coefficient) on a small circuit board. Think about a clever use of the power supply unit in case surprising results are observed with the NTC-resistor!

To Exercise D2

Observation of the temperature dependence of the (specific) resistance can be made by placing probes (copper wire, semiconductor material) in a heat bath up to ca. 100 °C and directly measuring the resistance value with a multimeter.

For copper one should determine the linear temperature coefficient of the resistance by graphical means. For the semiconductor probe, the energy gap between valence- and conduction electrons (specified in J and eV) can be determined by evaluation of a half-log plot of the resistance against 1/T.

Subject Area F

To Exercise F1

The *R*-*C* circuit is formed by a capacitor (0.1 μF / labeled RCK), a resistor (18 kΩ) and the function generator connected in series and excited with a square-wave voltage (≈ 50 Hz). The current $I = f(t)$ can be observed with the oscilloscope as a voltage drop across the resistor. Optimize the settings at the oscilloscope and make a table of *t*-*U* values. For evaluation make a suitable linearization of the measurements.

In addition, observe the current in the circuit when a small resistor (1 kΩ) is used and the voltage at the capacitor when a large resistor (1 MΩ) is used and interpret the results.

To Exercise F2

The *L*-*C* circuit is formed by the parallel connection of a coil (labeled ZAS), a capacitor (1 μF / labeled LCK), the function generator and the oscilloscope to observe the wave forms. The circuit is excited with a square wave voltage (≈ 50 Hz). The period is evaluated by measuring the time for as many periods as possible; to evaluate the damping constant, the amplitude values can be more exactly measured by shifting the curve to the fine scaled middle line on the screen of the oscilloscope.

In addition, make a drawing of the complete voltage spectrum.

Subject Area G

To Exercise G1 and G2

Use the 1 μF capacitor or the coil ZAS. Think about a suitable gradation of the frequencies to measure the frequency dependence of the alternating current resistance of the capacitor. Assess the measurements qualitatively (expected functional behavior) and quantitatively (parameter *C* or *L*) by appropriate graphical presentations.

To Exercise G3

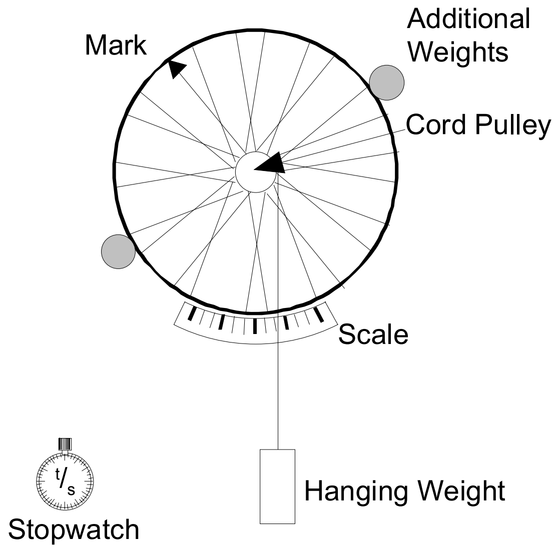
Make a voltage divider circuit using the 0.1 μF capacitor and the 18 kΩ resistor and measure the voltages across *R* and *C* as a function of frequency. Note in particular the pair of values where the voltages at the resistor and capacitor are equal (transfer frequency). Plot the values on double log paper and calculate the capacitance of the capacitor from the transfer point. Compare this value with a "direct measurement" using a multimeter.

Supplementary Questions

How large is the mean drift velocity of conduction electrons when a current of 1 A flows through a copper wire of 1 mm diameter (conduction electron density ca.1 per atom, i.e., 10^{29} m^{-3}). How large in comparison is the mean thermal velocity of the conduction electrons when these are considered as a classical gas? (Due to the *Pauli Exclusion Principle*, the thermal velocities lie actually an order of magnitude above this estimation).

ROTATIONAL MOTION

GPI



Key Words

Rotational motion of rigid bodies.

Torque; Moment of inertia, *Steiner's theorem*.

Equation of motion.

Frictional forces.

Aim of the Experiment

Investigating the equation of motion for the rotation of rigid bodies about a fixed axis; testing *Steiner's theorem*; investigating the contributions of friction.

Introduction to elementary measurement- and evaluation techniques and error calculations.

Literature

[1]: Chapter 2.2, 2.3

[2]: Chapter 10.2, 10.3

Exercises

Investigating uniformly accelerated motion

- Measurement of distance-time dependencies,
- Measurement of torque-time dependencies and
- Measuring frictional losses

for different moments of inertia (with and without additional masses).

1. Qualitative and quantitative testing of the equation of motion. Measuring the time dependence of the angle of rotation (with fixed torque) and the time dependence of the torque (with fixed angle of rotation). Determining the moment of inertia (with and without additional masses) of the wheel from the measurements and comparing the results with the calculated values from *Steiner's theorem*.
2. Discussing the influence of friction and frictional models (dependence of frictional forces and moments of friction on the various parameters of motion) from the results of the measurements.

Physical Principles

For the rotational motion of rigid bodies about a fixed axis, the equation of motion is analogous to that for translational motion

$$\vec{M} = I \frac{d\vec{\omega}}{dt} \tag{1}$$

with the torque $M = r \times F$, the moment of inertia I and the angular velocity $\omega = d\Phi/dt$. The general formulation of (1) with angular momentum L is:

$$\vec{M} = \frac{d\vec{L}}{dt} \tag{2}$$

from which follows the conservation of angular momentum for a closed system ($M = 0$). The moment of inertia is defined as:

$$I = \int_V r^2 dm \tag{3}$$

and can be calculated for homogeneous and symmetric bodies by a suitable decomposition in volume elements dV and the transformation dm = $\sigma \cdot$dV. If the axis of rotation lies a distance a from the center of gravity, one obtains as the moment of inertia *(Steiner's theorem)*:

$$I = I_s + m\,a^2 \tag{4}$$

where I_s is the moment of inertia of the body with respect to an axis parallel to the axis of rotation and passing through the center of gravity.

The law of motion for the time dependence of the angle of rotation follows from the integration of the equation of motion for a constant torque with the initial conditions ω_0 and Φ_0 :

$$\phi = \frac{M}{2\,I} t^2 + \omega_0\, t + \phi_0 \tag{5}$$

Friction

Friction occurs through interactions in the microscopic boundary or "contact" regions on the surface or inside materials (internal friction in fluids). The actual processes are often complex and difficult to describe analytically. They are then taken into account by a suitable hypothetical assumption (constant *static friction*, velocity proportional friction, dependence on the square of the velocity, ...).

With the bicycle wheels used for this experiment, bearing friction and air friction occur, whereby their influences in the framework of the given measuring accuracy (reading errors when measuring rotation angles, subjective errors when making time measurements with a hand stopwatch) with respect to the kinematics, lie at the detection limit and suitable measuring- and evaluation methods must be employed.

With the simple (but unrealistic?) assumption of a constant *moment of friction* M_R one has the possibility of varying the torque and plotting the data, suitably linearized, so that an additional constant portion of the torque can be recognized as an axis intercept. The law of motion (5) gives (for a fixed angle of rotation Φ_n and with $\omega_0 = \Phi_0 = 0$ as the linearized function $M(1/t^2)$:

$$M = M_R + 2\,I\,\phi_n\,\frac{1}{t^2} \qquad (6)$$

The influence of friction is made clearer when considering the energy loss of the hanging weight when the cord again rises after reaching its end point, whereby the initial height would again be attained if no friction were present. From the measurements of this energy loss (the difference in height of the hanging weight) as a function of the other parameters of motion, one can deduce more exact statements concerning the contribution of friction.

Representation of the Physical Principles

(as preparation for part of the report). Present a statement and short discussion of the equation of motion and its solution for the present case (uniformly accelerated rotational motion of a rigid body about a fixed axis).

Derivation of *Steiner's* theorem. Calculating the moment of inertia of a rigid cylinder.

Apparatus and Equipment

Rotating body (bicycle wheel with a pulley on the axel; see figure on the title page). cord, attached weight to provide a constant torque. Additional weights on the wheel to provide a defined changed of the moment of inertia. Set of weights; Balance.

Hand stopwatch (1/10 s).

Metal rule, sliding caliper.

Experiment and Evaluation

General Information

A small offset weight is attached to a spoke of the wheel to compensate for unbalance. If necessary, remaining unbalance is taken into account by suitable measurement techniques (measuring in both directions of rotation and averaging).

When measuring with additional weights, take note of the geometrically correct attachment of the weights If need be, also take into account the weight of the spring at the end of the cord and the diameter of the cord. (To select the number and position of the measuring points see also *GRAFISCHE DARSTELLUNGEN UND GRAFISCHE AUSWERTUNG VON FUNKTIONEN* in the annex to the GPI script).

The arrow marks at the wheels are fixed so that when the cord is completely unwound (reversal point of the hanging weight) the arrow points to the zero mark on the scale. Fractions of a rotation during reverse motion must be estimated.

To Exercise 1

Four series of measurements are to be made

$$t\,(\phi;\ M,\ I_0),$$
$$t\,(\phi;\ M,\ I_0 + I_z),$$
$$t\,(M;\ \phi_n, I_0),$$
$$t\,(M;\ \phi_n, I_0 + I_z)$$

and evaluated by plotting in a linear representation Φ against t^2 and M *against* $1/t^2$ respectively (determination of the moment of inertia from the slope of the straight line curves; determination of a constant fraction of the moment of friction).

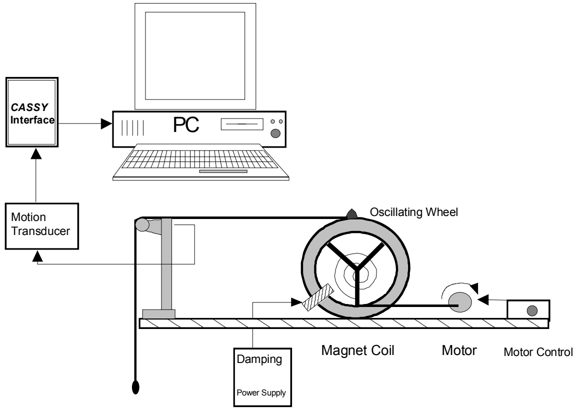
To Exercise 2

Calculation of the work done by friction for the individual measurements from the difference in height of the hanging weight after the weight rises and comparing and discussing the result with the other experimental data (total weight of the wheel or load force on the axel; mean velocity) in the frame work of simple models of friction.

Supplementary Questions

1. The accelerated falling mass does not act with its full weight on the pulley of the wheel. Why not? What does the complete equation of motion look like and how can one estimate the size of the expected maximum error?
2. How can one explain the difference between *static* and *dynamic* unbalance using a simple example of a rigid body (ideal dumbbell)?
3. How can one take into account the reversal of the hanging weight when considering the work done by friction?

HARMONIC OSCILLATIONS GPI



Key Words

Free and damped oscillations

Forced oscillations; Resonance behavior (absorption) and phase shift (dispersion).

Aim of the Experiment

Mathematical treatment of forced oscillations. Mathematical approach and stationary solution of the equation of oscillation. Resonance curve; Frequency dependence of the amplitude (absorption) and the phase shift (dispersion). General solution of the equation of oscillation and observation of the swinging behavior.

The experiment aims to consolidate and deepen ones physical understanding of forced and free oscillations using a classical teaching model of an oscillator (*Pohl's Wheel*).

Literature

[1]: Chapter 1.4, 4.1.2, 4.1.3

[2]: Chapter 20.3, 20.4, 21.1, 21.2

Exercises

1. Investigation of free and damped oscillations. Recording the amplitude as a function of time. Determining the eigenfrequency and the damping constant of the system.
2. Investigation of forced oscillations. Recording the amplitude as a function of time. Determining the eigenfrequency and the damping constant.
3. Qualitative observation of the phase shift between exciter and oscillator as a function of the excitation frequency.
4. Observation of the swinging behavior for the resonance case and for an excitation frequency close to the resonance.

Physical Principles

Oscillation phenomena are elementary features of nature. The *harmonic oscillator* is the simplest model of a system as the basis for the understanding of oscillating phenomena. A harmonic oscillator can be mathematically analyzed without too much difficulty and experimentally realized to a good approximation.

The Harmonic Oscillator

The prerequisite for harmonic oscillations is a system possessing two degrees of freedom for the energy and which is bound to a state of rest by a linear force law (linear restoring force). The following presentation gives a formal treatment of the problem with the solution of the equation of motion in the foreground. Consideration is given to a one-dimensional system with $x(t)$ as the variable (amplitude). The derivation is representative for the solution of analogous differential equations found in similar problems in physics (see experiments *ALTERNATING CURRENT CIRCUITS* and *COUPLED OSCILLATIONS*).

Free (undamped) Oscillations

The equation of motion for a linear restoring force is:

(1) $$m\,\ddot{\boldsymbol{x}} + D\,\boldsymbol{x} = 0$$

Differential equations of this type (*linear differential equations with constant coefficients*) are solved mathematically by complex exponential functions. As a trial solution select

(2) $$\boldsymbol{x}(t) = \boldsymbol{x}_0 \boldsymbol{e}^{i\,\omega\,t}$$ (where $\boldsymbol{x}(t)$ is a complex quantity)

Setting this function and its second derivative in (1) results in the *characteristic equation for the frequencies*. As a solution we find:

(3) $$\omega^2 - \frac{D}{m} = 0$$ or

$$\omega = \sqrt{\frac{D}{m}} = \pm\,\omega_0$$

Two *special* solutions with the frequencies + ω_0 and - ω_0 are obtained. These are termed the eigenfrequencies of the system.

The general solution of the differential equation is made up of the sum of both special solutions with two arbitrary constants, which may also be complex:

(4) $$\mathbf{x}(t) = \mathbf{C}_1\,e^{\,i\omega_0 t} + \mathbf{C}_2\,e^{-i\omega_0 t}$$

Finally the mathematical result must lead back to the physical problem. As a physical solution one takes from (4) those functions which are real for all times t. Since the exponential expression are complex conjugates, $\boldsymbol{x}(t)$ is real when $\mathbf{C}_1$ and $\mathbf{C}_2$ are also complex conjugates. Writing

(5) $$\mathbf{C}_1 = \frac{1}{2} A\,e^{\,i\beta}$$ and $$\mathbf{C}_2 = \frac{1}{2} A\,e^{-i\beta} = \mathbf{C}_1^*$$

it follows that:

(6)
$$x(t)=\frac{1}{2}A\left[e^{\,i(\omega_0 t+\beta)}+e^{-i(\omega_0 t+\beta)}\right]$$

or

(7)
$$x(t)=A_0\cos\left(\omega_0 t+\beta\right)$$

Writing the constants in the form

(8)
$$\mathbf{C}_1=\frac{1}{2}\left(A_1-\mathrm{i}\,A_2\right) \quad \text{and} \quad \mathbf{C}_2=\frac{1}{2}\left(A_1+\mathrm{i}\,A_2\right)=\mathbf{C}_1^*$$

gives:

(9)
$$\mathrm{x(t)}=\mathrm{A}_1\cos\omega_0\,\mathrm{t}+\mathrm{A}_2\sin\omega_0\,\mathrm{t}$$

Both formulae (7) and (9) of the equation of motion are equivalent. The constants ***A*** and ß or $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$ are determined by the initial conditions of the system $\boldsymbol{x}(t = 0) = \boldsymbol{x}_o$ and $\dot{\boldsymbol{x}}(t=0)=\dot{\boldsymbol{x}}_0$ and can be described by them. Thus the solution (9) takes on a clearer form.

Free, Damped Oscillations

The equation of motion, taking into consideration a velocity proportional frictional force is:

(10)
$$\mathrm{m}\cdot\ddot{\mathrm{x}}+\mathrm{k}\,\dot{\mathrm{x}}+\mathrm{D}\,\mathrm{x}=0$$

Using the same trial solutions as in the undamped case, one again gets two *particular* solutions with two frequencies. The trial solution (2) leads to a characteristic equation:

(11)
$$\omega=\mathrm{i}\frac{\mathrm{k}}{2\mathrm{m}}\pm\sqrt{\frac{\mathrm{D}}{\mathrm{m}}-\frac{\mathrm{k}^2}{4\mathrm{m}^2}}=\mathrm{i}\,\delta\pm\omega_1$$

With the following substitutions

(12)
$$\delta=\frac{\mathbf{k}}{2\mathbf{m}} \quad \text{and} \quad \omega_1=\sqrt{\frac{\mathbf{D}}{\mathbf{m}}-\frac{\mathbf{k}^2}{4\mathbf{m}^2}}=\sqrt{\omega_0^2-\delta^2}$$

the general solution has the form:

(13)
$$\boldsymbol{x}(t)=\mathbf{C}_1\,e^{-\delta t}\,e^{\,i\omega_1 t}+\mathbf{C}_2\,e^{-\delta t}\,e^{-i\omega_1 t}$$

The physically meaningful solutions are given by the condition $\boldsymbol{x}(t)$ real for all *t*. Since under strong damping conditions the frequency ω_1 in (12) can become imaginary, it is necessary to distinguish between cases when discussing the solutions.

$\delta < \omega_0$: Oscillating case

The frequency remains real and the exponential expressions with i ω_1 t are thus complex conjugate so that $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ must also be complex conjugate. The solution evolves up to the factor $e^{-\delta t}$ exactly as in the case of the undamped oscillation:

(14)
$$x(t)=A\,e^{-\delta t}\cos\left(\omega_1 t+\beta\right)$$

or:

(15)
$$\mathrm{x(t)}=\mathrm{e}^{-\delta\,\mathrm{t}}\left[\mathrm{A}_1\cos\omega_1\,\mathrm{t}+\mathrm{A}_2\sin\omega_1\,\mathrm{t}\right]$$

with the constants:

(16 a,b)
$$\mathbf{A}_1=\mathbf{x}_0 \quad \text{and} \quad \mathbf{A}_2=\frac{\delta\cdot\mathbf{x}_0+\dot{\mathbf{x}}_0}{\omega_0}$$

$\delta>\omega_0$: Creeping Case

The frequency becomes imaginary. One can transpose ω_0 and δ and bring the imaginary unit i in front of the total expression:

(17)
$$\omega=i\left[\delta\pm\sqrt{\delta^2-\omega_0^2}\,\right]=i\;\delta_{1,2}$$

The solution is then:

(18)
$$\boldsymbol{x(t)}=\boldsymbol{C}_1\,e^{-\delta t}+\boldsymbol{C}_2\,e^{-\delta_2 t}$$

Since both exponential parts are real, but of different magnitude, $\boldsymbol{x}(t)$ is only then real for all *t* when the constants $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ are also real. They again serve to match the initial conditions. Under this very strong damping, the motion looses its periodic character and passes over into a monotone, exponential decay to a state of rest. With increasing damping, the term δ_1 decays quickly indeed, the time constant δ_2, in contrast, becomes smaller and in the limit of infinite damping approaches zero. The frictional forces become so large that movement is hardly possible and the system creeps slowly to its state of rest.

$\delta = \omega_0$ and $\omega_1 = 0$: Aperiodic Limiting Case

Between the oscillating- and creeping case lies the special case $\delta=\omega_0$ with $\omega_1=0$. Direct substitution of $\omega_1=0$ in (14) or (15) gives only a description for the special initial conditions ***A*** or $\boldsymbol{A_1} \neq 0$ and $\dot{\boldsymbol{x}}=0$.

One gets the general solution by inspecting the limit of (15) with (16):

(19)
$$\boldsymbol{x(t)}=e^{-\delta t}\;\boldsymbol{x}_0\,\cos\omega_1 t+\left(\delta\,\boldsymbol{x}_0+\dot{\boldsymbol{x}}_0\right)\frac{\sin\omega_1 t}{\omega_1}$$

In the limit cross-over point:

(20)
$$\lim_{\omega_1\to 0}\frac{\sin\omega_1 t}{\omega_1}=t$$

and for the aperiodic limiting case one again gets a non-periodic motion:

(21)
$$\boldsymbol{x(t)}=\boldsymbol{x}_0 e^{-\delta t}+\left(\delta\,\boldsymbol{x}_0\right)t\,e^{-\delta t}$$

The aperiodic limiting case provides for the fastest return of an oscillating system back to its rest state.

This is of major importance in measurement technology. Measuring instruments often represent oscillating systems whereby aperiodic damping is the fastest method for the displayed value to reach equilibrium or a state of rest.

Forced Oscillations

The system is periodically excited with frequency Ω by an additional external force. By neglecting a special zero point for the time one can write without constraining the generality and without further phase constants:

(22) $$m\,\ddot{\boldsymbol{x}} + k\,\dot{\boldsymbol{x}} + D\,\boldsymbol{x} = \boldsymbol{F}_0 \cos \Omega t$$

Compared to the previous approaches, the differential equation has become *inhomogeneous*. The general solution (22) is formed by summing the solution (14) or (15) of the associated homogenous equation with an arbitrary particular solution of the inhomogeneous equation. As a particular solution of the inhomogeneous equation one assumes that for large times, the system also oscillates with the external frequency Ω and with constant amplitude whereby a phase shift appears compared to the initial excitation. The trial solution for this *stationary* case is thus:

(23) $$x(t) = A_s \cos(\Omega t + \phi)$$

Basically it is possible to correctly express the cos-function, as in (6), by complex e-functions and calculate the result. However, one gets the same result more simply with a general complex expression:

(24) $$\mathbf{F}(t) = F_0\, e^{i \Omega t}$$

and the trial solution

:

(25) $$\mathbf{x}(t) = A_s\, e^{\mathrm{i}(\Omega t + \phi)}$$

whereby the physical circumstances are mirrored by the real part of (24) or the magnitude and phase of (25). By differentiation, substitution and elimination of i Ω t one gets:

(26) $$\left(D - m\,\Omega^2 + \mathrm{i}\,k\,\Omega\right) A_s\, e^{i\phi} = F_0$$

and:

(27) $$A_s\, e^{i\phi} = \frac{F_0}{D - m\,\Omega^2 + \mathrm{i}\,k\,\Omega}$$

or setting D/m = ω_0^2 and k/(2m) = δ:

(28) $$A_s\, e^{i\phi} = \frac{F_0 / m}{\omega_0^2 - \Omega^2 + 2\,\mathrm{i}\,\delta\,\Omega}$$

Thus the amplitude of the stationary solution depends on the excitation frequency Ω. From (28) one gets the complex representations with respect to the relationships between magnitude and phase and real- and imaginary parts:

(29) $$A_s = A_s(\Omega) = \frac{F_0 / m}{\sqrt{\left(\omega_0^2 - \Omega^2\right)^2 + 4\,\delta^2\,\Omega^2}}$$ and:

(30) $$\tan\phi = \frac{-2\,\delta\,\Omega}{\omega_0^2 - \Omega^2}$$

Since in the general solution the damped term has practically decayed after a sufficiently long time (see swing behavior below), one designates the stationary solution as a forced oscillation in the narrow sense.

Resonance Curves

The behavior of the amplitude as a function of the excitation frequency ***A***(Ω) (resonance curve) and also the phase shift δ(Ω) is strongly dependent on the degree of damping. Examples of the behavior are given in the following figures. The position of the resonance curve is determined by the eigenfrequency (characteristic frequency) of the undamped system, the width is determined by the damping.

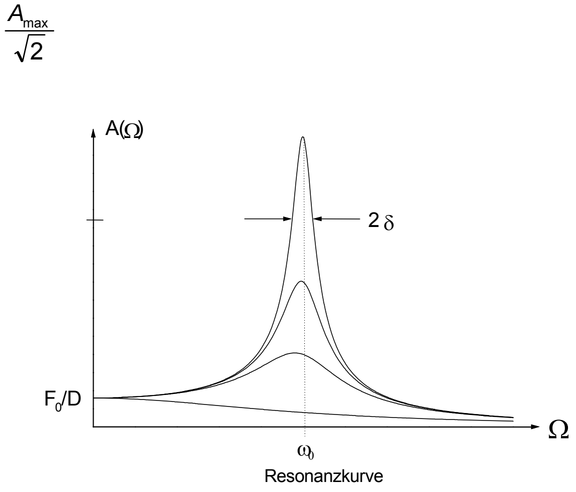
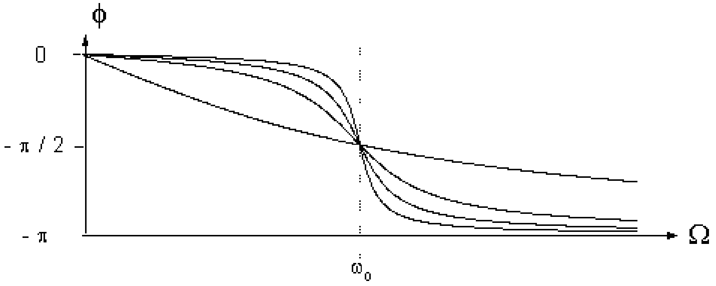


Figure: Resonance Curves from Top to Bottom for Damping Ratios δ/ω0 = 0.05; 0.1; 0.2 and 1



Phase Shift (Dispersion Curves)

Figure: Phase Shift (Dispersion Curves from Top to Bottom) for the same Damping Ratios.

Physically, the behavior is easy to understand, however, mathematically (from 30) only after taking into consideration the nature of inverse functions.

Under small damping conditions the oscillator amplitude can become very large (resonance catastrophe), whereby the transition region of the phase shift becomes very small (phase jump).

For small deviations $\omega_0 \approx \Omega$ and $|\omega_0 - \Omega| = \Delta\Omega$ (detuning), i.e. for comparatively narrow resonance curves under small damping conditions, equation (29) can be transformed and simplified to give the approximation:

(31) $$\omega_0^2-\Omega^2 \approx (\omega_0+\Omega)(\omega_0-\Omega) \approx 2\,\omega_0\,\Delta\Omega$$

(32) $$A_s(\Delta\Omega)=\frac{F_0/m}{2\,\delta\,\omega_0\cdot\sqrt{1+\left(\frac{\Delta\Omega}{\delta}\right)^2}}$$

In this approximation, the maximum of the curve lies at $\Omega=\omega_0$, and the amplitude achieves the maximum value:

(33) $$A_{\Omega=\omega_0}=A_{max}=\frac{F_0/m}{2\,\delta\,\omega_0}$$

For the case $\Delta\Omega = \pm\,\delta$ we have:

(34) $$A_{\Omega=\omega_0\pm\delta}=\frac{F_0/m}{2\,\delta\,\omega_0\sqrt{2}}=\frac{A_{max}}{\sqrt{2}}$$

The damping constant can be determined from the half-width of the curve at

$$A = A_{max}/\sqrt{2}$$

Swing Behavior

The general solution of forced oscillations consists of the general solution (e.g. 14) of the homogenous equation for the oscillating case and the particular real solution (25):

(35) $$x(t)=A\cdot e^{-\delta t}\cos(\omega_0 t+\beta)+A_s(\Omega)\cos(\Omega t+\phi)$$

At the beginning of motion, the system is impressed with the stationary term as well as excited with a free, damped oscillation. The superposition of both terms of (35) leads to a complicated behavior called the swing behavior. Clear relationships are found for the case - small damping ($\delta \ll \omega_0$) and with an excitation frequency close to the eigenfrequency ($\Omega \approx \omega_0$). If one selects the initial conditions x_o=0 und $\dot{x}=0$, then to a good approximation we have:

(36) $$x(t)=A_s\left[\cos(\Omega t+\phi)-e^{-\delta t}\cdot\cos(\omega_0 t+\beta)\right]$$

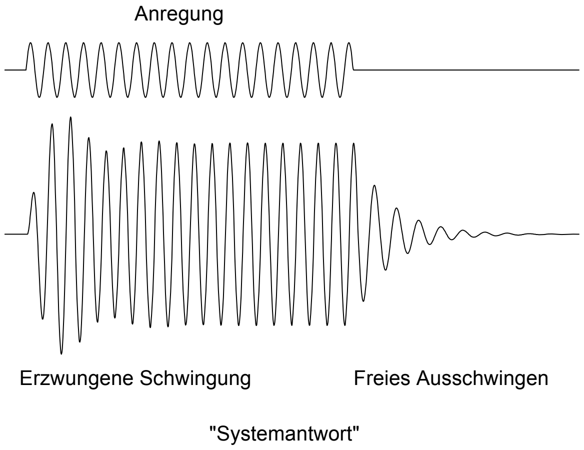
The superposition of both oscillations results in a *beat* with half the difference frequency $|\Omega - \omega_0|/2$, which, however, decays with $e^{-\delta t}$, with the motion slowly going over into the stationary state.

The largest possible amplitude in the swing behavior is reached when both cos-terms have opposite signs due to the difference of both frequencies. Under very small damping conditions the *switch-on peak* can come close to the value 2$\boldsymbol{A}_s$, i.e. almost double the magnitude of the stationary amplitude.

For the case $\Omega = \omega_0$ the swing behavior is simplifier further. The phase shift ϕ is then $\pi/2$ and from (36) we have:

(37) $$x(t)=A_s\left(1-e^{-\delta t}\right)\sin\omega_0 t$$

After switch-on, the system performs an oscillation with exponentially increasing amplitude and an amplitude working against the limit $\mathbf{A}_s$ of the stationary case. The following figure shows the relationship of the swing behavior of the stationary oscillation and the free, damped decay together with the excitation oscillation.



Presentation of the Physical Principles

(As preparation for part of the report): A short description of how the problems are approached and the results of the calculations and the physical interpretation in the framework of the practical exercises.

Apparatus and Equipment

Copper wheel acting as an oscillating system, held on ball bearings and connected to a spiral spring. The system is damped by an eddy current brake. Periodic excitation through a motor drive and eccentric-and-rod mechanism (*Pohl's Wheel*, see sketch on title page).

Motion measurement transducer: Small pulley, with thread and weight placed between the arms of a photo-sensor. Operating unit to convert the time dependent pulses at the output of the photo-sensor into analog signals proportional to amplitude, velocity and acceleration.

Computer supported measuring system (*Cassy-Lab-System*) for data collection (velocity-time values) and evaluation. Hand stopwatch.

Experiment and Evaluation

Pohl's wheel can be set in motion by hand or periodically excited by means of a motor drive. Damping is set in six steps (0, 1, 2, 3, G = limit case, K = creep case) using the eddy current brake and the associated power supply unit.

The measurements for the exercises should be carried out with damping stage 2. The transducer records the amplitude. A thread attached to the wheel is guided over the periphery of the wheel and then over a small pulley. The pulley with holes runs between the arms of a photo-sensor. The output of the photo-sensor delivers a stream of pulses whose frequency is proportional to the velocity of the wheel. The pulses are electronically processed and the output of the operating unit delivers analog voltage signals for amplitude (***s***), velocity (***v***) and acceleration (***a***). The velocity signal (***v***) is suitable for the measurements. A dc voltage component is superimposed on the output signal, which especially interferes with the subsequent log-presentation of the data. This dc voltage component can be suppressed by connecting a capacitor in the line of the floating potential between transducer and *CASSY-Lab.-System*.

Attention: When evaluating the results pay attention to whether the calculations are done with the frequency *f* or angular frequency *ω*.

Exercise 1

The measured data (amplitude as a function of time) are recorded with the aid of the *CASSY-Lab-System* by a computer. Refer to the general information on *CASSY* in the script and special information on using the program in the lab bench script.

Exercise 2 and 3

The torsion pendulum is periodically excited with the motor. The speed of the motor is continuously variable.

The frequency is set with a potentiometer, however, the display only shows arbitrary units fort he frequency. The real values are determined by the *CASSY-Lab-System* and at the same time, the constancy of the amplitude is checked (decay of the swing behavior).

The amplitude values can be read directly from the scale of the torsion pendulum or determined with the data acquisition system. One must wait a sufficient length of time for each measurement point until the swing behavior has decayed.

The resonance curve is first recorded with measurement points in coarse steps (for about each full rotation of the frequency setting at the potentiometer) and plotted during the experiment (amplitude as a function of frequency in scale divisions of the potentiometer). After plotting the rough form of the curve more exact intermediate values can be determined in the interesting regions (rising- and falling edges, maximum). Due to the steepness of the curve in the resonance region, it is important, when setting the potentiometer values to take into account the mechanical play of the adjusting knob.

The phase shift between the excitation and the oscillation of the wheel can be qualitatively observed by means of the pointer at the wheel and the drive rod. The observations are to be compared with the theoretical prediction.

Exercise 4 (done without damping)

In order to observe the swing behavior outside the resonance, the frequency should not be too strongly detuned to enable clear observation of the typical switch-on spikes. The amplitude-time dependence is registered with the *CASSY-Lab.-System*. For the case of non-resonant excitation, compare the exponential behavior at the begin of swing and during decay with the expected value.

CASSY

SWITCH ON

Voltage Supply SENSOR-CASSY (12V)

Switch computer on

=>Log on with *Username*: **„praktgast“**

=>*Pass word* **„PhysPrakt“**

Start Cassy-Lab by clicking the CassyLab icon (if necessary switch power supply off/on again, or restart program in case of problems)

=> Close the Enable (freischaltung) window (software is already enabled)

=> **Settings**

=> Click the active channel (input-/output) (color changes to red)

=> Now make the sensor input settings (also with F5)

=> Make parameter settings

- ***e.g.*** Trigger (supports the comparability of different measurements)
- **e.g.** Repeat measurement

=> Right mouse key (before, during or after the measurement)

- ***e.g.:*** zoom / thereafter, „disable zoom“
- ***e.g.:*** change axis: log-or x^2 coordinates
- ***e.g.:*** set mark and measure difference (bottom left).

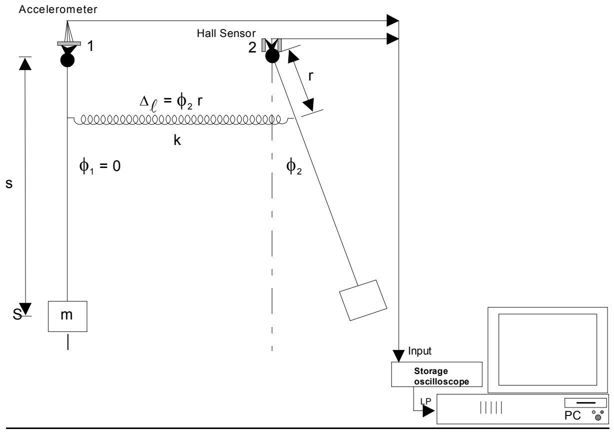
....

=> **F9 starts and stops the measuremen**t (the button is colored if repeat measurement is set).

=> Left mouse key marks points on curves. At the same time, values in the table are marked. Moving the arrow up and down marks the next point or value in the curve or table respectively;

Print: click icon button (prints curve or table) Desk jet HP 600

COUPLED OSCILLATIONS | GPI



Key Words

Coupled oscillations, coupled pendulums; normal oscillations (basic- or fundamental oscillations); beats.

Aim of the Experiment

Classical experiment for a quantitative discussion on coupled oscillations. Coupled oscillations form an important extension of the oscillations of isolated systems. Furthermore, they are closely related to wave phenomena. The experiment has, as an addition goal, the exemplary handling of an eigenvalue problem.

Literature

[1]: Chapter 4.1.1

[2]: Chapter 22.1, 22.3

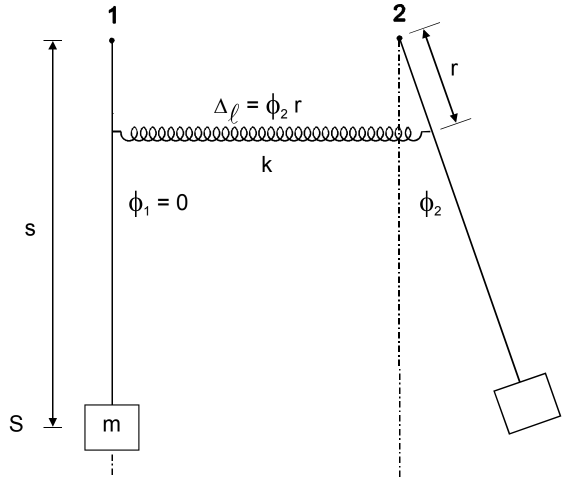
Exercises

1. Alignment of the individual pendulums (gravity pendulum) to the same swing time. Measuring the natural frequencies and comparing the results with the value calculated from the dimensions of the pendulum.
2. Excitation of the symmetric and anti-symmetric normal oscillation and measurement of the natural frequency as a function of the coupling strength. Calculation of the coupling factor.
3. Excitation of beats for the different couplings and calculation of the coupling factor from the beat frequency.
4. Calculation of the spring constant of the coupling springs from the plot of the coupling factor against the square of the coupling points (distance of the coupling points from the fulcrum of the pendulum). Direct determination of the spring constant from the measurement of the swing time of free oscillator with an added weight. Comparing and discussing the results.

Physical Principles

For the following formal treatment of the problem, we shall investigate the simplest case of two coupled pendulums of the same length and mass (symmetric coupling; see figure). The structure of the problem of coupled harmonic oscillators is always the same and the results of this special simple example can be carried over, without difficulty, to other cases.

The treatment of two coupled oscillators corresponds to a large extent to that of a simple oscillating system and does not involve any fundamental difficulties. The complete formal execution and interpretation is, however, much more substantial and tends to become tortuous. For this reason, the following presentation is purposely held short to highlight the essential features of the methods used in solving the problem and deliberately foregoes a detailed explanation of all aspects. It presumes good knowledge of the simple harmonic oscillator (see experiment *HARMONIC OSCILLATIONS).*



Equation of motion

The torsional constant M_g as a result of gravity ($\sin\phi = \phi$ to a first approximation) and the coupling moment M_k for pendulum 2 with pendulum 1 in the state of rest (spring constant *D*; coupling distance *r*) are:

(1a,b) $$M_g = -m\,g\,s\,\phi_2 \quad \textit{und} \quad M_k = -D\,r^2\,\phi_2$$

The (coupled) equation of motion for two equal pendulums with moment of inertia *I* is given by:

(2a) $$\ddot{\phi}_1 + \frac{m\,g\,s}{I}\,\phi_1 + \frac{D\,r^2}{I}\left(\phi_1 - \phi_2\right) = 0$$

(2b) $$\ddot{\phi}_2 + \frac{m\,g\,s}{I}\,\phi_2 + \frac{D\,r^2}{I}\left(\phi_2 - \phi_1\right) = 0$$

The following abbreviations are used:

(3a,b) $$\omega_o^2 = \frac{m\,g\,s}{I} \quad \text{and} \quad f = \frac{D\,r^2}{m\,g\,s}$$

where ω_o^2 is the frequency of the individual free pendulums. The ratio of the coupling quantity Dr^2 to the torsional constant mgs is called the *coupling factor f*. The (coupled) equations of motion of the system are then:

(4a) $$\ddot{\phi}_1 + (1+f)\,\omega_o^{\,2}\,\phi_1 - f\,\omega_o^{\,2}\,\phi_2 = 0$$

(4b) $$\ddot{\phi}_2 - f\,\omega_o^2\,\phi_1 + \left(1+f\right)\omega_o^2\,\phi_2 = 0$$

This is a linear system of equations treated with the formalism of matrix- and vector algebra. In matrix notation one gets (where we write **vectors in bold type** and **<u>matrices are additionally underlined</u>**):

(5) $$\ddot{\phi} + \underline{\Omega}\cdot\phi = 0 \quad \text{with}$$

(5a) $$\underline{\Omega} = \begin{bmatrix} \left(1+f\right)\omega_0^2 & -f\,\omega_0^2 \\ -f\,\omega_0^2 & \left(1+f\right)\omega_0^2 \end{bmatrix}$$

The homogeneous linear differential equation (5) with constant coefficients (analogous to the case of a simple harmonic oscillator) is solved by a trial solution with complex exponential functions (see script *HARMONIC OSCILLATIONS*):

(6) $$\phi(t) = \mathbf{Z}\,e^{i\,\omega\,t}$$

where the constants **Z** form a vector. For the second derivative one gets:

(7) $$\ddot{\phi} = -\,\omega^2\,\mathbf{Z}\,e^{i\,\omega\,t} = -\,\omega^2\,\underline{\mathbf{E}}\,\mathbf{Z}\,e^{i\,\omega\,t}$$

where **<u>E</u>** is the unit matrix. From the equation of motion we then have:

(8) $$\left(\underline{\Omega} - \omega^2\,\underline{\mathbf{E}}\right)\mathbf{Z} = 0$$

This (homogeneous, linear) characteristic system of equations has only a non-trivial zero solution (**Z=0**) when the determinant in the bracket vanishes:

(9) $$\begin{vmatrix} \left(1+f\right)\omega_0^2 - \omega^2 & -f\,\omega_0^2 \\ -f\,\omega_0^2 & \left(1+f\right)\omega_0^2 - \omega^2 \end{vmatrix} = 0$$

Equation (9) has four solutions (in the general case, from *N* coupled oscillators, 2*N* solutions), given by the *eigenvalues* ω_j. Substituting ω_j in (8) gives (up to an arbitrary factor) the associated *eigenvectors* $\mathbf{Z}_j$:

(10a) $$\omega_1 = \omega_0 = \omega_s \qquad \mathbf{Z}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \mathbf{Z}_s$$

(10b) $$\omega_2 = -\,\omega_0 = -\,\omega_s \qquad \mathbf{Z}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \mathbf{Z}_s$$

(10c) $$\omega_3 = \omega_0\,\sqrt{1+2f} = \omega_a \qquad \mathbf{Z}_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \mathbf{Z}_a$$

(10d) $$\omega_4 = -\,\omega_0\,\sqrt{1+2f} = -\,\omega_a \qquad \mathbf{Z}_4 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \mathbf{Z}_a$$

After collecting real valued functions we find that the amplitude ϕ_i of the *i*-th oscillator is represented by the sum of *N normal oscillation*s:

(11) $$\phi_i(t) = \sum_{j=1}^{N} A_{ij}\cos\left(\omega_j\,t + \delta_j\right)$$

Note that the phase δ_i is not dependent on *i* and that the A_{ij} for fixed *j* are given by the eigenvector $\mathbf{Z}_j$ (up to a common factor). Thus there are exactly 2*N* free constants, determined by the initial conditions $\phi_i(0)$ and $\dot{\phi}_i(0)$.

For the special case of two equal oscillators we have for the *normal frequencies*:

(12a,b) $$\omega_1 = \omega_s = \omega_0 \quad \text{and} \quad \omega_2 = \omega_a = \omega_0\,\sqrt{1+2f}$$

The associated amplitudes obey the conditions:

(13a) $A_{1s} = A_{2s}$ (*symmetric oscillation*)

(13b) $A_{1s} = -\,A_{2s}$ (*anti-symmetric* oscillation)

In the general case, (11) leads to complex, non-periodic motions (already for *N*=2), however, by selecting suitable initial conditions one can excite pure normal frequencies. All oscillators then swing with a pure sinusoidal motion (up to the sign) with the same phase and an energy exchange between the systems does not take place.

<u>Beats</u>

Beats are understood as phenomena occurring by the superposition of oscillations of comparable frequencies, i.e. for $\Delta\omega << \omega$. Weakly coupled, equal pendulums are an example. With the help of the addition theorem for trigonometric function, equation (11) can be represented as a product of a fast oscillation with slowly changing amplitude:

(14a) $$\phi_1(t) = \phi_0\,\cos\Delta\omega\;t\,\cos\omega t = A_1(t)\cos\omega t$$

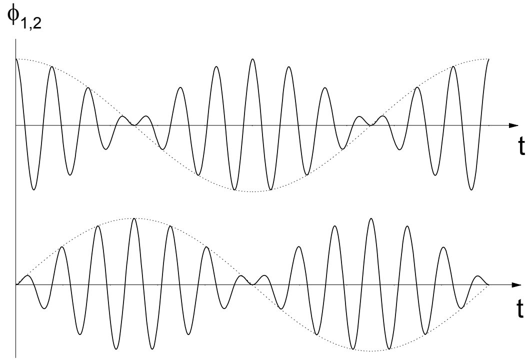
(14b) $$\phi_2(t) = \phi_0\,\sin\Delta\omega\;t\,\sin\omega t = A_1(t)\sin\omega t$$

with frequencies

(15a,b) $$\omega = \frac{\omega_a + \omega_s}{2} \quad \text{and} \quad \Delta\omega = \frac{\omega_a - \omega_s}{2}$$

and initial conditions $\phi_{10} = \phi_0$, $\phi_{20} = 0$ and $\dot{\phi}_{10} = \dot{\phi}_{20} = 0$.

Beats result when the energy is alternately and completely transferred from one pendulum to the other. Phase jumps occur at the zero-crossover of the slowly changing functions (see figure below); the energy flow between oscillators changes direction at these positions.



Presentation of the Physical Principles

(As preparation for part of the report): A short description of how the problems are approached and the results of the calculations, especially the calculation of the frequencies and the relative amplitudes of the normal oscillations and a detailed physical interpretation in the framework of the practical exercises.

Apparatus and Equipment

Two equal gravity pendulums; coupling spring;

Hand stopwatches;

Hall Effect angle recorder or accelerometer (G-sensor) with operating unit;

Storage oscilloscope PCS100; pc; printer.

Experiment and Evaluation

The measurements are made on two gravity pendulums coupled by a spring. The pendulums are made of metal rods with a weight (m_1) screwed to each end (see figure below and sketch on the title page). The weight can be moved to adjust the swing time and is fixed by a counter nut (m_2).

The dimensions of the pendulums are:

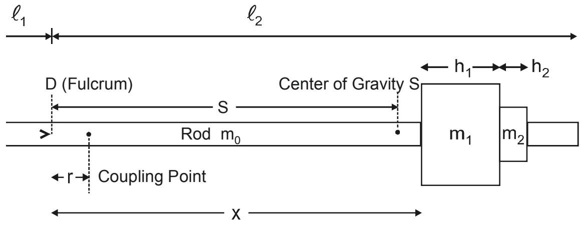
Rod $\ell_1 = (0.026 \pm 0.001$ m
$\ell_2 = (1.069 \pm 0.01)$ m

Distance x adjustable ($x \approx 0.995$ m)

$m_o = (1.048 \pm 0.002)$ kg

Weight $h_1 = (0.0593 \pm 0.0002)$ m
$m_1 = (1.359 \pm 0.001)$ kg

Counter nut $h_2 = (0.0180 \pm 0.0002)$ m
$m_2 = (0.074 \pm 0.001)$ kg



The moment of inertia of a linear mass distribution of length ℓ with respect to the fulcrum at one end of the mass is:

$$I = \frac{1}{3} m \ \ell^2 \tag{16}$$

The eigenfrequencies and the frequencies of the normal oscillations are measured with stopwatches. Ensure sufficient accuracy.

Evaluation of the beats (plot of amplitude as a function of time) is done with a small Hall- effect sensor in a constant magnetic field (permanent magnet) mounted at the fulcrum of one of the pendulums. Since the Hall voltage is proportional to the sine of the angle between the magnetic field and the probe, it can be used as the signal for the displacement for small amplitudes. A small operating unit delivers the output signal. The signal is recorded by means of a storage oscilloscope PCS100 and pc. Operating instructions are found in the lab script.

The Hall sensor with downstream operational amplifier is prone to interference and as an alternative an accelerometer (G-sensor) with operational amplifier can be used for angle measurements.

One should try out which of both angle transducers delivers the best interference -free results.

To Exercise 1

Before beginning the measurements, the swing times of the pendulums must be so aligned that the relative difference is $\leq 10^{-3}$.

The swing times are to be measured and compared with the value calculated from the dimensions of the pendulums.

To Exercise 2

When conducting the measurements, ensure that the amplitudes are so selected that the coupling spring is still under tension even the pendulums are closest to each other during the swing phase. The distance of the coupling points from the fulcrum points is given in the lab script.

To Exercise 3

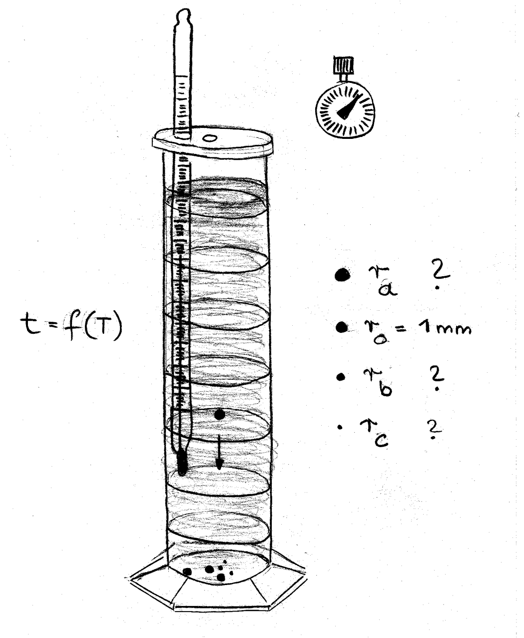
Beats are excited for three coupling points and recorded with the PCS100.

To Exercise 4

The coupling factors are calculated from the measured frequencies. For the quantitative assessment of the measurement (comparison of experiment and theory) and for evaluation, the coupling factors are plotted against the square of the coupling points (distance of the coupling point from the fulcrum). The spring constant of the coupling spring is determined from the gradient of the curve together with the torsional constant.

A small weight is available which can be used to excite oscillations in coupling spring. The spring constant is again calculated from the swing time.

FALLING BALL VISCOSIMETER GPI



Key Words

Laminar Flow.

Internal friction, Viscosity; Drag, *Stoke's*- and *Hagen-Poiseuille Law*.

Turbulent flow and *Reynolds number*.

Aim of the Experiment

Investigating the drag, internal friction, viscosity and laminar flow around a ball falling in a liquid; *Stoke's Law*. Temperature dependence of the viscosity.

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Literature

[1]: Chapter 3.3

[2]: Chapter 16.2 and 18.1

Exercises

1. Measuring the velocity of steel balls, with known and unknown radii, falling through a liquid as a function of the temperature of the liquid.
2. Investigating the dependence of the viscosity of glycerol as a function of temperature. Determining the viscosity of the glycerol at 20 °C and comparing the result with the literature value.
3. Determining the unknown radii of steel balls from the measurements and comparing the results with direct measurements made with a micrometer.
4. Setting up and solving the equation of motion with the boundary conditions $v(t=0) = v_0 = 0$ and $v(t\to\infty) = v_\infty$ and estimating the time and distance respectively where the ball sinks with almost constant velocity.

Physical Principles

Liquids (continuous media) have the property of *laminar* or *turbulent* flow. The frictional forces arising in laminar flow are determined by the viscosity η (coefficient of internal friction). *Stoke's Law* applies for the frictional force R when a ball moves in a viscous fluid:

$$R = -6\,\pi\,\eta\,r\,v \tag{1}$$

where r is the radius and v the velocity of the ball.

If a ball drops under the influence of gravity in a liquid, then the force of gravity G, diminished by the amount of the buoyancy force A, works against the drag (frictional) force R. After a certain time and independent of the initial velocity and because of the velocity dependence of the drag, a state of balance is reached in which the sum of all forces vanishes:

$$G + A + R = 0 \tag{2}$$

The ball then falls with constant velocity from which one can calculate the viscosity.

Temperature Dependence

In liquids, the internal friction originates from the action of intermolecular forces and decreases with increasing temperature. In many cases, the temperature dependence follows a functional progression given by:

$$\eta(T) = A\,e^{\frac{B}{T}} \tag{3}$$

Presentation of the Physical Principles

(As preparation for a part of the report): Aside from the method of measurement (measurement equations), a short presentation should be given on the themes internal friction and laminar flow and the definition of viscosity.

Equipment

Stand cylinder with ring markings and thermometer; filled with glycerol (see figure on the title page). Refrigerator. Steel balls of various sizes. Tweezers. Paper and 2-Propanol to clean the balls Stopwatch. Metal rule. Micrometer.

Experiment and Evaluation

The measurement of the temperature dependence of the viscosity throws up a number of problems: The temperature, as a state variable, presupposes thermodynamic equilibrium, which is very difficult to achieve in practice. The following experimental procedure is thus prescribed as a compromise between effort and result:

The stand cylinders are stored in a refrigerator. At the start of the experiment they are taken out and exposed to room temperature so that the temperature of the glycerol takes on changing values. This warming up is a dynamic process without thermodynamic equilibrium. However, one can, as an approximation, presume that the systematic errors in the values are about the same for all measurements so that the temperature dependence of viscosity can be reasonably well observed.

The experimental procedure is as follows: The cylinders are taken out of the refrigerator and during a certain time interval (about 1 hour), and thus over a certain temperature range (about 6 to 10 K) repeated measurements are made of the falling times of the different balls t(r_0), t(r_a), t(r_b), t(r_c) one after the other and the corresponding temperatures. All other parameters must remain unchanged (falling distance of the balls, position of the thermometer, etc). Observe and take into account the accelerating phase of the balls when determining the falling distance. It is recommended to protocol the initial- and final temperatures for each falling time and to take the average for the subsequent evaluation. One then gets a table of values for the falling time as a function of temperature and ball radius.

Any attempt to achieve a uniform temperature (e.g. by stirring the glycerol) is pointless and brings about conditions making further experimental work impossible (e.g. air bubbles in the glycerol).

Stoke's Law applies rigorously only for the movement of a ball in an infinitely extended medium. In a narrow cylindrical tube, the resistance is increased due to boundary interference. For this reason, the balls should fall, as close as possible, in the middle of the cylinder. Furthermore, the measurement must be stopped an ample distance from the bottom of the cylinder.

Before beginning the experiment, the balls must be cleaned from any remaining traces of old glycerol (why?). At the end of the experiment, the balls are removed from the bottom of cylinder using a magnet.

The experiment is evaluated by plotting the falling time as a function of temperature on single log paper. As a check, it is recommended to plot the results during the course of the experiment. Later, a second axis can be drawn along the ordinate with the actual viscosity values derived from the falling time of the known balls.

The radii of the unknown balls are found by determining the ratio of the falling times to that of r_0, provided one assumes that all other conditions remain unchanged. For viscosity, this may not be taken for granted since temperature changes can not be influenced. However, comparative data can be obtained from the plot by interpolation with the aid of best fit lines.

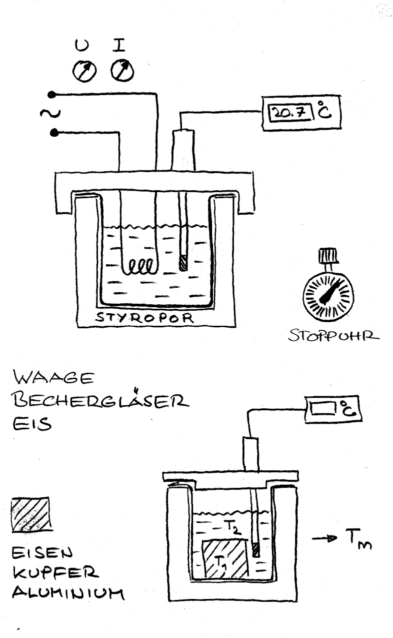
Supplementary Questions

The transition from laminar- to turbulent flow is described by the *Reynolds number* R_E:

$$R_E = \frac{\rho\, r\, v}{\eta} \tag{4}$$

where ρ is the density of liquid. In which range are the *Reynolds numbers* in this experiment? Does this prove that we are dealing with laminar flow?

SPECIFIC HEAT CAPACITY GPI



Key Words

- Internal energy and heat;
- 1st. Law of Thermodynamics. Heat capacity;
- *Du-Long-Petit Law.*
- Phase transitions, Heat of fusion.
- *Joule heat*

Aim of the Experiment

Investigating the heat capacity of solids and the heat of melting of water (ice). Introduction to elementary measurement- and evaluation techniques taking into account systematic errors.

Literature

[1]: Chapter 5.1, 5.2.3, 5.6.1, 5.6.2, 6.3.3

[2]: Chapter 34.8 and 34.15, 37.2

[3]: Chapter 3.2.3

Exercises

1. Determining the specific heat of water.
2. Determining the specific- and molar heat capacity of Aluminium, Iron, Copper and Plexiglas and comparing the results with the **Dulong-Petit Law**.
3. Determining the heat of fusion of ice.

Physical Principles

Internal Energy and Heat

In a general physical system, the unordered energy (statistical energy, heat) is coupled with the ordered binding energy to form the *internal energy U* as the total energy of the system. Because of the statistical exchange, the internal energy cannot be decomposed or separated into components (such as potential energy E_{pot} or kinetic energy E_{kin} of the charge- or center of mass of the system). The character of the statistical energy lies in the fact that it is a part of the ensemble nature of the particles and quanta of the system, where the microscopic states fluctuate continuously and generate the Uncertainty of the system. This uncertainty is a physically important state variable and expressed as the *entropy S* of the system.

The internal energy is associated with the state variables of the system. It can be changed through *work* d*W* and *heat* d*Q*.

The change of internal energy through work is tied to a change in volume:

(1) $$dW = p\,dV$$

Because heat is coupled with the ordered energy forms as mentioned above and because of its statistical nature it cannot not be defined as a part of the *energy balance* of a system but only as a form of exchange d*Q*. This exchange is connected with the entropy of the system:

(2) $$dQ = T\,dS$$

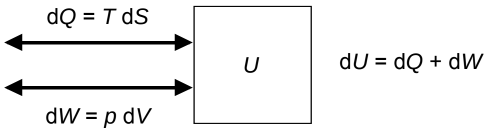
The temperature represents the *thermodynamic potential*. It describes the thermal energy qualitatively as a statistical parameter of the distribution or occupation of the states of the system respectively.

Heat is a non-integral function of a system and a state variable "*Q*" does not exist. Equation (2) represents the definition of heat as a form of energy, in which in integral form, is totally converted to a *quantity of heat* ΔQ after a change of state has occurred.

The *1. Law of Thermodynamics* includes the internal energy as a part of the total energy and heat as a form of energy exchange in the conservation of energy theorem:

(3) $$E_{gesamt} = E_{kin} + E_{pot} + U = const$$ und

(4) $$dU = dQ + dW$$



Heat Capacity

Heat causes either a change in temperature or a change in the internal structure of the system (phase transition; see below). In the first case the *heat capacity C* is defined as the ratio of exchanged heat to temperature difference:

(5) $$C = \frac{dQ}{dT}$$

In general, the heat capacity is dependent on the temperature, i.e. $C = f(T)$, and in addition, on the change of state connected with the process. Solids and liquids have very small expansion coefficients so that pressure and volume can be considered as constant and the exchange of work resulting from an accompanying change in volume can be neglected. C is then independent of the special change of state. Furthermore, at sufficiently high temperatures, the heat capacity, to a good approximation, can be taken as independent of T:

(6) $$C = \frac{dQ}{dT} \approx \frac{\Delta Q}{\Delta T} = const$$

Temperature Equilibrium

If two bodies of different temperatures T_1 and T_2 are brought into thermal contact with one other, an exchange of heat takes place until a common (mixing-) temperature T_m is reached. Because of energy conservation, the heat given up by the hotter body must equal the heat taken up by the colder body:

(7) $$c_1\ m_1\left(T_m - T_1\right) = c_2\ m_2\left(T_2 - T_m\right).$$

Specific Heat Capacity and the **Dulong-Petit Law**

The heat capacity of a system is proportional to quantity. If the value of the heat capacity is referred to the mass (unit kg) it is termed the *specific heat capacity c* and if the value is referred to the amount of material ν (unit Mole) it is termed the molar heat capacity C_M:

(8) $$C = c\ m = C_M\ \upsilon$$

The molar heat capacity at sufficiently high temperatures (room temperature) is for most solids about 25 J/(mol K). This behavior is called the *Dulong-Petit Law*.

Phase Transitions

Phase transitions are changes in the internal structure of the system, in which, at constant temperature, the entropy changes and discontinuities occur in the temperature-entropy diagram. Simple examples are the conversions between the aggregate states gas, liquid and solid.

The heat given off or absorbed at a phase transition point does not change the temperature of the system (the heat capacity is apparently infinite), but compensates the energy by changing the ordered- and binding states of the system. It is termed *latent heat* and must be taken into account in the energy balance just as with the heat of fusion and heat of vaporization in phase transitions.

Joule Heat

When an electric current passes through a conductor, energy is released as heat to the conductor (and the surroundings). The amount of heat converted is:

(9) $$\Delta Q = U\ I\ \Delta t$$

where U is the voltage, I the current and Δt the time interval.

Presentation of the Physical Principles

(As preparation for a part of the report): Short presentation on internal energy, heat and heat capacity. A Supplementary explanation on heat conduction for different materials. A short presentation of the thermodynamic relationships involved in the melting process. An Explanation of the *Dulong-Petit Law*. A listing of the equations relevant for the measurements.

Equipment

See title page: Simple calorimeter made out of an aluminium container with styrofoam isolation; Test materials (aluminium, iron, copper, plexiglass).

Semiconductor-resistance-digital thermometer (1/10 K).

Heater (wire wound). Power supply unit (transformer with variable output voltage); Two digital multimeters for current- and voltage measurements; Cables. Stop watch (1/10s).

Various measuring beakers. Weighing balance. Ice (ice box in the lab room, ice mill in the stair case corridor).

Experiment and Evaluation

(Preliminary experiment to exercise 1 - Water equivalent)

Since the calorimeter is involved in all exchange processes, its heat capacity must be taken into account and determined in a preliminary experiment. For this purpose, cold water ($m_W \approx 300$ g, $T_1 \approx 0$ °C) is filled in the empty calorimeter which is at room temperature T_R. The heat capacity of water C_K can then be determined from the temperature increase to the value of T_m using equation (7) giving:

(10) $$C_K\left(T_R - T_m\right) = c_W\ m_W\left(T_m - T_1\right)\ .$$

Because the specific heat of water c_W is as yet unknown, one calculates the ratio $C_K/c_W = m_Ä$. The value represents the mass of a fictitious amount of water possessing the same heat capacity as the calorimeter; it is called the *water equivalent* of the calorimeter. This value is used in the equations for the following experiments by taking it into account in the respective mass balance determinations:

11) $$m = m_W + m_Ä\ .$$

Exercise 1 (Specific Heat of Water)

The water in the calorimeter of the preliminary experiment is used. A certain amount of heat ΔQ is applied to the water by means of a small electrical heater. The conversion of electrical energy into thermal energy (*Joule Heat,* see experiment *DIRECT CURRENT/ ALTERNATING CURRENT*) in the time interval Δt is given by:

(12) $$\Delta Q = U\ I\ \Delta t\ ,$$

where I is the current and U the voltage at the heater.

For reasons of accuracy, the temperature increase should be at least 10 K. Start- and end temperature should be symmetrical about the room temperature to keep systematic errors, resulting from heat exchange with the surroundings due to insufficient thermal isolation of the calorimeter, to a minimum.

Measurements

When measuring the specific heat capacity of water (amount of water about 300 g) by electrical heating, the resulting temperature interval should be about 10 K and lie symmetric about the room temperature. Information on the electrical set-up is found in the lab bench script. To evaluate the results it is advantageous to protocol the temperature as a function of time (about every minute) during the heating phase.

Exercise 2 (Specific Heat of Metals)

The measurement of the specific heat capacities of metals is by means of mixing experiments. The calorimeter is filled with very cold water (about 200 g; add ice to cool down further; then remove the ice or let it melt completely), the metal probes, at room temperature, are then placed in the calorimeter.

Exercise 3 (Heat of Fusion)

The heat of fusion is determined from the cooling caused by melting ice in a water probe. When setting up the experimental equations, three aspects must be considered in the heat exchange process: for ice, the take up of the heat of fusion and the subsequent warming from T_0 = 273.15 K (= 0 °C) to the mixing temperature T_m and for the water probe, the cooling down to the mixing temperature.

The water probe is again the same water used in the calorimeter from the heating experiments. The mass of ice should be about 50 g (two large ice cubes). The ice from the cool box must be kept for a while in a water bath in order to reach a temperature of 0 °C.

Supplementary Questions (optional)

1. According to the *Equipartition Theorem* the mean thermal energy of a particle is ½ kT per degree of freedom (*Boltzmann's constant* k). How many degrees of freedom per atom correspond to the value of the *Dulong-Petit Law*?
2. In addition to the systematic errors caused by heat conduction and heat radiation, a not completely closed calorimeter also exhibits systematic errors caused by convection, evaporation and condensation. Which arguments can be used for a favorable selection of the temperature interval for the experiments?
3. Thermal equilibrium within a test body is characterized by a thermal diffusion time τ_D proportional to $\rho \cdot c/\lambda$ (density ρ; specific heat capacity c; coefficient of thermal conductivity λ). The coefficient of thermal conductivity λ is about 400 W/(K·m) for copper and 0.2 W/(K·m) for Plexiglas at room temperature. How large is the expected ratio of the diffusion times? Is this consistent with the observations and how can one explain observed deviations?

Literature values

Specific heat capacity of water at 20 °C:

$$c_W = 4.1818 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Specific heat of fusion of water:

$$\Gamma_E = 333.7 \frac{\text{kJ}}{\text{kg}}$$

Specific heat of vaporization of water at

p = 1.0132 bar and T = 373.15 K

$$\Gamma_{V1} = 2255.5 \frac{\text{kJ}}{\text{kg}}$$

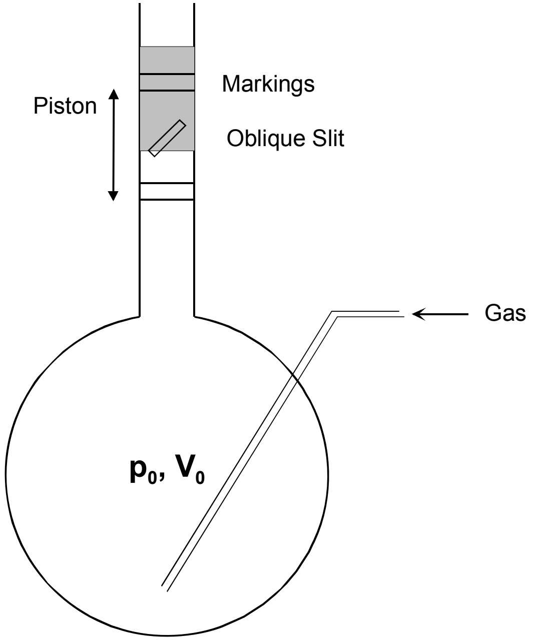
Specific heat of vaporization of water at

p = 1.0132 bar and T = 273.15 K

$$\Gamma_v = 2500.5 \frac{\text{kJ}}{\text{kg}}$$

(Source: KOHLRAUSCH; Praktische Physik 3; p.43 B.G.Teubner Stuttgart).

ADIABATIC INDEX GPI



Key Words

Internal energy; 1. Law of thermodynamics; Heat capacity. General equation of state of an ideal gas; *Poisson-Equation* (Adiabatic equation). Kinetic gas theory, Degrees of freedom. Harmonic oscillations.

Aim of the Experiment

A simple experimental method, whose theoretical principles give a good introduction to thermodynamics and kinetic gas theory. From the results of the experiment, one can draw conclusions concerning the molecular structure of the investigated gases.

Literature

[1]: Chapter 5.1, 5.2, 1.4.3

[2]: Chapter 34.14 with 34.16, 35.1 with 35.4

Exercises

1. Determining the ratio of the specific heat capacities $c_p/c_V = \kappa$ for air according to the method of *Clement-Desormes*.

2. Determining the value of κ for a monatomic gas (Argon), a diatomic gas (N_2) and a triatomic gas (CO_2) by measuring the natural frequency of a gas oscillator.

   Comparing the measured results with each other and with the values from the kinetic gas theory of an ideal gas.

Physical Principles

Adiabatic and *reversible* changes of state, i.e. those occurring without an exchange of energy with the surroundings and without the formation of entropy, are described, for ideal gases, by the *Poisson equations*. One of these is:

$$p\,V^{\kappa} = const \quad (1)$$

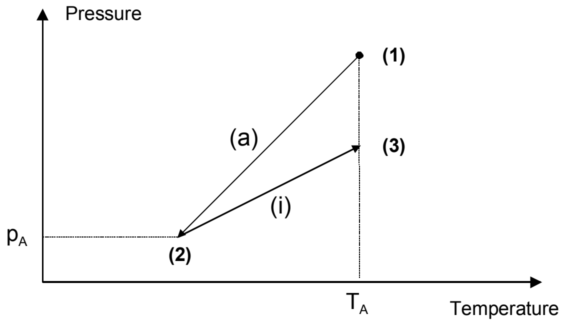
Further equations for other pairs of variables, such as (2), can be derived from (1) by introducing the general equation of state. The value $\kappa = c_p/c_V$ representing the ratio of the specific heat of the gas at constant pressure c_p and at constant volume c_V is called the *adiabatic index* or simply *kappa*. According to kinetic gas theory, κ is dependent on the number of degrees of freedom of the gas molecule (an example for a monatomic gas is $\kappa = 5/3 \approx 1.67$).

The experimental determination of κ by measuring c_p and c_V is made difficult by the fact that the heat capacity of the gas container is generally much higher than that of the gas itself. An elegant solution to this problem is to utilize adiabatic changes of state, as, e.g. in sound propagation (see experiment *SOUND WAVES*). In the method used here, according to *Flammersfeld-Rüchert* and *Clement-Desormes*, the adiabatic condition is satisfied by making the changes of state occur in very short intervals, so that energy exchange with the walls of the container can be practically neglected.

Method according to *Clement-Desormes*

The investigation of the changes of state for the determination of κ requires the measurement of temperature or temperature differences. The classical method of *Clement-Desormes* is based on the idea, the difficult measurement of small temperature changes in a gas be replaced by a pressure measurement, where the gas itself serves as the thermometer substance (gas thermometer).

A volume of gas V is enclosed in a container such that by a momentary change of state the energy exchange with the walls of the container may be neglected while over a longer time span, heat- and temperature equilibrium can take place. The difference between the gas pressure p inside the volume and the external air pressure p_A is measured by an open U-tube manometer. The measured difference in height Δh is proportional to the pressure difference. One investigates an adiabatic change of state (a) from $(p_1, T_1=T_A)$ with $p_1 > p_A$ and T_A = room temperature to ($p_2 = p_A$, T_2) and a subsequent *isochore* (V = const) change of state (i) to (p_3, $T_3 = T_A$).



The experiment begins by producing a slight overpressure in the container. After temperature equilibrium is reached ($T_1 = T_A$), a stop-cock is opened for a short time to allow the gas to relax adiabatically to the outside pressure ($p_2 = p_A$). The out flowing gas performs work against the outside air pressure and reduces is internal energy and temperature. The drop in temperature can be calculated from the *Poisson equation*. Subsequently, the gas (V = const.) warms up to T_A, where the rise in pressure to p_3 is used to determine the change in temperature with the aid of the general equation of state.

Under the given conditions, pressure- and temperature changes are small compared to their absolute values and the derivation of the equation for kappa is made relatively simple by considering the differential approximation d$p \approx \Delta p$. The associated *Poisson equation* for the adiabatic change of state (a) is:

$$p^{1-\kappa}\, T^{\kappa} = const \tag{2}$$

Setting up the total differential and rearranging gives:

$$\left(1-\kappa\right)\frac{\mathrm{d}p_{(a)}}{p} + \kappa\,\frac{\mathrm{d}T_{(a)}}{T} = 0 \tag{3}$$

For the isochore (V = const.) change of state (i), one gets from the general equation of state:

$$\frac{p}{T} = const \qquad \text{and} \tag{4}$$

$$\frac{dp_{(i)}}{T} - p\,\frac{dT_{(i)}}{T^2} = 0 \quad or \quad \frac{dT_{(i)}}{T} = \frac{dp_{(i)}}{p} \tag{5}$$

Inserting (5) in (3) with d$T_{(i)}$ = - d$T_{(a)}$ gives:

$$\left(1-\kappa\right)\frac{\mathrm{d}p_{(a)}}{p} - \kappa\,\frac{\mathrm{d}p_{(i)}}{p} = 0 \qquad \text{or} \tag{6}$$

$$\kappa = \frac{\mathrm{d}p_{(a)}}{\mathrm{d}p_{(a)} + \mathrm{d}p_{(i)}} \approx \frac{\Delta h_1}{\Delta h_1 - \Delta h_3} \tag{7}$$

where Δh_1 and Δh_3 are the difference in height readings of the manometer at the initial state (1) and at the final state (3).

The method of *Clement-Desormes* boils down to a surprisingly simple prescription (7). But because of the small pressure- or height differences respectively, and because of systematic errors (dQ = 0 not realized, leakiness of the apparatus), the accuracy of the method is, in general, not good and not sufficient to distinguish between the κ-values for monatomic, diatomic or polyatomic gases.

The Method of *Flammersfeld-Rüchert*

More exact measurements are based of the oscillation method of *Rüchert*. In this method, a volume of gas in a container is closed by a moving piston which is made to oscillate in a glass tube. The natural frequency is determined from the restoring force, dependent on the volume and pressure of the gas, and the mass of the piston and can be calculated with the aid of the adiabatic equation. However, the free oscillations are dampened after a few periods so that the measurements again are not of sufficient accuracy.

A solution to this problem was a modification of the experiment by ***Flammersfeld***, in which stationary oscillations were achieved by **parametric auto-control**. A week flow of gas is fed into a small opening (oblique slit in the neck of the flask, see diagram on the title page) near the middle position of the piston between the ring markings. The flow of gas serves to compensate for the gas- and energy loss (raising the piston) due to leakage around the piston and the effect of damping. The flow of gas leads to a considerable reduction of friction (*Bernoulli* forces) by building a layer of gas between the piston and the cylinder wall. The prerequisite is that the fundamental frequency of the oscillator is only insignificantly changed by the auto-control mechanism. The strong damping without the flow of gas can be clearly observed when the gas flow is cut off and the oscillations fade away practically immediately.

Detailed observation shows a rather complicated movement. If the piston is below the slit opening, the oscillations are distorted by the fact that the pressure resulting from the normal adiabatic compression continuously rises because of the inflow of gas. The movement of the piston above the slit opening (in the limiting case, taken as a large hole) is described by a free fall with strong friction. The surprisingly good results for κ in comparison to the expected- or literature values (test via reference measurements) show, however, that the errors due to the reduced friction remain low and hardly affect the fundamental frequency.

In the following derivation of the oscillating frequency, dx is the displacement of the piston and dF is the force on the piston with respect to the rest position, S and m are the cross-sectional area and mass of the piston and V and p, the volume and pressure of the enclosed gas. From (1) and forming the total differential we have:

$$\frac{\mathrm{d}p}{p} + \kappa\,\frac{\mathrm{d}V}{V} = 0 \tag{8}$$

The restoring force constant is given by:

$$D = -\frac{\mathrm{d}F}{\mathrm{d}x} = -\frac{S\,\mathrm{d}p}{\mathrm{d}x} = \kappa\,\frac{p\,S^2}{V} \tag{9}$$

The fundamental frequency is then:

$$\omega_0^2 = \frac{D}{m} = \kappa\,\frac{p\,S^2}{m\,V} \qquad \text{and thus} \tag{10}$$

$$\kappa = \frac{4\,\pi^2}{\tau^2}\,\frac{m\,V}{p\,S^2} \tag{11}$$

where τ is the observed period.

Presentation of the Physical Principles

(as preparation for part of the report): Short definition of the specific- and molar heat capacity of an ideal gas taking into account the respective change of state. Derivation of the dependence of κ on the degrees of freedom of an atom and molecule respectively.

Derivation of the adiabatic equation (***Poisson equation***).

Apparatus and Equipment

Apparatus of *Clement-Desormes*

Large, isolated glass flask with a stop cock and liquid manometer to read the pressure differential with respect to the outside air pressure. A colored ethanol-water mixture is used as the manometer liquid. The pressure in the flask is increased by using a small rubber-ball hand pump.

Apparatus of *Flammersfeld*

Glass flask with precision cylinder neck and plastic piston. Argon, nitrogen and carbon dioxide in gas bottles. Photo sensor with counter. Electric stop watch (1/100 s). Barometer.

CAUTION: Please handle the apparatus with care!

The glass flasks are sensitive to breakage, especially the area around the slit opening of the gas oscillators.

Note: The steel gas bottles, under high pressure, present a potential danger and require careful handling. Use the bottles only after instructions from the tutor! Take note of the information in the bench script before beginning the experiment!

Measurement using the *Clement-Desormes* Apparatus

The rubber pump is used to produce an overpressure Δh_1 of 80-100 mm H_2O-column. After the increase in pressure one has to wait until the temperature of the gas has again reached room temperature T_1 and the overpressure has stabilized. Finally, the stop cock is opened for a short time to allow for adiabatic relaxation. The opening time must be suitably selected (about 1 s). A too short or too long opening time leads to systematic falsification of the results. Subsequently, the (isochore) rise in pressure is observed and the maximum value is noted as the experimental result Δh_3. Repeat the measurement for control purposes and for error estimation.

Only one experimental setup is available and must be correspondingly shared by the groups.

Measurement using the *Flammersfeld* Apparatus

A photo sensor with counter and a hand-held stop watch are used to measure the times of 100 τ periods (a series of at least 10 measurements for each gas).

The measurement of the air pressure in the lab is by means of a barometer mounted on the wall. Experimental data (working volume, mass of piston, and diameter of piston) is found in the bench script.

Three experimental setups are available, one respectively for argon, nitrogen and carbon dioxide. Each group cycles through the three experiments.

SOUND WAVES GPI

Key Words

Sound waves; Sound propagation in gases and solids, sound velocity. Standing waves.

Aim of the Experiment

Investigation of sound propagation in gases (air) and solids. Relationship between sound propagation and material constants.

Measurement and recording of transient and periodic processes and computer supported experimentation.

Literatur

[1]: Chapter 4.2, 4.4, 4.5

[3]: 23.4, 24.3

Exercises

1. Determining the velocity of sound in air by measuring the elapsed time.
2. Observing the resonances in a column of air with closed and open end by varying the excitation frequency. Calculating the velocity of sound and the ratio of the specific heat $c_p/c_V = \kappa$ of air (isentropic index or adiabatic coefficient).
3. Determining the velocity of sound in metals from the elapsed time and the fundamental oscillating frequency of the rod clamped at two different points. Calculating the modulus of elasticity of the metal.

Physical Principles

Sound Waves and Propagation Velocity

In an extended elastic medium, a local excitation („perturbations“, deflection from the rest state, pressure variations) leads, via elastic forces to interaction with neighboring volume elements. A wave is thus generated from the excitation and propagates through the medium (*Sound Wave*). The propagation velocity c (phase velocity, sound velocity) is determined by the restoring force (restoring constant D) and the inertia of the accelerated mass (density ρ).

$$c = \sqrt{\frac{D}{\rho}} \tag{1}$$

In a solid, where each volume element has a defined state of rest, longitudinal pressure waves or transverse shear waves can be formed, whereby the restoring constants are given by the modulus of elasticity E or the shear modulus G.

Pressure waves are only present in gases and fluids and the restoring constant is equal to the modulus of compressibility K.

In gases, an energy exchange between individual volume elements hardly takes place because of the relatively short period of sound oscillations and the poor thermal conductivity, so that the adiabatic equations (*Poisson Equations*) are valid. Taking the derivative of the *Poisson equation* $p(V;\kappa)$, one finds for the modulus of compressibility K:

$$p\,V^{\kappa} = const \tag{2}$$

and

$$K = V\,\frac{\mathrm{d}p}{\mathrm{d}V} = -\,\kappa\,p \tag{3}$$

where κ is the ratio of the specific heats c_p/c_V (*isentropic index* or adiabatic coefficient). Thus for the velocity of sound we have:

$$c = \sqrt{\kappa\,\frac{p}{\rho}} = c(T) \tag{4}$$

The velocity of sound is pressure independent, since inertial- and restoring terms (density and compressibility) depend on pressure in the same way. The velocity is, however, temperature dependent due to the additional temperature dependency of the density.

Standing Waves

In a limited volume, a number (ordinal number n) of stationary oscillating states (*standing waves*) are generated by reflection and interference when the wave length λ is in certain ratio to the length ℓ of the resonator.

For a resonator closed at one end one finds:

$$\ell = \left(n - \frac{1}{2}\right)\frac{\lambda}{2} \tag{5a}$$

and for a resonator closed at both ends:

$$\ell = n\,\frac{\lambda}{2} \tag{5b}$$

If the wave length λ and frequency υ are known, the velocity of sound is calculated using the fundamental relationship for waves:

$$c = \lambda\,\upsilon \tag{6}$$

The resonances represent the possible *excitation states* of the system which absorb the excitation energy even with a pulsed excitation. A percussion-type excitation first travels back and forth in the resonator in the form of a shock wave. However, Fourier decomposition shows that the wave consists of a whole spectrum of excitation frequencies. The higher frequencies and resonant harmonics are strongly damped due to the high oscillations so that after a sufficient time only the fundamental (sine) wave remains.

In exercise 2 one can observe and investigate the different behavior of both limiting cases (pulse propagation, resonant states) by selecting the appropriate time window. Sharp pulses are observed shortly after the shock excitation (percussion) and their delay times can be measured. The fundamental wave is observed a certain time after the excitation (here: a few seconds).

Presentation of the Physical Principles

(As a part of the preparation for the report): A short presentation of sound propagation in gases. Calculation of the temperature dependence of the velocity of sound. A discussion of standing waves and resonances in extended media under various boundary conditions (air columns, metal rods supported in different positions along their lengths).

Equipment

Microphone, percussion device (clapping sticks) with trigger cable and metal rule.

Resonance tube with loud speaker and microphone. Waveform generator with counter. Multimeter to measure the microphone output signal.

Metal rod with two support clamps.

Storage oscilloscope PCS 100

Experiment and Evaluation

Exercise 1

The percussion device with built-in contact is connected via a cable to a microphone and the microphone signal is routed to the input of the storage oscilloscope. The percussion device generates at the same time as the percussion signal a short trigger pulse at the microphone output. Since the electric propagation velocity is negligible compared to the velocity of sound, one measures the time from the start of the trigger pulse (= start of the x-scale) to the start of the sound percussion signal. The distance is measured between the percussion device and the microphone.

Vary the distance from the percussion device to microphone.

Check to see whether you can recognize sound reflections (in simple cases they can be used in the evaluation of the results).

Characteristics of the Trigger Signal: short rising edge << 1ms and a slow falling edge without fine structure. As a result, the minimum measuring distance between clapping device and microphone is <~ 30 cm.

Characteristics of the Percussion Signal: fast rising oscillation (T~ ¼ ms) of higher amplitude. Only a few ms long but may become longer due to reflection from the wall or contact bounce.

Implementation:

- Connect: percussion device-microphone-PCS100
- Switch on: Microphone, PCS100, PC
- Symbol: double click OsciPcs100 ⇨ Setup window: pre-settings (Oscilloscope: PCS100, Function Generator: none, LPT Port Address: 378 LPT1) acknowledge with OK
- Oscilloscope settings first [Ocilosc.] ⇨[1ms]⇨[30mV]⇨[DC]⇨[TriggerOFF]⇨ RUN] Result: Low noise signal
- Whistle into the microphone ⇨ sine waves are observed

Measuring the Velocity of Sound in Air

→[Trigger ON] →[Single (shot)]

Slide the trigger level with the cursor one full vertical division upwards.

(Trigger grid =1/2 div.)

Note status button [single]: Status= │stopped │triggered │waiting for trigger│. If the result is not satisfactory, select other settings (volts/time/trigger level) →[Single]

Next measurement

Sound propagation time: 0 to the start of the percussion signal. NOTE for short distances, the percussion signal is superimposed on the falling trigger edge but still observable. Do not place the percussion device on the table when generating a sound pulse. Sound waves through the table travel faster than through air!

If the phone jack of the percussion device is not properly inserted in the microphone socket or is defective then the percussion contact does not trigger but instead the sound pulse triggers. As a result, the response level of the microphone is not at 0 but higher by a few „cm“. The fault can be eliminated by making a new connection or exchanging the microphone. If need be, one can also make measurements without the electrical connection percussion device – microphone by connecting two microphones to the oscilloscope input, setting the trigger level as low as possible and placing one microphone very close and one very far from the percussion device. In this case ensure that distances to sound reflecting surfaces are kept as far as possible. The signals are then observed on the same channel (front and back) but can be distinguished with a little luck.

Exercise 2

Standing waves are excited in a resonance tube (with and without end cap) using a waveform generator and a loud speaker. The standing waves are detected by means of a microphone. The microphone output voltage is measured with a multimeter.

Remove the phone jack of the percussion device from the microphone and place the microphone close to the side opening of the tube. Measure the microphone voltage with the multimeter. A parallel measurement with the oscilloscope (in RUN-Mode) is also possible.

The function generator must be set to sine wave and the symmetry knob to off (fully left), otherwise harmonics are generated which could lead to a false allocation of the resonances.

The resonance frequencies are measured as a function of the ordinal number. For the evaluation of the results according to (5a, b) one should develop a numerical method satisfying the high accuracy of the measurements.

When discussing the results, one must decide whether the tube should be considered as having an open or closed end due to the loud speaker mounting.

The temperature must be taken into account when calculating the velocity of sound according to (3), where, to a good approximation, air may be regarded as an *ideal gas*. A thermometer is available in the lab to measure the temperature.

Exercise 3

The experiment and measurements are first made with the rod clamped in the middle. A microphone is placed very close to one end and a small wooden hammer is used to strike the other end of the rod to generate the oscillations. Observe the change in waveform as a function of time. The initial distribution (to and fro oscillation of the shock wave) is obtained with PCS100 – settings [Trigger ON], [Single]-shot, [1ms], [30ms], or [10mV] and trigger level about 1 div above 0V.

Later distributions are obtained in [RUN]-mode by a suitable selection of the trigger level: the lower the trigger level the later the last triggering.

The asymptotic oscillations are obtained in [RUN]-mode with [Trigger OFF] and [10mV] by observing the decaying signal and freezing the sine-shaped signal by clicking [RUN] (toggle between run and stop)

The asymptotic oscillation with the lowest damping is the fundamental wave for the rod clamped in the middle and the first excited wave with the rod clamped at ¼ and ¾ of its length respectively.

Faults:

When the oscilloscope no longer reacts even at trigger level 0 or [Trigger OFF] and ⇨[Run]: Reset: close the PCS100 programme and start again.

Special effects: To highlight reflected sound waves: Direct the microphone not to the percussion device but to the wall and select a suitable large area without obstacles in the way.

How do the wave forms change after the percussion?

When is the fundamental wave clearly observed?

Plot characteristic diagrams for the limiting cases. The signals of the PCS100 are plotted using an additional computer and printer. Both these units are located near the door to R. 2.08 and must be switched on.

The diagrams are plotted by clicking the file menu in the PCS100 window and clicking Print.

Observe and discuss the oscillations also for the rod clamped twice in a sensible way.

The density of the rod must be measured (balance and sliding calliper in room 2.05).

Supplementary Questions

Why are waves of higher frequency more strongly damped than those of lower frequency?

RADIOACTIVE DECAY GPI

Half-life of ^{108}Ag and ^{110}Ag

Key Words

Structure of the nucleus. Radioactive decay; Decay scheme, Energy (quantum energy), Energy spectrum. Decay time; Decay law, Lifetime and half-life. Nuclear reactions. Counting statistics; Radiation detection.

Aim of the Experiment

Phenomenological treatment of nuclear decay. Introduction to nuclear radiation detection and measurement techniques.

Literature

[4]: Chapter 47, 49

RADIATION PROTECTION NOTICE in annex V of the GP I lab script.

Exercises

1. Common exercise for immediate evaluation: Measuring the natural ion dose rate and the ion dose rate at the outside of the neutron generator with an ion dose rate monitor (*Hand-Monitor*). Conversion to the equivalent dose value per year in mSv/year and mrem/year.
2. Measuring the zero-effect with a Geiger-Müller counter.
3. Activation of the radio isotopes ^{108}Ag and ^{110}Ag. Measuring the time dependence of the decay rate for different activation times. Determining the decay constants and half-lives of both silver isotopes.
4. Calculation of the saturation counting rates for both isotopes and the ratios of their cross-sections for neutron capture reactions. Comparison with literature values (see nuclear tables at the work bench).

Physical Principles

Dosimetry

See *RADIATION PROTECTION NOTICE* in the annex V of the GP I lab script.

Activation via Nuclear Reactions

Natural silver is composed of 51.83 % Ag-107 and 48.17 % Ag-109 (*natural isotope mixture*). The activation of both instable radio isotopes Ag-108 and Ag-110 occurs through (n,γ)-reactions, where after the silver isotopes are converted to cadmium isotopes via ß$^-$ decay:

(1a) $$^{107}Ag\,(n,\gamma)\,^{108}Ag^{*} \longrightarrow {}^{108}Cd^{*} + \beta^{-} + \bar{\nu}$$

(1b) $$^{109}Ag\,(n,\gamma)\,^{110}Ag^{*} \longrightarrow {}^{110}Cd^{*} + \beta^{-} + \bar{\nu}$$

Isotopes in the excited state are labeled with a star (*).

The slow neutrons required for activation are generated by an Americium-Beryllium neutron source (*neutron generator*). The α-radiation of the Am-241 source generates high energy neutrons by a (α,n)-reaction with beryllium. These are slowed down by elastic collisions in a paraffin moderator:

(2) $$^{9}\mathrm{Be}\,(\alpha, \mathrm{n})\,^{12}\mathrm{C}$$

Radioactive Equilibrium (Mother-Daughter System)

During activation, radioactive nuclei are produced at an approximately constant rate since the number of stabile mother nuclei N is very large compared to the number of converted nuclei and thus is practically constant. The simultaneous decay, however, depends on the momentary number of radioactive nuclei *n*(*t*) and at first increases. For the number of radioactive nuclei, the following equation applies:

$$dn = dn\Big|_{Reaktion} + dn\Big|_{Zerfall}$$

(3) $$= \sigma\,\phi\,N\,dt - \lambda\,n\,dt$$

where σ is the cross-section for the reaction, ϕ is the neutron flux density and λ is the decay constant. Integrating (2) gives:

(4) $$n(t) = \frac{\sigma\,\phi\,\mathrm{N}}{\lambda}\cdot\left(1 - e^{-\lambda t}\right)$$

For the build up in activity during activation *A*(*t*) = λ·*n*(*t*) one gets:

(5) $$A(t) = \sigma\,\phi\,\mathrm{N}\left(1 - e^{-\lambda t}\right) = A_s\left(1 - e^{-\lambda t}\right)$$

For large times, equilibrium is established between generation and decay. The obtainable activity A_S is called the saturation activity.

Activity after Activation

After activation is stopped, the decay proceeds according to the simple decay law and the instantaneous activity (5) represents the initial activity.

Radiation Detection

The decays are detected by registering the high energy ß-particles (emitted according to equations 1a,b) with a thin walled *Geiger-Müller tube* and counter. The functioning of the G-M tube is explained in Annex IV *KERNSTRAHLUNGSDETEKTOREN* of the GP I lab script.

Counting Errors in Radioactive Decay

The error Δ*n* for the number of events *n* in a stochastic process is given by (see annex VI *STATISTIK* of the GP I lab script):

(6) $$\Delta n = \sqrt{n}$$

Apparatus and Equipment

Geiger-Müller tube in lead shielding with counter.
Silver cylinder. Am-Be neutron generator. Stopwatch.

Experiment and Evaluation

Pay attention to the *RADIATION PROTECTION NOTICE* (*STRAHLENSCHUTZANLEITUNG)* in annex V of the GP I lab script during the experiment!

To Exercise 1 (Dose Rate)

An integrated measuring unit with a Geiger-Müller counter (*Hand Monitor*) is available to measure the ion dose rate. The conversion to the equivalent dose rate is done by taking into account the basics of dosimetry (see *RADIATION PROTECTION NOTICE* (*STRAHLENSCHUTZANLEITUNG)* in annex V of the GP I lab script).

To Exercise 2 (Zero Effect - Background)

Radiation detection at the silver cylinder is by means of a Geiger-Müller counter. An automatic counter is available to measure the count rate (events per second). The counter operates periodically in 10 s intervals: Zero reset (reset); measurement made during about 9 s; Display of the measured value for about 1 s. The experiment consists in a running observation and recording of the measured values.

Before beginning the experiment, background (zero effect) measurements, over a suitable time, must be made without a radioactive source. The background count must be taken into account later, as a correction to the actual measurements.

To Exercise 3 (Decay Curves)

The silver cylinder is activated in one of the central channels of the neutron generator for two different activation times (1 min and 12 min). When the cylinder is removed from the generator at the end of the respective activation time, the count cycle of the G-M tube is started and the cylinder placed over the tube (see lab bench script for information on operating the counter).

The synchronization between the end of activation and start of the counter (the time axis)) must be exactly kept to otherwise the observed initial activity is strongly distorted due to the short half-life of Ag-110.

The measuring time must be over 12 min. for all activation times even when one gets the (false) impression that the activity has decayed to the zero effect after a short time. The half-life of the long-lived isotope Ag-108 is about $2^1/_2$ minutes and one requires observations over 5 half-lives to get a clear determination of the progress of the separation of both components.

The experiment is evaluated by first correcting the measured values for background counts and then plotting on half-log paper. The behavior of the curve should show two components corresponding to the different decay constants of both isotopes. For large times, one can assume that the short-lived component has decayed completely. This region is evaluated by drawing a best fit straight line from which one can determine the decay constant and the saturation activity $A_s = A(t=0)$ of Ag-108.

Finally, the long-lived portion, corresponding to the values of best fit line for Ag-108, is subtracted from the measured values for short times and these are again plotted on half-log paper. (The curves for both activation times can be presented in common diagrams).

Literature Values

(*Seelmann-Eggebert*, *Pfennig*, *Münzen*; NUKLID-KARTE; Kernforschungszentrum Karlsruhe. See also the Nuclear Chart in the lab room).

Half-Lives

(7a) $T_{1/2}$ (Ag-108) = 2.41 min

(7b) $T_{1/2}$ (Ag-110) = 24.6 s

Cross-section

(8a) $\sigma(\text{Ag-107} \rightarrow \text{Ag-108}_{\text{Ground state}}) = 34.2$ b (Barn)

$= 34.2 \cdot 10^{-28}$ m^2

(8b) $\sigma(\text{Ag-109} \rightarrow \text{Ag-110}_{\text{Ground state}}) = 89$ b

GAMMA SPECTROSCOPY GPI

Key Words

Nuclear structure. Radioactive decay; Decay scheme, energy (quantum energy), energy spectrum. Interaction of γ-radiation with material; photo effect, *Compton effect*, pair formation effect. Counting statistics. Radiation detection.

Aim of the Experiment

Introduction to the phenomena of nuclear decay and the different types of radiation. Introduction to the associated measuring techniques.

Literature

[4]: Chapter 47, 48

See also *STATISTICS, NUCLEAR RADIATION DETECTORS, RADIATION PROTECTION NOTICE AND DOSIMETRY* in the annexes IV, V and VI of the GP I lab script.

Exercises

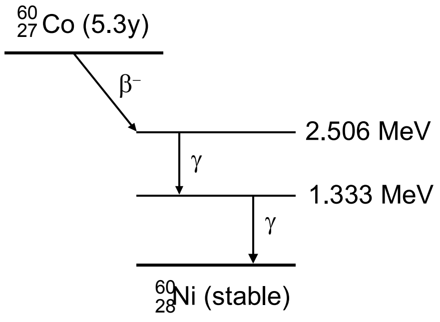
1. Common exercise for immediate evaluation: Measuring the natural equivalent dose rate and the dose rate of the ^{60}Co probe 0.5 m from an integrated equivalent dose rate meter. Conversion to the equivalent dose value per year in mSv/year and mrem/year.
2. Recording the γ-spectra of ^{60}Co, ^{137}Cs, ^{22}Na and ^{241}Am and calibration of the spectrometer (calibration curve).
3. Determining the energy of the e^+-e^- annihilation radiation and comparison with *Einstein's equation* ($E=mc^2$).
4. Determining the resolution of the spectrometer for the γ-lines of ^{137}Cs.
5. Determining the maximum energy transfer in *Compton scattering* (*Compton edge*) for the γ-lines of ^{137}Cs and comparison with the theoretical value from the scattering formula.
6. Checking the absorption law and determining the attenuation coefficient and half-value thickness for iron and lead for the 0.662 MeV-γ radiation of ^{137}Cs.

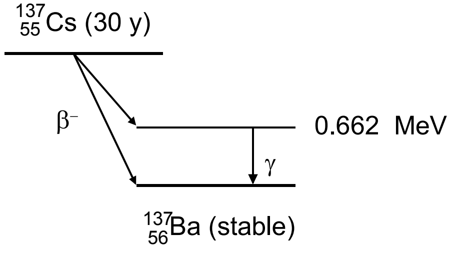
Physical Principles

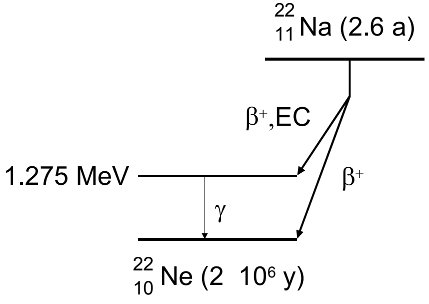
Dosimetry

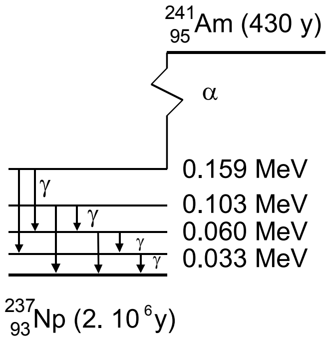
See *RADIATION PROTECTION NOTICE AND DOSIMETRY* in annex V of the GP I lab script.

Decay scheme of the radiation sources provided









Interaction of γ-radiation with mater

Under the assumption that the interaction probability is proportional to the thickness d*x* of the absorber and that a quantum of radiation is lost to the radiation field in an interaction process, one gets an exponential law for the intensity:

$$I = I_0 \, e^{-\mu x} \tag{1}$$

The absorption coefficient μ (*linear attenuation coefficient*) depends essentially on the energy of the radiation and the density of the absorbing material. The quantity μ/ρ (density ρ) is approximately constant for a given radiation energy and is designated as the *mass attenuation coefficient*.

Three processes are responsible for absorption:

Photo Effect

In the photo effect, the γ -quantum is stopped by an electron (of the inner shell) of an atom and its energy is completely transferred to the electron. The electron is ejected from the shell.

The atom reorganizes itself whereby an electronic transition into the hole of the inner shell takes place accompanied by the emission of characteristic Röntgen radiation.

Compton Effect

The *Compton Effect* (*Arthur Holly Compton*; 1892-1962; Am. physicist) is an inelastic scattering process of γ -photons on free- or weakly bound electrons. The energy T transferred to the electron is dependent on the scattering angle Θ:

$$T = \frac{E_0}{1 + \frac{m_0 c^2}{(1 - \cos\Theta) E_0}} \tag{2}$$

where E_o is the energy of the γ -photon. The maximum energy transfer occurs in back scattering ($\Theta = 180^0$).

Pair Formation Effect

In pair formation, radiation materializes corresponding to the *Einstein equation* (*Albert Einstein*; 1897-1955; dt./am. physicist):

$$E = m c^2 \tag{3}$$

Pair formation occurs when a γ -photon, of sufficient energy, interacts with the field of the nucleus to produce an electron-positron pair.

Pair formation is the opposite process to the transformation of an electron-positron pair in radiation energy (annihilation radiation), where for reasons of energy conservation two γ-photons are produced. This annihilation radiation can be observed on $ß^+$-radiation, e.g., emitted by Na-22.

Radiation Detection and Spectrometer; Resolution

See *NUCLEAR RADIATION DETECTORS* in annex IV of the GP I lab script.

Counting Errors in Radioactive Decay

The error Δn of a number *n* of events in a stochastic process is given by (see *STATISTIC* in annex VI of the GP I lab script):

$$\Delta n = \sqrt{n} \tag{4}$$

Apparatus and Equipment

NaI-Scintillation spectrometer with high- and auxiliary voltage supply. PC-supported multi-channel analyzer.

Iron- and lead absorbers of various thicknesses.

Experiment and Evaluation

Pay attention to the *RADIATION PROTECTION NOTICE* (*STRAHLENSCHUTZANLEITUNG)* in annex V of the GP I lab script during the experiment!

To Exercise 1 (Ion Dose Rate)

An analog ion dose rate meter (*Hand Monitor*) is available to measure the ion dose rate. Make the measurements over a certain time span in order to estimate the measurement error. The evaluation should be made straight after the measurement and applies to the whole group.

Spectrometer and Multi-Channel Analyzer

To operate the spectrometer, the high voltage (of the secondary electron multiplier), the power supply of the A/D converter (*Peak-Detector-Interface*) and the computer must be switched on.

The high voltage (coarse adjustment) must be set to the mark on the unit (ca. 700 V).

The multi-channel program in the computer start automatically after switch on, the measurement- and program function are self explanatory. Additional information is found in the lab bench script. Before stating the measurements, make yourself familiar with the functioning of the multi-channel analyzer.

The amplification of the system is adjusted at the peak-detector-interface with slide-switch (1×) and rotary knob (2..6) such that the 1.33 MeV line of Co-60 is visible at the right edge of the screen.

To Exercise 2 and 3 (Calibration of the Spectrometer; e^+-e^-)

Calibration of the spectrometer and determination of the e^+-e annihilation radiation (using a Na-22 source) is done by recording the individual spectra of the sources and determining the position (channel number k) of the respective γ -line. The statistically exact position of a line is given by its center of moment $\bar{k}$ (mean value, 1. Moment):

$$\bar{k} = \frac{\sum n(k) k}{\sum n(k)} \tag{5}$$

when *n*(*k*) is the respective channel content (number of events). The measurements begin with the 662 keV-line of Cs-137. For this line, the center of moments shall be calculated over several channels in the neighborhood of the maximum and compared with the position of the maximum during the experiment.

The calibration curve of the spectrometer is made by plotting the measured channel numbers of the lines against the energy. The energy of the e^+-e annihilation radiation is read from the curve.

In order to get a descriptive picture of a spectrum, the run of the complete spectrum of Cs-137, as displayed on the screen, shall be sketched in the protocol book.

To Exercise 4 (Resolution)

The complete 662 keV-line is recorded and plotted later point for point. The measured values are fitted to a bell-shaped curve (with a curve template) and then the full width at half maximum is read from the curve. The relative resolution is determined directly from the channel number, independent of the calibration.

To Exercise 5 (*Compton Edge*)

The position of the *Compton edge* is estimated from the middle of the falling edge after the *Compton plateau*.

To Exercise 6 (Absorption Law)

Checking the absorption law is also done with the 662 keV-line of Cs-137. Various iron- and lead absorbers are available and placed between source and detector.

In these measurements, the intensity must be taken into account which can be directly determined using the function *Fläche* (area) of the multi-channel analyzer.

When conducting this experiment, take into account all possible systematic errors, their minimization and their possible influence on the results. If necessary, use the functions *Impulsgrenze* (pulse limit) or *Zeitgrenze* (time limit) of the multi-channel analyzer.

The attenuation coefficient and half-value thickness are evaluated by plotting the measurements on half-log paper.

ANNEX I
ERROR CALCULATIONS
GPI

Real variables, due to the nature of things possess a distribution with a stochastic character and the measurement of such values is, due to this character and other unavoidable shortcomings of the measurement procedure only approximate. Even when the measured values are represented by numbers these have not the properties of exact values in the mathematical sense but represent random single values of distributed variables.

In all experimental work this leads to the application of statistics in quantitative investigations whereby the statistical methods of measurement applied to physics are known as error calculations. In the frame of the basic lab course in physics, simple methods of error calculations are considered and implemented to give one an understanding of error calculations and to impart a critical, statistical treatment of measured results.

Error calculations belong to the elementary methods of the physicist and a discussion of errors and preliminary investigations on achievable accuracy are always in the forefront when designing an experiment. This is because conducting an experiment only makes sense when the errors are sufficiently small in order to give a meaningful answer to the question posed.

Basic Statistics

Statistical Principles

Statistical considerations and methods depend on two elementary principles:

(1) Statistical statements indicate *ensemble properties*, whereby one cannot draw a conclusion from an individual case to the collective.

(2) Statistical statements are probability statements with finite accuracy and (thus correlated) finite certainty.

Empirical Situation

The uncertainty of physical quantities themselves and the additional unavoidable random and systematic deviations due to the measurement method lead to a distribution of measured values. The centre of the measured value distribution must not necessarily coincide with the centre of the distribution of quantities. Thus measured results can only be approximately determined and exact values remain out of reach. For this reason, the results obtained from the practical data using the methods of mathematical statistics are termed *estimates* whereby complete estimates take place in intervals (*interval estimates*), which represent the position (value) and the spread of quantities (error).

Random Errors: Error Distribution and Error Interval

Measured values have a *normal distribution* due to random influences; this is an empirically observed fact. Normal distributions are distinguished by two parameters; the expectation value μ describes the *position* and the standard deviation σ the *spread* of the distribution, whereby 68 % of all values of the distribution are found in the interval $(\mu \pm \sigma)$ about the expectation value. Conversely, for an arbitrary value of the distribution, e.g. a measured value x_i, then for a similar interval $(x_i \pm \sigma)$ the statistical probability is also 68 %, that the expectation value is within this interval. These types of intervals are designated as *interval estimates* for the quantity X and are called confidence- or *error intervals*.

The interval $(x_i \pm \sigma)$ is the complete result of a measurement; the interval radius itself is called *error* Δx. It is a measure of the expected deviation and thus represents an *accuracy* in the framework of a given probability, the *statistical certainty*. Here, a type of "uncertainty relation" exists between the accuracy and the certainty: the more accurate a statement is made i.e., the smaller the estimate of the error interval, then the less the certainty of the statement, i.e. the probability that the given interval encompasses the expectation value.

For clarification we repeat: in physics and measurement technology the underlying measure of error is the simple standard deviation, where the statistical certainty is 68 % (≈2/3) with still a remaining *probability of error* of 32 % (≈1/3). (In other specialities this is not acceptable for special reasons, and higher certainties are implemented, as in the biosciences and medicine where typically a three-fold standard deviation is used encompassing a statistical certainty of 99.7 %).

The error intervals resulting from measurements are to be considered basically as *homogenous* i.e. the resulting value taken as the mean of the interval is not more probable (and hence, not better) than any other value of the interval.

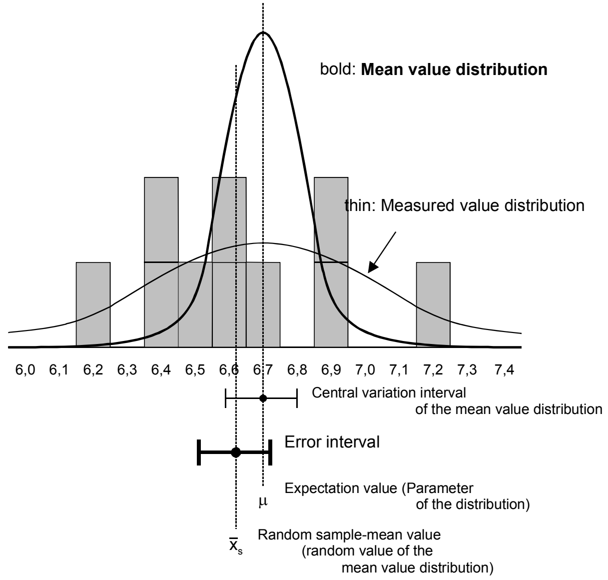
Systematic Errors

Aside from random errors *systematic* influences appear which lead to certain, one-sided deviations from the actual values, as e.g. a bent pointer, tilted scales or calibration errors. Systematic errors are essentially avoidable, but difficult to recognise (random errors are recognised by their spread). In the scope of the basic lab course, no difference is made between random- and systematic errors and they are handled equivalently.

Example of a Distribution of Measured Values

Graphical representation of the histogram (rectangular area) of a collection of data (random sample) and the associated (hypothetical) distribution function.

The relationships displayed show an example with a low resolution in comparison to the standard deviation of the distribution function, which often occurs in problems of measurement, and a very small random sample (10 measurement values). One clearly sees that such a low collection of data can hardly be considered as a random sample in the statistical sense and only represents a very coarse *approximation* of the distribution function.



Closing Comparisons

Any conclusion from results follows from quantitative comparison: from dependencies in the variation of a parameter, between theory and experiment, through comparison with available data (literature values). Three types of comparisons have been specified in the sense of *statistical test*:

Results are assessed as (unrestricted) identical when their *(simple) error intervals* encompass each other.

Results are assessed as compatible when they encompass each other in the scope of the three-fold *error interval.*

Results are first considered as significantly different when the deviation exceeds the three-fold error margin.

Resolution

Many measurement methods show no spread and deliver stable values. This is because each measurement method and each number combination (scale, instrument display) possess a limited resolution, below which the values cannot be separately resolved or represented. All real values have *discrete* character and behave in a stabile manner when the spread lies under the given limit of resolution.

In this case, error estimation must take into account the scale resolution (ability to read the scale) or the number representation (digital displays) and, if necessary other circumstances of the measurement. Here, one speaks of *practical error estimation*.

With analog instruments (scale/pointer) one can, within the scale division, usually read a further estimate of error and the error depends on the number of scale divisions and parallax influences and is of a subjective nature:

$$\text{Analog display}: \Delta_{\text{Est.}} = 0.1 - 0.5 \text{ scale div.}$$

Now 0.5 scale divisions represent an upper limit when the estimation of an intermediate value is not possible and hence, a whole display interval must be taken as the error interval.

With digital instruments the error estimation is taken as (±) 1 in the last digit of the display:

$$\text{Digitall display}: \Delta_{\text{Est.}} = 1\,\text{d} \quad (\text{digit})$$

Equipment Requirements: Nominal Error

Errors present in each measuring instrument are listed by the manufacturer in the respective manual. These must be taken into consideration by the user and are designated here as nominal errors.

Multimeters (*U/I/R/C/L*) are typical pieces of lab equipment possessing nominal errors. Analog multimeters are characterized by their *quality class* which give the absolute error as a percentage of the measurement range:

$$\text{Quality class}: \quad \Delta x = \frac{k}{100} \cdot \text{Measurement range}$$

The quality class is (aside from other labels) is given as a small number (in the range of around 0.5 to 3) together with the type of current (= or similar for dc; ≈ for ac) on the scale display of the instrument.

With digital instruments, the error comprises a relative component in % of the *measured value* (% v. M.) and an absolute component in *digits* (d; unit of the last digit):

$$p\,\%\ \text{v.M.} + n\,\text{d}:$$

$$\Delta x = \frac{p}{100} \cdot \text{Measured value} + n \text{ Unit of last digit}$$

In the lab course, the nominal errors for the instruments are listed in the respective lab bench scripts if they make a dominant contribution and must be taken into consideration with the measured results.

Terms

The expected deviations are themselves designated as *absolute errors* and generally written with a capital delta; the absolute error is used as a measure for the comparison of quantities:

$$\Delta x = \text{Absolute Error} \quad (:= \text{measure of comparison})$$

In *error propagation* (see below) the absolute error is required for additive relationships (+/-).

Furthermore, as a *measure of accuracy* the *relative error* is also used and gives the relationship of the error to the measured quantity. The relative error is written with a small delta:

$$\delta x = \frac{\Delta x}{x} = \text{Relative Error} \quad (:= \text{measure of accuracy})$$

The relative error is a ratio and dimensionless. In the respective range of values, the relative error is usually given as a percentage (1% = 0.01).

For small values, powers of ten are used ($\delta x = 3 \cdot 10^{-5}$ for $\delta x = 0.00003$).

In error propagation (see below), the relative error is required for multiplicative relationships (x/÷).

Structure of Error Calculations

In general an experiment consists of the actual *measurement* comprising the recording of measured values and the determination of the results, the final *evaluation* with the arithmetical calculation of the results or functional relationships. The associated error calculations comprise the determination of the *measurement error*, the calculation of the error in arithmetical evaluations, termed *error propagation* and the *error determination* in the evaluation of functions.

Measurement Error

Measurement error can be basically ordered into three categories. In practice, usually only one of the contributions dominates and can be considered on its own (see below: *contributing* und *non-contributing errors*).

To any measurement one usually has a preceding *setting* (measuring with a ruler, activating a stopwatch, adjusting or setting optical equipment, setting an additional "parameter", e.g. a temperature), whereby, in the following no difference is made whether the error of the measured value only comes from the measurement procedure or additionally from the required settings.

Control Measurements

To recognize the behaviour of the distribution or spread of measured values it is fundamental to repeat measurements or in continuous measurements, to observe a value over a sufficiently long time.

If the value shows a (clear) spread, then from a statistical point of view a *random sample* must be upraised, i.e. a series of measurements made and statistically evaluated (see the following section).

On the other hand, if the values are constant, the spread is not resolved and the error must be estimated from the resolution and other measurement conditions.

Random Sample Estimation (measurement series and error of spread)

One essentially cannot make any (statistical) predictions from a (single) measured value of a fluctuating quantity; especially one cannot give the standard deviation of the measured value distribution as a measure of error. From a random sample (series of measurements) taken as an approximation of the distribution one can not only calculate approximate values for the expectation value and the standard deviation of the distribution itself, but also the standard deviation of the *mean value distribution* from internal statistical relationships. As a result one then has the (single) mean value with its error. When x_s are the random sample measured values and n their number, then the mean value x is given by:

$$x = \bar{x}_s = \frac{1}{n}\sum x_s$$

The standard deviation $\sigma_{\bar{x}}$ of the mean value distribution as an error depends on the spread (standard deviation σ) of the measured quantity itself and the scope n of the random sample:

$$\Delta x = \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

The standard deviation σ of a distribution is the square root of the mean quadratic deviation σ^2 (*variance*), whose construction corresponds to that of the mean value:

$$\sigma^2 = \frac{1}{n}\sum (x_i - \bar{x})^2$$

Since the series of measurements as a random sample gives only an approximation of the distribution, the standard deviation also can only be approximately calculated. The usual designation for the approximate value is to suffix the standard deviation with n-1 since one divides by n-1 instead of n:

$$\sigma^2 \approx \sigma_{n-1}^2 = \frac{1}{n-1}\sum (x_s - \bar{x})^2$$

Hence, the error of the mean value is then

$$\Delta x \approx \sqrt{\frac{\sum (x_s - \bar{x}_s)^2}{n\,(n-1)}}$$

In simple standard measurement procedures typical for the basic lab course, distributions are seldom observed and statistical evaluations as above are rare.

Error Propagation

Linear Combinations of Distributions

Error relationships are based on the rules of interrelations between distributions, whereby, a compensating effects comes about when in individual cases positive and negative deviations come together and reduce the summed deviation. In the elementary case of a linear combination of two statistically independent distributions X_i and Y_i yielding a summed distribution $(aX + bY)_i$ the mean values and variances (mean quadratic deviation) add corresponding to their linear construction:

$$\overline{a x_i + b y_i} = a\bar{x} + b\bar{y}$$

and

$$\sigma_{ax_i+by_i}^2 = a^2\,\sigma_x^2 + b^2\,\sigma_y^2$$

Gaußian Error Propagation Law

The value z is given as a function of measured variables $a, b, c, \ldots$:

$$z = f(a,b,c,\ldots)$$

For error calculation, the error in measurements Δa, Δb, Δc, ... are considered as small deviations of the measured results a_0, b_0, c_0, ... and to a good approximation the error function is expanded as a linear Taylor series:

$$z \approx z_0 + \frac{\partial f}{\partial a}\Delta a + \frac{\partial f}{\partial b}\Delta b + \frac{\partial f}{\partial c}\Delta c + ...$$

When constructing the standard deviation as error Δz the constant contribution z_0 is dropped and one gets the *Gaußian Error Propagation Law*:

$$\Delta z = \sqrt{\left(\frac{\partial f}{\partial a}\Delta a\right)^2 + \left(\frac{\partial f}{\partial b}\Delta b\right)^2 + \left(\frac{\partial f}{\partial c}\Delta c\right)^2 + ...}$$

The partial derivatives represent *weighting factors* for the contribution of the individual measured quantities. The addition of the squares and taking the square root takes into account the character of the distribution with the probability of compensation of deviations (the total error is smaller as the linear sum of the components).

The Gaußian Error Propagation Law makes one important qualification. It assumes that the measured variables are statistically independent and non-correlated.

Elementary Rules

The partial derivatives can make the error calculations rather cumbersome and for arithmetic relations it is easier to handle error propagation "from the inside to outside" in pairs corresponding to the procedure of arithmetic calculations of the result itself:

$$\Delta(a +/- b) = \sqrt{(\Delta a)^2 + (\Delta b)^2}$$

Hence, with addition or subtraction the absolute errors "add" (quadratic error and root). For multiplication and division we have:

$$\delta(a \times/\div b) = \sqrt{(\delta a)^2 + (\delta b)^2}$$

That is with multiplication and division the relative errors "add".

In powers (roots) the relative error is multiplied by the exponent:

$$\delta(a^r) = r\,\delta a$$

For values of higher functions the error must be calculated by taking derivatives. The use of the elementary rules also assumes statistical independence of the individual quantities so that they are only suitable for error evaluation where each measured quantity enters only once.

Maximum Error

If statistical independence cannot be assumed and measured quantities taken as correlated, then the above combination rules for distributions are invalid. In this case the most unfavourable case is taken that all measured quantities deviate in one direction and up to the limit of the error interval. The propagation law for the maximum error is then:

$$\Delta z = \left|\frac{\partial f}{da}\right|\Delta a + \left|\frac{\partial f}{db}\right|\Delta b + \left|\frac{\partial f}{dc}\right|\Delta c + ...$$

Estimation of the Limiting Value

If error functions are very complicated a limiting value (error limit) can be calculated by using the limits of the measured value. One must take into account in which manner the individual quantities go into the expression (summand/subtrahend; numerator/denominator). The error is then the difference between the limiting- and resulting value:

Result $z = f(a, b, ...)$
Upper limit $z_+ = f(a_\pm, b_\pm, ...)$
Error $\Delta z = z_+ - z$

Direct Measured Quantity

Before beginning error propagation calculations the error equation must be ascribed to the direct measured quantity in order to recognize erroneous error contributions caused by redundant quantities in intermediate values or correlations. Trivial example:

As an exercise, one is to calculate the ratio of two forces $G = m \cdot g$. In the preceding individual calculations of G_1 and G_2, the relative error of the gravitational acceleration g goes into the ratio G_1/G_2 twice, whereas in reality the gravitational acceleration cancels out.

Graphical Evaluation of Functions

Experimental investigations and evaluation of functions covers the qualitative assessment of function types and the quantitative determination of the parameters (axis intercept and slope as an example for a linear function). For the qualitative assessment a graphical representation is always required which at the same time is a powerful tool for the quantitative determination. The evaluation of parameters can, when the type of function is given, also be done numerically (linear regression for straight line; best fit calculations for arbitrary functions); however, the numerical effort is relatively high. Moreover, the graphical evaluation of functions gives one practical experience in the critical assessment of run of the function.

A graphical evaluation is possible for linear functions or „linearized“ functions; the methods are described in annex II

With these functions, error estimation is simple and uses the principles described above for limiting values. Aside from the *line of best fit* with the parameters *a* (axis intercept) and *m* (slope) one also constructs a limiting line which delivers the limiting values a_G and m_G of the parameters, and thus the error. The results are then:

$a = (a \pm \Delta a)$ with $\Delta a = a_G - a$ and
$m = (m \pm \Delta m)$ with $\Delta m = m_G - m$

Details concerning the construction of lines of best fit and limiting lines, on the linearization of functions and calculations are found in annex II.

Methodological Supplements

Representing the Results (result interval)

A clear and concise form of representing the results as a result interval is to specify the interval middle (result) and the interval radius (error):

(Example) $I = (27.4 \pm 0.5)$ mA

This type of representation is preferred in the lab course. An alternative representation is to write the error in brackets directly after the result; the example above is then:

I = 27.4(5) mA.

For values from reliable sources where errors are not specified, the usual convention is to assume an error of 1 (one unit) *in the last digit*.

Contributing and Non-contributing Errors

Errors are also estimated values with accuracies usually of one or more orders of magnitude lower than the result. The aim of an error calculation can therefore be confined to capturing the essential part of an expected deviation while ignoring small contributions; especially with respect to error propagation and additional, systematic rounding-up in the calculation procedure (see below).

Small error contributions can be ignored in error propagation thus reducing time and effort.

Errors are to be considered as small when they are half an order of magnitude (factor 3) or more lower than the contributing errors.

Rounding of Errors

For the same reasons as above, i.e. the low accuracy of the error value itself, it is not meaningful to give the error with more than one *counting digit*, whereby the accuracy of the error in the most unfavourable case is reduced by a factor 2 equivalent to 100 % (in the range of values between 1 and 2).

In doing so, the given statistical reliability must not be compromised:

Error intervals may only be *rounded up*!

If evaluations are not calculated through to the end but intermediate values noted and included latter then errors should be specified to two digits to prevent rounding errors making an increasing contribution.

Number Representation of the Results

The number representation of the results cannot feign to have better accuracy or possess a higher resolution than that given by the error. The steps in calculation (multiplication/division) often result in an excess of digits which must be then (“properly”) rounded.

The numerical value of the result and error must end in the same order of magnitude (decimal power).

A consistent result after calculating a resistance [U = (9.13 ± 0.06) V; I = (243 ± 3) mA] would be:

(Example) R = (37.6 ± 0.9) Ω

It would be false to write the result to more than one digit after the decimal point since these, with respect to the error, these places can take on any arbitrary numerical value and are thus meaningless;

(Example) *False:* R = (37.5720 ± 0.8) Ω

On the other hand, zeros are also counting numbers and must not be suppressed when needed for numerical resolution. If a voltage with an error of 0.06 V is accurately set to 9.00 V it would be false not to write down the zeros:

(Example) False: U = (9 ± 0.06) V

Conclusion

Errors can always be estimated!
Often, however, situations arise, where this cannot be done in a purely schematic way but requires a differentiated and critical examination of the circumstances surrounding the measurement. Basically, the assumption is false that errors cannot be estimated because they are too small or too large!

ANNEX II
GRAPHICAL REPRESENTATIONS

GPI

and

GRAPHICAL EVALUATION OF FUNCTIONS

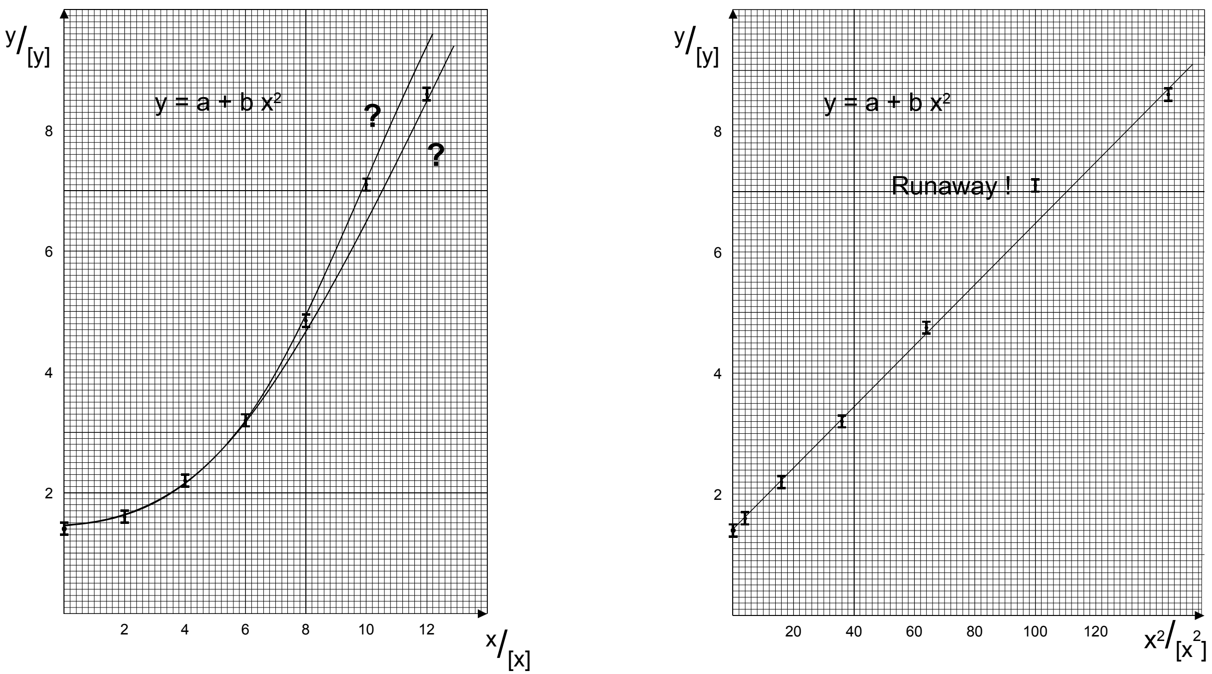
A picture says more than a thousand words.

Humans possess an amazing visual capacity. This is the basis for the power of graphical representation which makes possible the visualization of information.

Graphical representations serve to display measured results, allow judgements and comparisons, make possible the efficient evaluation of parameters and provide a practical form of presenting numerical relationship.

Moreover, graphs are useful tools for the momentary control of a measurement. In many cases it is advantageous to plot measured values at the same time as recording them. For example, a jump in an otherwise smooth curve could indicate an error in measurement and allows an immediate check of the circumstances which would be more difficult at a latter time or even impossible (error in reading an instrument, switchover to another measuring range, sudden change in measuring conditions). Furthermore, the immediate assessment of a measurement helps better to select parameters, makes possible to recognize the scattering of values and allows the quick evaluation of intermediate results.

The use of calculators and computers cannot be considered a substitute for graphs but graphs are often the prerequisite for the meaningful use of computers. Selecting models for best fits, eliminating runaway results, for example, can be seen at a glance.



Example 1: Qualitative check of a power functions by a linearized representation.

In the direct graph (above) a clear decision cannot be made as to the progress of the curve.

The linearized curve (right) clearly shows the probable run of the curve. If the curve had been drawn during the measurement the supposed runaway result could have been recognized and checked in good time.

Graphical Representations

The explanatory power of a graph strongly depends on its content and form.

To this belongs: the selection of the r*epresented quantities*, the *coordinate system* (right-angle coordinates, polar coordinates), *axis division* (linear, logarithmic), *axis scale* and range and of course a clean and clear construction. A graph must be constructed in such a way that the data is correctly and accurately rendered.

Graph Paper

Graphs constructed for the lab reports must be drawn on (original) graph paper with suitable axis divisions:

- *mm paper for linear curves*,
- *simple- and double log paper* for exponential- and power functions
- *Polar coordinate paper* and *probability paper* for polar functions and normal distributions etc.

Axis Scale and Division

The axis scale and the intervals (division) must be selected such that the measured data can be completely and clearly plotted.

In the framework of finite drawing accuracy, information and accuracy must not be lost through the selection of scale and division.

If the represented quantities show a distinct dependence on each other, then the area available should be used to the fullest. If a relatively stable quantity is plotted then a suitable compromise should be found. A small scale would make it difficult to spot possible tendencies and a too larger scale would make things unclear due to the scattering of data.

The graph must make it possible to completely assess the data.

With an expected zero-point line, the zero point must be drawn in order to observe and assess the run of the (extrapolated) fit curve at the zero point.

The division of the axes should be simple.

For example, one, two or five units per cm graph paper for linear plots. Complicated divisions make reading and recording laborious and are often the cause of errors.

Axis Labelling

The main divisions of the axes must be labelled with the measure numbers, the whole axes are specified by the symbol for the represented quantity divided by the unit.

Each measured value is a product of the measured number and unit so that the numbers on the axes represent the quotient out of quantity and unit. For clarity it is recommended to clearly separate symbol and unit labels (numerator and denominator). For example:

$$v \Big/ 10^{-2}\,\frac{\mathrm{m}}{\mathrm{s}} \quad \text{and not} \quad 10^{2}\,\frac{v}{\mathrm{ms}^{-1}}$$

$$\frac{1}{T} \Big/ 10^{-3}\,\mathrm{K}^{-1} \quad \text{and not} \quad \frac{10^{3}\,\mathrm{K}}{T}$$

It is incorrect to set units in square brackets.

Quantities a placed in square brackets to designate their units, e.g. [p] = mmHg.

Marking measured values

Measured values are marked by points, crosses or small circles.

Errors can be directly represented in the form of error bars. In general it is sufficient to draw only a few representative error bars.

Curves (Fit Curves, Theory Curves)

Solid curves in graphical representations can have different meanings (in general two). The exact meaning of a solid curve must therefore be explained in the representation.

Fit curves can be drawn as solid lines and these may be considered as functional curves whose qualitative form is given by some underlying theory or model whose parameters fit well (minimum of the quadratic deviation) to the measured values

Pure theoretical curves can also be drawn as solid lines, whereby the function is determined by model considerations.

It is not meaningful to connect measured values from point to point as if drawing a polygon! For thing, this contradicts the statistical randomness of the individual measured values and at least in the macroscopic world, physical processes in general are continuously differentiable i.e. "smooth".

Legend (labelling)

The graph must have a sufficient and unambiguous explanation of the relationships investigated (legend)!

To this belongs not only the higher-order labelling of the data (e.g. "Rotational Motion; Angle-Time Law") but also references for the report or a more detailed description of the measuring conditions (e.g. "Test series II; without additional weights").

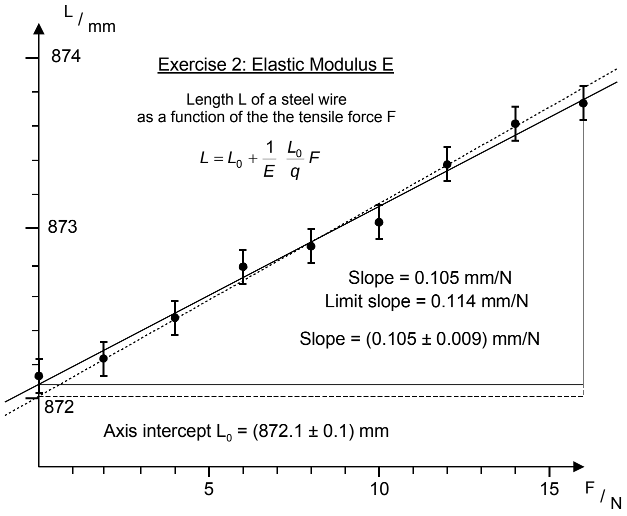
GRAPHICAL EVALUATION OF FUNCTIONS

The numerical evaluation or fitting of a function to a random sample of scattered measured values requires much computing effort. The most common task is to fit a linear function (*linear regression*).

This can be carried out simply, clearly and with good results with the help of a graph by drawing a line of best fit taking into consideration the location and distribution of the points (see example 2 below).

The power of this „visual mean value calculation" is made clear when compared with numbers churned out with the numerical method.

In the numerical method, it should also be mentioned that the construction of a curve is essential to assess the match of the fit to the data points since the method fits a line to the data even when the values show no linear characteristics.



Example 2: Graphical Evaluation of a Linear Function (for clarity no mm graph paper is shown)

Constant error $\Delta L = 0.1$ mm and $\Delta F = 0.001$ N (weight m·g; balance error 0.1 g). The error ΔF can be neglected in the framework of ΔL and drawing accuracy. Qualitatively, a linear relationship is well given. The scattering of the measured values are in agreement with the error bars. Slope:

$$\text{Slope} = \frac{(873.76 - 872.08)\ \text{mm}}{(16-0)\ \text{N}} = 0.105\ \text{mm}/\text{N}$$

$$\text{Limitslope} = \frac{(873.84 - 872.02)\ \text{mm}}{(16-0)\ \text{N}} = 0.114\ \text{mm}/\text{N}$$

Higher functions can also be graphically filled by suitable linearization (see below).

Non-linearizable functions can only be evaluated numerically with a large amount of computer effort. The basis for all fit procedures is the variance.

Parameter Estimation of Linear Functions

The random scattering of measured points should follow a normal distribution:

The line of best fit is selected such that the measured point lie symmetric about it, whereby single far-off points need not be considered since they have a large error probability (runaway point).

If aside from random scattering a systematic tendency is recognizable (e.g. a slight curvature) then additional criteria must be considered in relation to the physical problem of the measurement method (see supplementary notes).

A fit curve is best made with a Plexiglas rule having a black line in the middle and possessing no markings or divisions.

Estimates for the parameters of the function (axes intercepts, slope) are then determined by the fit line:

The axis intercept can be read directly at the cut point of the fit line with the y-axis.

The slope is determined by fixing a right-angle triangle with the fit line and calculating the tangent.

A large triangle should be selected in order to hold read errors as small as possible. The slope triangle should be drawn in the graph to check the evaluation.

When determining the parameters note that, in general, the variables are dimensioned quantities and correspondingly, the intercept and slope have units.

Graphical Error Estimation

A limit line is used for the graphical determination of error.

The limit line is a line which is just compatible with the measured values taking into consideration scattering and error bars. Fixing the position of the limit line requires a critical inspection of the measured values with respect to these.

The limit line directly marks on the ordinate (y-axis) the error interval Δy. The error in slope is the difference between the fitted slope and the limit line slope.

In the scope of error considerations, a measurement is taken to be consistent when the error bars and the mean scattering are about the same. Error bars which are too small indicate false or marginal error estimation or an unconsidered source of error. Error bars which are too large indicate incorrect error estimation or additional systematic errors. For example:

If one measures a voltage (e.g. as a function of the load current), then the scattering of the values will be considerably smaller than the error given by the quality class of the measuring instrument. The quality class describes essentially the systematic calibration error of the instrument. It therefore follows for the error of the parameter that the axis intercept is afflicted with the full uncertainty as given by the quality class. The slope, however, can be given with a better accuracy resulting from the scattering since a systematic shift of the points upwards or downwards does not influence the slope.

In general, for the above reasons, the fixing of the limit line can be oriented more to the scattering of the measured points whereby, as an additional criterion the number of measured values must be taken into consideration (corresponding to the increase in accuracy of the mean value with the number of points). But:

A correct allowance for the number of measured points is visually difficult and graphical evaluations deliver, as a rule, errors which are too large.

If higher demands are placed on the comparability and accuracy of error predictions then numerical evaluations must be made.

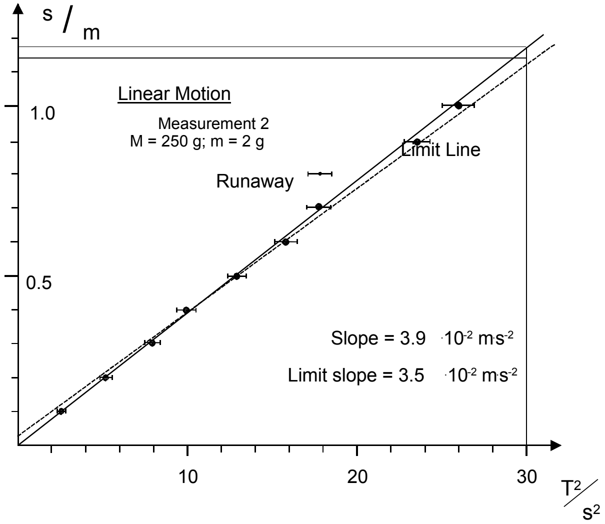
Evaluation of Non-linear Functions

Non-linear functions must often be checked and evaluated, however, a visual assessment and graphical analysis is only possible for linear relationships.

In many cases a linearization of the function or curve can be achieved by selecting suitable variables (substitution) by a transformation.

Variable Substitution

The parabola s = ½ a t^2 is linearized, when s is not plotted as a function of t, but rather as a function of t^2 (see example below). The "Projection" $K = K_0 \cos \omega t$ is linearized (in the range of ωt from 0 to $\pi/2$) when K is plotted against $\cos \omega t$ and not t.



Example 3: Evaluation of a quadratic function via a linearized plot

Constant measurement error $\Delta s = 0.001$ m (negligible) and $\Delta t = 0.04$ s. The error of t^2 is $\Delta t^2 = 2\, t\, \Delta t$. Since a zero-point line is expected and, within the errors, is obviously seen, the fit line and limit line go through the zero point.

$$\text{Slope} = \frac{(1.17 - 0)\ \text{m}}{(30 - 0)\ \text{s}^2} = 3.90\,10^{-2}\ \text{m s}^{-2}$$

$$\text{Limit slope} = \frac{(1.14 - 0)\ \text{m}}{(30 - 0)\ \text{s}^2} = 3.80\,10^{-2}\ \text{m s}^{-2}$$

Simple Logarithmic Representation

Exponential functions of the form

$$y = C\, e^{-k\,x}$$

must be analysed quite often. Exponential functions can be linearized by taking the log of both sides:

$$\ln y = \ln C - k\,x \qquad \text{or} \qquad \log y = \log C - k\,x\,\log e$$

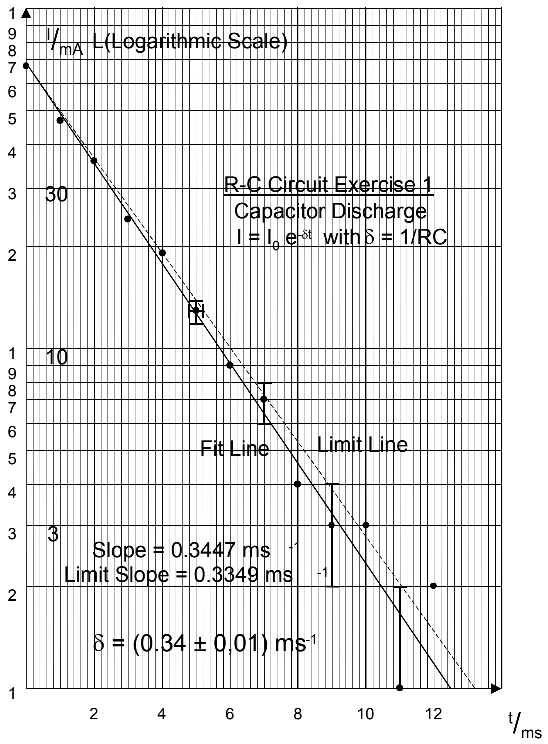
A linear function then results when ln y (or log y) are substituted as variables.

Simi-log paper is used to draw these functions so that the y-values can be directly plotted without calculating the logarithm.

The log paper is based on logarithmic decades, whereby one unit of the log scale always corresponds to a decade of the dependent variable.

The absolute scaling of the log axis is (as with a linear division) undetermined and must be fixed in relation to the order of magnitude of the measured values.

Evaluation and error determination are basically as described above. However, attention must be paid to two important points (sources of error).



Example 4: Evaluation of an exponential function by a semi-log plot.

Log of current plotted against time during capacitor discharge. The measurement error for current and time are 1 mA and 0.5 ms respectively. To calculate the slope the axis intercepts for I and t are required.

$$\text{Slope} = \frac{\ln 1 - \ln 68 = \ln \frac{1}{68}}{12.24\ \text{ms}}$$

$$\text{Limit Slope} = \frac{\ln 1 - \ln 68 = \ln \frac{1}{68}}{12.60\ \text{ms}}$$

When determining the difference quotient one must take into account that the logarithmic scale substitutes the taking of logs of the variable and the log axis carries the numerical value of the variable.

Thus, to calculate the difference quotient one can only use (two) y-values and the logs of these have to be taken. Since this calculation is independent of the graphical representation, one selects natural logarithms to directly obtain the exponential coefficient (*k*).

The construction of the limit line also requires special consideration:

By "stretching" the scale "downwards" the scattering of the measured values and the size of the error bars increases strongly (with constant errors) as the values become smaller. The logarithmic scale *weights* the measured points with their relative errors, i.e. with the accuracy (in the logarithmic scale equal distances are represented by equal factors). Correspondingly, the position of the fit line and the limit line must be oriented more to the larger measured values with their better accuracy (larger weighting).

Double Logarithmic Representation

Power functions with arbitrary exponents can be linearized using double log paper.

Number and Position of Measured Points

As a fundamental problem in the experimental verification of functional dependence, consideration must be given to the number and location of measured points, whereby resolution and time are limiting factors.

With simple curves (linear- or linearized functions) and low scattering ten measured values can be sufficient (from a statistical standpoint a very low number).

When selecting the position of the points consideration should be given to whether the function is to be qualitatively checked or just quantitatively evaluated. For a qualitative check an equidistant position of the measured values is helpful. If (in the other case), for a linear progression, only the slope of a line is to be determined it is better to have the measured values over a large interval with half the values in the upper and lower interval respectively.

Supplementary Information

In many cases a formalistic and uncritical application of the basics discussed above is only conditionally possible. In the following a few cases will be considered and examples given for matching graphical representations to special requirements.

In an experiment a linear relationship is expected. The measurements, however, show a curved progression. If the curvature is distinctively systematic then one must assume that a quantity considered as constant shows a dependence on the measured quantity. Very unsystematic curvatures are more difficult to explain and also indicate errors in measurement or evaluation.

In the measurement of a vapour pressure curve (in a suitable logarithmic representation) one expects a straight line for the plot of the vapour pressure as a function of temperature, where the heat of vaporisation enters as a constant in the slope. Since, however, the heat of vaporisation is dependent on the temperature one gets a curved plot. For such a case one can draw a tangent at a selected point whose slope gives the heat of vaporisation for that particular temperature.

In other cases a best fit line, taking all points into consideration, can be appropriate and thus represents an average. Corresponding average values are then determined from the slope.

In both cases special evaluation must also be taken into consideration for error analysis (in averaging, for example, by drawing a limit line covering the whole curvature).

Unexpected curve forms make a critical discussion of the measurement necessary. They always have a reason which must be found out. An uncritical statement ("something went wrong") misses the whole point of deriving conclusions from measurements and observations.

It can also happen that the amount of points is clearly divided into several regions either each having different slopes or mutually shifted with respect to one another. Again one has to decide whether the observed processes are themselves responsible (e.g. superposition of various contributions which could then be separated by a suitable analysis), or whether unconsidered effects (e.g. switching of measurement range) or errors in the evaluation are the cause. In any case the observed behaviour ort he assumed error must be explained by a critical discussion.

Graphical representations are often used to compare different processes in a common diagram. If the collections of points to be compared lie far apart then the representation or comparison is unsatisfactory. A large scale must be used to provide a common plot of the measured values which, however, results in very low sensitivity for the individual measurements.

Example: For water and a certain type of glass one is to plot the refractive index as a function of wavelength (dispersion). Both materials are only slightly dependent on the wavelength but have a large difference between each other. One obtains a suitable plot by selecting an elongated scale and shifting both measurement one above the other (superimposed) and giving each measurement its own axis.

Superposition, omitting a part of an axis or changing scales belong to the conventional methods of comparative graphical representations.

ANNEX III
Dimensions and Units (SI)

GPI

Physical Quantities

A fundamental concept of physics is the *physical quantity*. It is a property ascribed to observable and measurable phenomena by which one can describe physical processes. Simple physical quantities are length, time, velocity, force, work, electric voltage, magnetic field strength, temperature, and so on.

One differentiates between *base quantities*, which are axiomatically fixed, and *derived quantities*, which are defined in terms of the base quantities.

All quantities are on equal footing and convention has fixed the base quantities. The international system of units (*SI*; *Système International d'Unites*), is founded on the following seven base quantities:

Length L
Mass m
Time t
Electric current I
Temperature T
Amount of substance ν
Luminous intensity I

The names and symbols of the quantities are also internationally agreed upon. A compilation is given below.

Dimensions and Units

Physical quantities have qualitative properties as well as quantitative characteristics.

The quality of a physical quantity is determined by its *dimension*. The dimensions of the base quantities correspond to themselves (length, mass, time, electric current, temperature, etc.).The dimensions of the derived quantities are founded on the product of powers of the base dimensions. The dimension of acceleration a is thus *length* divided by $time^2$, of force F (= *mass* × *acceleration*) *mass* times *length* divided by $time^2$, of work W (= *force* × *distance*) *mass* times $length^2$ divided by $time^2$.

To designate the dimension of a quantity (e.g. A) one writes:

$$\dim \mathrm{A} = \frac{\text{Mass Length}}{\text{Time}^2}$$

The quantitative specification of a physical quantity is through its *number value* and *unit*. The unit fixes a comparative quantity and the number states how often this unit is contained in the described quantity:

$$\text{Quantity} = \text{number value} \times \text{unit}$$

The units have names and are also specified by letters. The unit of time is the second (1 s or short s). One uses square brackets to specify an (arbitrary) unit of a quantity. If one wants a pressure to be expressed in mm mercury, one writes

$$[p] = \text{mm Hg}$$

Système Internationale d'Unites

A *system of units*, founded on independent base units, and where derived quantities are defined in terms of these base units is termed a *coherent system of units*.

In 1948 the 9. General Conference of Weights and Measures (CGPM)with it’s Resolution 6 asked the International Conference of Weights and Measures (CIPM) to conduct an international study of the measurement needs of the scientific community. The resulting system was called *Système Internationale d'Unites* or *SI* for short and has found application in all areas of physics, especially experimental physics and in the legislation of national states with respect to weights and measures and is used in the lab course (nearly) exclusively.

The SI consists of the *base units*, and the *derived units*. The following presents an overview of the SI with the *prefixes for submultiples and multiples* and selection of derived quantities.

The base units of SI were expanded by one unit to include the physiological measure for light (the candela) in the field of photometry. This quantity would not have been required for a pure physical system of units.

Definitions of the Base Units

- 1 Meter is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second. (Hence, the meter is an implicit base unit, constructed from the actual base units time and velocity of light).
- 1 Kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram (a cylinder of about 39 mm in diameter and the same height made of an alloy consisting of 90 parts platinum and 10 parts iridium stored in Paris).
- 1 Second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the Cesium 133 atom.
- 1 Ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} Newton per meter of length.

- 1 Kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (The zero point of the Celsius scale is defined exactly to 273.15 K).

  In everyday life the Celcius scale remains as the temperature scale. When using both scales note:

  Temperatures can be given in both K or °C, whereby the unit Kelvin is preferred.

  Conversion is given by the equation: T/°C = T/K + 273.15.

  Temperature differences are basically given in K (e.g. 30.7 °C – 21.3 °C = 9.4 K).

- 1 Mol is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12. (This number is called Avogadro's number).
- 1 Candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 x 10^{12} Hz and that has a radiant intensity in that direction of 1/683 watt per steradian.
- Dimensionless Derived Units

Angle	α	$\alpha = \frac{\text{Arc}}{\text{Radius}}$	$1\,\frac{\text{m}}{\text{m}} = 1\,(\text{rad} = \text{Radian})$
Solid angle	Ω	$\Omega = \frac{\text{Area}}{\text{Radius}^2}$	$1\,\frac{\text{m}^2}{\text{m}^2} = 1\,(\text{sr} = \text{Steradian})$

Prefixes for Submultiples and Multiples

10^{-18} 10^{-15} 10^{-12} 10^{-9} 10^{-6} 10^{-3}	a- (Atto-) f- (Femto-) p- (Pico-) n- (Nano-) µ- (Mikro-) m- (Milli-)	10^{-2} 10^{-1} 10^{1} 10^{2}	c- (Centi-) d- (Deci-) D- (Deka-) h- (Hekto-)	10^{3} 10^{6} 10^{9} 10^{12} 10^{15} 10^{18}	k- (Kilo-) M- (Mega-) G- (Giga-) T- (Tera-) P- (Peta-) E- (Exa-)
		Used little in science and should be avoided			

Mecnanics, Thermodynamics, Hydrodynamics

Velocity	***v***	$\vec{v} = \frac{d\vec{r}}{dt}$	$1\,\frac{\text{m}}{\text{s}} = 1\,\text{m s}^{-1}$
Angular velocity	ω	$\omega = \frac{d\Phi}{dt}$	$\frac{1}{\text{s}} = 1\ \text{s}^{-1}$
Acceleration	***a***	$a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$	$1\,\frac{\text{m}}{\text{s}^2} = 1\,\text{m s}^{-2}$
Angular acceleration	α	$\alpha = \frac{d\omega}{dt} = \frac{d^2\Phi}{dt^2}$	$\frac{1}{\text{s}^2} = 1\,\text{s}^{-2}$
Frequency	ν	$\upsilon = \frac{1}{T}$	$\frac{1}{\text{s}} = 1\,\text{s}^{-1} = 1\,\text{Hz}\,(\text{Hertz})$
Angular frequency	ω	$\upsilon = \frac{2\,\pi}{T}$	$\frac{1}{\text{s}} = 1\,\text{s}^{-1}$
Momentum	***p***	$p = m\,\vec{v}$	$1\,\frac{\text{kg m}}{\text{s}} = 1\,\text{kg m s}^{-1}$
Angular momentum	***L***	$\vec{L} = \vec{r} \times \vec{p}$	$1\,\frac{\text{kg m}^2}{\text{s}} = 1\,\text{kg m}^2\ \text{s}^{-1}$
Force	***F***	$F = m\,\vec{a}$	$1\,\frac{\text{kg m}}{\text{s}^2} = 1\,\text{N}\ (\text{Newton})$
Torque	***M***	$\vec{M} = \vec{r} \times \vec{F}$	$1\,\text{N m}$
Moment of Inertia	***I***	$I = \int_V r^2\,dm$	$1\,\text{m}^2\ \text{kg}$
Work Energy	***W*** ***E***	$W = \int F\,d\vec{s}$	$1\,\text{N m} = 1\,\text{J}\ (\text{Joule})$
Power	***P***	$P = \frac{dW}{dt}$	$1\,\frac{\text{J}}{\text{s}} = 1\,\text{W}\ (\text{Watt})$
Density	ρ	$\rho = \frac{dm}{dV}$	$1\,\frac{\text{kg}}{\text{m}^3} = 1\,\text{kg m}^{-3}$
Pressure	***p***	$p = \frac{F}{A}$ older units	$1\,\frac{\text{N}}{\text{m}^2} = 1\,\text{Pa}\ (\text{Pascal})$ 1 bar (Bar) = 10^5 Pa 1 Torr = 1.333 hPa
Heat capacity	***C***	$C = \frac{dQ}{dt}$	1 J K^{-1}
Spannungskoeffizient	β	$p = p_0\,(1 + \beta\,t)$	1 K^{-1}
Dynamic Viscosity	η	$F = \eta\,A\,\frac{dv}{dz}$ ältere Einheiten	$1\,\frac{\text{N m}}{\text{m}^2\,\text{m/s}} = 1\ \text{Pa s}$ 1 P (Poise) = 0,1 Pa s
Kinematic Viskosity	ν	$\upsilon = \frac{\eta}{\rho}$ older unit	$1\,\frac{\text{m}^2}{\text{s}} = 1\,\text{m}^2\ \text{s}^{-1}$ 1 St (Stokes) = 10^{-4} m^2 s^{-1}

Electrodynamics

Electric charge	Q	$Q = \int I\,dt$	1 A s = 1 C (Coulomb)
Electric field	E	$\vec{F} = Q\,\vec{E}$	$1\ \frac{\text{N}}{\text{C}} = 1\ \text{V m}^{-1}$
Electric voltage	U	$U = \int \vec{E}\,d\vec{s}$	$1\ \frac{\text{N m}}{\text{A s}} = 1$ V (Volt)
Magnetic field (magnetic excitation)	H	$\oint \vec{H}\,d\vec{s} = \int \vec{j}\,d\vec{A}$	$1\ \frac{\text{A}}{\text{m}} = 1\ \text{A m}^{-1}$
Magnetic field (magnet. Flux density)	B	$U_{ind} = -\frac{\text{d}}{\text{d}t}\left(\int \vec{B}\,\text{d}\vec{A}\right)$	$1 \frac{\text{V s}}{\text{m}^2} = 1\,\text{T}$ (Tesla)
		older unit	1 G (Gauß) = 10^{-4} T
Magnetic flux	Φ	$\Phi = \int \vec{B}\,\text{d}\vec{A}$	1 V s = 1 W (Weber)
Resistance	R	$R = \frac{U}{I}$	$1 \frac{\text{V}}{\text{A}} = 1\,\Omega$ (Ohm)
Impedance	Z	$Z = \frac{U_0}{I_0}$	$1 \frac{\text{V}}{\text{A}} = 1\,\text{V A}^{-1}$
Capacitance	C	$I_C = -C\,\frac{\text{d}U}{\text{d}t}$	$1 \frac{\text{A s}}{\text{V}} = 1\,\text{F}$ (Farad)
Inductance	L	$U_L = -L\,\frac{\text{d}I}{\text{d}t}$	$1 \frac{\text{V s}}{\text{A}} = 1\,\text{H}$ (Henry)

Radioactivity

Decay constant	δ	$A = A_0\,e^{-\delta t}$	$\frac{1}{\text{s}} = 1\,\text{s}^{-1}$
Activity	A	$\text{A} = \frac{\text{Decay}}{\text{Time}}$	$\frac{1}{\text{s}} = 1\,\text{Bq}$ (Bequerel)
		older unit	1 Ci (Curie) = $3.7 \cdot 10^{10}$ Bq

Recommended Symbols for Physical Quantities

Space and Time

x, y, z	Cartesian coordinates	T	Time
r	Position vector	T	Period
L, s	Distance	ν, f	Frequency
A, S	Area	ω	Angular frequency
V	Volume	τ	Decay constant
$\alpha, \beta, \ldots$	Plane angle		(time constant)
$\ldots, \Theta, \Phi$		V	Geschwindigkeit
ω, Ω	Solid angle	ω	Angular velocity
k	Kreiswellenzahl	a	Acceleration
α, δ	Decay constant	α	Angular acceleration
	(Damping coefficient)	g	Gravitational acceleration
		c	Vacuum velocity

Mechanic

M	Mass	ε	Expansion
ρ	Density	E	Young's modulus
P	Momentum	G	Shear modulus
L	Angular momentum	μ	Poisson number
I, J	Itertia	η	Dynamische viscosity
F	Force	ν	Kinematic viscosity
G	Weight	E	Energy
M	Torque	E_p, V	Potentielle energy
p	Pressure	E_k, T	Kinetic energy
σ	Axial stress	W, A	Work
τ	Shear stress	P	POwer

Thermodynamics

Q	Wärmearbeit	β	Spannungskoeffizient
T	Kelvin temperature	λ	Heat conductivity
t	Celsius temperature	A	Temperature conductivity
S	Entropy	C	Heat capacity
U	Internal energy	c	Specific heat capacity
F	Free energy	κ, γ	Isentropic index c_p/c_V
H	Enthalpy		

Electricity and Magnetism

Q	Charge	B	Magnetic field strength
ρ	Space charge density		(Mag. flux density)
σ	Surface charge density	Φ	Mag. flux
V, Φ	Electric potential	μ_0	Mag. Field constant
U, V	El. voltage	μ	Permeabiliy
E	El. Field strength	M	Magnetization
D	Dielectric displacement	χ_m, κ	Mag. Susceptibility
ε	Dielectric constant	J	Mag. Polarization
ε_0	El. Field constant	R	Resistance
P	El. polarisation	ρ	Specific resistance
p	El. dipole moment	γ, σ	Conductivity
I	(Charge-) Current strength	Z	Impedance
j	Current density	C	Capacitance
H	Magnetic field strength	L	Self-induction coefficient
	(magn. excitation)	S	Poynting-vector
		A	Mag. vector potential

Atomic- and Nuclear Physics

e	Elementary charge	A	Mass number
N	Principal quantum number	Z	Atomic number
L, l_i	Angular momentum quan. num.		(Charge number)
S, s_i	Spin quantum number	N	Neutron number
M, m_i	Magnetic quantum number	λ	Decay constant
J, j_i	Total angular momentum	τ	Mean life
I, J	num. of the electron shell	$T_{1/2}$	Half life
F	Nuclear spin quan. num.	A	Activity
	Total angular momentum quan.		
	num. of a particle		

The Greek Alphabet

A number of capital- and some small letters (ι, ο, υ) correspond with Latin letters or are so similar that they are not used as symbols.

Α	α	Alpha	Ι	ι	Iota	Ρ	ρ	Rho
Β	β	Beta	Κ	κ	Kappa	Σ	σ	Sigma
Γ	γ	Gamma	Λ	λ	Lambda	Τ	τ	Tau
Δ	δ	Delta	Μ	μ	Mu	Υ	υ	Upsilon
Ε	ε	Epsilon	Ν	ν	Nu	Φ	ϕ, φ	Phi
Ζ	ζ	Zeta	Ξ	ξ	Xi	Χ	χ	Chi
Η	η	Eta	Ο	ο	Omicron	Ψ	ψ	Psi
Θ	θ	Teta	Π	π	Pi	Ω	ω	Omega

Physical Constants

Vacuum light velocity	$c =_{Def} 2.997922458 \cdot 10^{8}$ m s^{-1}
Gravitational constant	$\Gamma = 6.673(3) \cdot 10^{-11}$ N m^{2} kg^{-2}
Avogadro's number	$L = 6.0220921(62) \cdot 10^{23}$ mol^{-1}
Mol volume	$V_M = 22.41383(70) \cdot 10^{-3}$ m^{3} mol^{-1}
Gas constant	$R = 8.31441(26)$ J mol^{-1} K^{-1}
Boltzmann constant	$k = 1.380652(43) \cdot 10^{-23}$ J K^{-1}
Electric field constant $\varepsilon_0 = \frac{1}{\mu_0 c^2}$	$\varepsilon_0 = 8.854200352... \cdot 10^{-12}$ As/Vm
Magnetic field constant $\mu_0 = 4\pi \cdot 10^{-7} \frac{\mathrm{V\,s}}{\mathrm{A\,m}}$	$\mu_0 = 1.256637061... \cdot 10^{-6}$ A s V^{-1} m^{-1}
Elementary charge	$e_0 = 1.6021829(22) \cdot 10^{-19}$ C
Specific electron charge	$e_0/m_e = 1.7588115(24) \cdot 10^{11}$ C kg^{-1}
Planck's constant	$h = 6.626124(13) \cdot 10^{-34}$ J s
Rest mass of the proton	$m_P = 1.6726355(17) \cdot 10^{-27}$ kg
Rest mass of the electron	$m_e = 9.1094634(99) \cdot 10^{-31}$ kg
Classical electron radius	$r_e = 2.8179378(70) \cdot 10^{-15}$ m

Gravitational acceleration for the lab building (1. floor)

$\beta = 52^{0}\ 27'\ 35(5)''$; $h = 59(3)$ m $g = 9.812777(5)$ m s^{-2}

ANNEX IV
NUCLEAR RADIATION DETECTORS

GPI

Nuclear Radiation Dectors and Measurement Electronics

Geiger-Müller Counter (Cascade tube)

When ionizing radiation passes through matter inelastic collisions cause electrons to be knocked out of their shells and the atoms become ionized. If this occurs by applying a voltage within an electric field then the free charge carriers (electrons and ions) move resulting in a current which can be detected.

Counters are gas filled tubes with a coaxial thin metal wire arrangement functioning as an electrode. The gas is usually an inert gas (Ne, Ar, He). With the *Geiger-Müller counter* (GM counter; *Hans Geiger*; 1882-1945; German physicist; *Ernst-Wilhelm Müller*; *1911; German physicist; 1951 Prof., FU Berlin) the applied voltage is so high (typically 500 V), that the primary charge carriers resulting from the radiation (*primary ionization*) are strongly accelerated in the electric field and ionize further atoms in the gas (*secondary ionization*). The result is an *avalanche of charged particles*. The magnitude of of which is independent of the primary ionization, and thus also independent of the energy of the incident radiation. The ionization process is interrupted by the addition of a *quenching gas* so that no continuous gas discharge takes place.

The short pulse of current in the counter circuit resulting from a collision causes a short voltage drop (pulse) across an external resistor which is coupled out through a capacitor. The pulses of the GM tube are so high that they can be feed directly to a counter without further amplification.

With constant radiation intensity, the counting rate is dependent on the applied voltage (tube characteristic). The *operating voltage* is the lower critical voltage for avalanche formation. It includes a range in which the counting rate depends little on the voltage (*Geiger plateau*) and within this a *working point* with which the tube is operated. A slight increase in the count rate within this plateau is caused by an increase in the effective sensitivity and with rising voltage an increase in the number of discharges finally resulting in quenching, i.e. a continuous electric pulse in the counter.

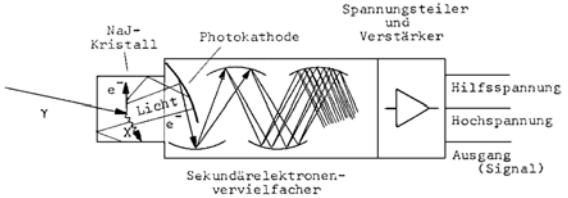
After an event (ionization) is registered the field strength in the tube drops for a short time due to screening of the negative potential of the tube wall. Screening is caused by the slowly drifting space charge cloud of the positive ions and in addition by the voltage drop across the external resistor. During this *dead time* no further quantum of radiation can be registered.

The advantage of the GM counter lies in its simple construction and the large output pulses resulting from gas amplification. The disadvantages include missing *energy resolution* and the limitation of low counting rates because of the comparatively large dead time.

Sodium Iodide Scintillation Detectors

Sodium Iodide scintillation detectors (NaI detectors) serve to detect γ radiation and function on the principle of the (internal) photo effect. In a NaI crystal the incoming γ photon is stopped by the photo effect and the total energy is transferred to the electron (primary electron)). The electron is released and causes, together with the subsequent X-ray photon a cascade of further electronic excitation processes. The spontaneous decay of these processes creates a pulse of light equivalent to the primary energy.

The light created in the NaI crystal passes through an optical window to the photomultiplier tube (PMT). The first part of the PMT contains the photocathode which produces electrons when light strikes its surface. The electrons are electro-statically accelerated to further plates called dynodes creating a multiplication effect. The resulting charge is finally amplified. The figure below shows the principle of the NaI scintillation detector.



The complete process is linear so that at the output of the system a pulse appears whose height and energy is proportional to the incident γ-photon, thus creating a *line* (*Photoline*; *Photopeak*).

Aside from the photo effect, Compton processes also occur in the crystal, whereby inelastic scattering of a γ-photon takes place. For the case that the scattered photon leaves the crystal only a part of the energy is transferred to the system and detected. Depending on the scattering angle, the Compton electrons have different energies and hence produce pulses of different heights forming a Compton plateau in the spectrum and breaks off at a scattering angle of 180° (*Compton edge*).

Conversely, Compton scattering processes also occur in the detector material outside the crystal and because of geometric conditions the back scattered γ-photons enter the crystal and are detected as backscatter peak. This line is clearly seen in the spectrum of the experiments performed in the course.

The pulses created by the NaI detector are comparatively broad due to the statistical process in the photomultiplier tube. The resolution of radiation measuring instruments is taken as the full width at half maximum of a peak:

$$\text{Resolution} = \frac{\text{FWHM}}{\text{Peak Position}} \tag{1}$$

The resolution of NaI detectors are around a few percent (5-10 %).

Ionization Counters

Simple detectors without energy resolution can be used when it is sufficient to register the integrated spectrum, whereby further processing of the pulses is by means of digital counters (see experiment *RADIOACTIVE DECAY*) or through analogue integration with a suitable time constant. Here, the pulses are converted to a voltage proportional to the mean pulse rate and displayed by a voltmeter or plotted (example: ion dose meter, in experiment *RADIOACTIVE DECAY*).

Spektrometer

Energy resolving systems with proportional counting tubes, scintillation detectors or solid state detectors are referred to as *Spectrometers:* They deliver for each absorbed quantum of radiation (α-, β-particles, X-ray or Gamma photon) an electric pulse whose height is proportional to the energy absorbed. The width and shape of the pulse (typically 1 μs or a few volts) is determined by the time-constant devices in the amplifying stages of the system. Certain interaction processes (e.g. internal photo effect) absorb the total quantum energy, so that the pulse height corresponds directly to the energy of the radiation (photo line). The lines (peaks) of a spectrum are recognized by the accumulation of pulse of equal amplitude. Other processes (e.g. the Compton effect) absorb only a part of the quantum energy and deliver smaller pulses with a certain distribution creating a additional continuous *background* in the spectrum. It is recommended in experiment *GAMMA SPECTROSCOPY* to observe the pulses of the scintillation detector on the oscilloscope using slow and fast time deflection.

Further processing of the pulses (where the height contains the energy information) can be performed with various technical methods.

Single Channel Analyser (SCA)

If the aim of an experiment is to measure the time dependence of a certain transition, for example, i.e. a certain line then this can best be done with a SCA. A SCA possesses two adjustable voltage thresholds and then gives out a logic pulse (square-wave of fixed height and width) when the pulse height at the input of the *window* lies between both thresholds. The output pulses can then be counted or evaluated by other means. A disadvantage of the SCA is that only one line can be observed and that the window also “sees” the continuous background. Recording a full spectrum is thus tedious and time consuming.

Multi Channel Anylyser MCA

Recording a full spectrum is simpler and more elegantly performed using a MCA. A MCA consists of an analog-digital converter (ADC) and a large number of downstream counters (typically 256 to 4096 and more). The ADC converts the height of an incident pulse into a number. At first the pulse charges a capacitor via a diode thus fixing the peak height. Thereafter, the capacitor is discharged at constant current, and the time taken for complete discharged is determined by counting clock pulses (typically 100 MHz), so that the number of these pulse corresponds to the peak height and hence, the absorbed radiation energy. This number is interpreted as an address of a counter (channel) and ist contents is increased by one. Thus the counters deliver a distribution function of the radiation intensity verses energy, which, for example, can be represented on a screen or output numerically. The MCA also incorporates a µ-computer for process and time control and a digital-analog converter (DAC) to convert channel addresses and contents into analog voltage signals for display on a screen (or plotter).

Multi channel analysers substantially ease the measurement and evaluation of spectra. The intensities of lines can be determined by the numerical integration of peaks whereby, background contributions can be determined from the spectrum outside the lines. In general, the evaluation of a spectrum must also take into account the sensitivity of the detector which can be strongly deoendent on the energy of the incident radiation.

The application of MCA’s is not only restricted to nuclear spectroscopy: they are used in modern measurement technology for many applications. Some examples: With a *TPHC* (Time-to-Pulse-Height-Converter) one can generate pulses whose height is proportional to the time difference between a start- and Stopp pulse. Further evaluation in a MCA delivers then, for example, the life time of an intermediate state in a gamma cascade (Start and Stop: MCA on the respective peaks) or an excited atomic state (Start: excitation laser pulse, Stop: radiation emitted by the atom). In the *Sampling Technique* a fast switch is used to *cut out* pulses from a continuously changing voltage*;* the MCA delivers then the associated amplitude spectrum. Also important are applications where the MCA is not operated in the the pulse-height-analysis mode (PHA) considered so far but rather in the multi-channel-scaling mode (MCS). Here, incoming (unit-) pulses are counted for a certain time in the individual channels thus, recording the dependence of the pulse rate on time or on a time modulated quantity. This has found wide application in the *Signal Averaging Method*, where many measurement cycles are added to reduce statistical errors. In this respect mention should be made of the *Sampling Oscilloscope* in which the sampling technique is combined with an MCA operating in MCS mode. The real-time representation of high frequency signals using oscilloscopes becomes expensive from about 50 MHz onwards and finds its technical limitation at around 1000 MHz. Since it is possible, however, to manufacture much faster trigger- and sampling circuits, the momentary value of a periodic recurring signal can, for different delay times, be stored sequentially after a trigger pulse. The curve can then be displayed on a "slow" oscilloscope.

ANNEX V
RADIATION PROTECTION and DOSIMETRY

GPI

Information on the dangers of ionizing- and non-ionizing radiation and possible protection measures and the fundamentals of dosimetry.

Radiation Protection Information

Danger to health and life can result from exposure to ionizing- and non-ionizing radiation. Protection against such radiation is regulated in the *Strahlenschutzverordnung (StrSchV)* which serves the protection of persons, equipment and the environment. Formal compliance to radiation protection measures is especially important because humans do not possess any sense organs for the perception of radiation.

Effects of Radiation

The most important types of radiation are α-, ß-, γ-, X-ray and neutron radiation. They differ greatly in their characteristic properties with respect to effects and protection.

α- and ß-particles undergo electrostatic interactions with the electron shell and atomic nucleus when passing through matter. The resulting strong interaction and stopping ionizes the material. Neutrons are uncharged. They can move relatively freely in material until captured. Neutrons are therefore hard to stop and do not ionize. X-rays und γ-rays are electromagnetic waves of very short wavelength, i.e. high frequency and high energy. They have a large range in matter and also ionize.

The damaging effects of radiation to health lie in the destruction of microscopic structures of organic substances (atoms, molecules, cells) and the resulting consequences for the organism. One differentiates between *somatic damage* (bodily damage) and *genetic damage*. Somatic damage can have direct consequences such as the destruction of cells and tissue and even skin burns or also indirect consequences such as changes in cells which can lead to diseases such as cancer.

With respect to the effects of radiation, a difference is also made between external radiation exposures through *exposition* (radiation) or *contamination* and internal radiation exposure through *incorporation* (inhalation, intake via the gastrointestinal tract or other applications in the field of medical diagnosis or therapy).

Fundamentals of Radiation Protection

Because of the dangers involved, great care must be taken when working with radiation generating equipment and radioactive substances. The important fundamentals of radiation protection (*28 StrSchV*) are:

- Avoid any unnecessary exposition or contamination of people, goods and environment.
- Keep as low as possible any exposition or contamination of people, goods or environment taking into account the state of science and technology under consideration of all facts of the individual case even when exposition or contamination is lower than the limits set by the radiation protection authorities.

The main protection measures against external radiation are good *shielding*, large *distance* from the radiation source and a short *stay* in the vicinity of the radiation.

α-radiation, due to its size and charge is strongly stopped and has a very short range (typically 40 mm in air and 0.03 mm in aluminium). The total energy of the radiation is, however, given up in a very short distance and, under certain circumstances, can lead to severe damage. α-radiation is relatively harmless with respect to external exposure since it is stopped in the dead layer of skin. On the other hand, it is extremely dangerous when incorporated since the inner organs have no protective layers.

β-radiation has a similar behaviour, however, with much larger range (typically 2 m in air und 1 mm in aluminium). External exposure also requires protection measures, especially for the eyes, since the lenses can become opaque under the effects of strong β-radiation. The radiation can be easily shielded using a few mm of Plexiglas or aluminium.

X-rays and γ-rays can penetrate large distances in material and a difficult to shield. Full absorption is not possible, only an exponential attenuation corresponding to the thickness of the absorber. Heavy substances such as lead or uranium are suitable for shielding purposes. The half value thickness of lead (attenuation of 50%) is about 15 mm for the1.2 MeV radiation of Co-60.

Neutrons are especially dangerous. Since they are neutral particles they experience no electric interaction forces and can easily pass through matter. Because of their similar mass they substitute easily for hydrogen nuclei in collisions thus destroying the hydrogen atom. Because the hydrogen atom is a central building block in organic substances this results in serious changes in the labile, chemical equilibrium of physiological systems. Heavy atomic nuclei have a large cross-section for neutron capture reactions resulting in nuclear reactions which also lead to serious damage to molecular- and chemical structure. Neutrons are difficult to shield; the best are substances containing collision partners with similar mass, such as protons as hydrogen nuclei in hydrogen rich substances like water or paraffin.

Dosimetry

Activity

When considering danger due to radiation one must differentiate between cause and effect. The cause is a radiation field that in radioactive decay processes is defined as the number of decays per unit time and termed *Activity*:

(1) Activity A = Number of decays/ Time

Thus the unit of activity is:

(2) $$[A] = 1\frac{\text{Decay}}{\text{s}} = 1\text{Bq} \quad (\text{Bequerel})$$

Energy Dose

If one places a probe, e.g. an organic body in a radioactive field a part of the radiation will be absorbed resulting in physical or physiological effects. Another part passes through the body without interaction and hence, without radiation damage. A measure of the radiation exposure of a probe or body is the radiation energy absorbed per unit mass. This quantity is termed the *energy dose* D_E:

(3) D_E = *Absorbed Energy/ Mass*

The fraction of absorbed radiation energy, i.e., the dose values depend on the type of radiation and the energy and always refer to a certain substance. The unit of energy dose is:

(4) $$[D_E] = \frac{1\text{Joule}}{\text{kg}} = 1\text{Gy} \quad (\text{Gray})$$

(An older unit 1 rad = 0.01 Gy). The dose per unit time is called *dose rate.*

Ion Dose

The energy dose is difficult to measure since the relevant amounts of energy a very low (a lethal radiation dose would increase the body temperature of a person by about 2 mK corresponding to the amount of heat in a sip of coffee). For this reason the *ion dose* D_Q was introduced as a practical measurement. The ion dose is a measure of the charge generated in air due to ionization by radiation:

(5) D_Q = *Generated charge/Air mass*

The unit of the ion dose is:

(6) $$[D_Q] = 1\frac{\text{C}}{\text{kg}}$$

An older unit for D_Q is 1 R (Röntgen) = $2.58 \cdot 10^{-4}$ C/kg.

The ion dose can be directly measured with *ionization chambers*. The unit of the ion dose rate is

1 C/(kg·s) = 1 A/kg.

The relationship between energy dose and ion dose depends in general, on the energy of the radiation and material. For muscle tissue soft tissue a good approximation independent of energy is:

(7) $$D_E = 40\,\frac{\text{Gy}}{\text{C/kg}}\,D_Q$$

Equivalent Dose

The biological effects of radiation and radiation damage are also influenced by the type of radiation apart from the energy. This is expressed by an additional dimensionless factor called the *radiation weighting factor* w_R:

Type of radiation	w_R
γ-, β-rays	1
Slow neutrons	5
α-rays, fast neutrons	10

The product of energy dose and *radiation weighting factor* gives the equivalent dose D_H. Since w_R is dimensionless, the units of both quantities are the same and 1 J/kg for the case of equivalent dose is given the unit name *Sievert*:

(8) $D_H = w_R\ D_E$ and

(9) $$[D_H] = 1\frac{\text{J}}{\text{kg}} = 1\,\text{Sv} \quad (\text{Sievert})$$

An older unit of equivalent dose is 1 rem = 0.01 Sv.

Typical Radiation Exposures

There exists a *natural radiation exposure* from terrestrial sources (radiation from radioactive materials in the earths crust) and from cosmic sources. For Germany the exposure is about 1.1 mSv per year. (The fraction from *fallout* from nuclear weapons testing is at present less than 1 % of the natural radiation). In certain regions of the Black Forrest, the natural exposure is about 2.7 mSv, and there are regions on earth where the terrestrial exposure is around 20 mSv per year. Detailed investigation on the effects of radiation have been conducted on the populations of these areas, however, no significant consequences could be documented.

According to the radiation protection regulations, the permissible increase in exposure of the general public due to radiation generating equipment (X-ray units, reactors, accelerators etc.) must not be more than 0.3 mSv per year.

Typical radiation doses in X-ray diagnosis are about 1 mSv for a simple exposure (thorax) up to about 100 mSv for multiple X-ray contrast exposures. Typical organ doses in radiation therapy (cancer therapy) can be in the thousands Sv. The legal limit for occupationally exposed persons is 50 mSv per year for whole body exposure. The lethal dose (100 % lethality) is around 8 Sv whole body exposure.

On the lab benches where experiments with radioactive materials are conducted, dose measuring instruments are available to record the natural exposure and the additional exposure due to the radioactive sources.

ANNEX VI
STATISTIC

GPI

Statistical Variation

Physical processes run stochastically because of the quantized nature of physical systems. Processes become deterministic in the rigorous sense for the limiting case of infinitely many or in the practical sense for a large number of elementary contributions, so that the expected variations approach zero. This is the situation in *macroscopic* physics, where statements are made, for example, with respect to "infinitely" many particles of a volume of gas or "infinitely" many oscillating charges of a radiating antenna.

For the specification of systems determined by finite many or a comparatively low number of atomic systems, there is no rigorous deterministic prediction, and the results show (statistic) variations with a distribution of values.

Analogously, if the number of observations is infinite and not the number of participating microscopic systems then one again can make a rigorous predication, however, only for the *Parameter* of the distribution, as e.g. the mean value or the standard deviation.

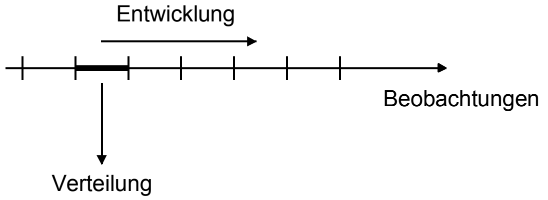
Furthermore, the variations only become evident experimentally when they stand out from the sensitivity of the measuring equipment. Actual series of measurements then deliver an approximation of the expected distribution function.

Classical examples, also in the scope of the basic lab course, are the decay of excited atomic or nuclear states, radioactive decay or the stochastic interaction of a particle or radiation with matter.

In these cases a typical consideration is the *limited observation*, i.e. a limited observation time or a limited interaction distance for which then the stochastic process can be described by probabilities with respect to time and distance (transition probability, collision probability).

From a mathematical statistics point of view this represents a *Bernoulli Experiment* with a two-valued output: *decay/collision or not*, *yes or no*, *0 or 1*, *black or white*, *heads or tales*. In this script distribution functions will be developed in relation to *Bernoulli-Experiments*.

Here we must consider two questions. First is the question on the distribution of the results within such an observation and secondly the progression of a finite ensemble due to such processes.



Exponential Development

Consider first of all time development using radioactive decay as an example (Experiment *RADIOACTIVE DECAY*). When λ is the decay probability (transition probability) of a nucleus with respect to time then the mean number of decays of an ensemble of n radioactive nuclei in an observation time Δt is:

(1) $$\bar{n} = p\,n = \lambda\,\Delta t\,n$$

With regards to a finite quantity of nuclei $\bar{n}$ represents a decrease. To a good approximation, this quantity is taken as large, and n as a continuous variable. From (1) we then get a differential equation for the time development of the ensemble:

(2) $$dn = -\lambda\,n\,\mathrm{dt}$$

With the solution

(3) $$n = n_0\,e^{-\lambda t}$$

This is the time law of radioactive decay, whereby one also gets an exactly corresponding result for the case of the absorption law of radiation through matter when one as a model, considers stochastically distributed radiation quanta and collision partners and "singular" collision processes in which a quantum of radiation is removed from the radiation beam.

Binomial Distribution

In the previous case where the number of events in an observation is described in "integral" form by a mean value let us now as the question as to the "differential" distribution of the number of events. As a generalization, the decay probability for an observation time λ Δ*t* is written as *p* so that the complementary event is expressed as (1-*p*). "Conducting" the single experiment N times (with N nuclei) one gets from the multiplication law for statistically independent events as the probability that in *n* certain cases (the decays) appears, and in the remaining (*N*-*n*) cases it does not:

(4) $$p^n\,(1-p)^{N-n}$$

However, in general we are not interested in which certain cases the events appear only in their total number. The value (4) is then to be multiplied with the number of possibilities to select *n* in which an event occurs from *N* experiments. This number is given by the binomial coefficient:

(5) $$\binom{N}{n} = \frac{N!}{n!\,(N-n)!}$$

The sought after distribution is then the *binomial distribution*:

(6) $$B(n) = \frac{N!}{n!\,(N-n)!}\,p^n\,(1-p)^{N-n}$$

The discrete *decay variable n* describes the actual possible results, and the distribution (6) their probability values (e.g. for the number of nuclear decays in an observation interval).

The binomial distribution corresponds to the *normalization axiom* for probabilities. The summation over all possible events, i.e. over all *n*, gives the binomial power function:

(7)

$$\sum_{n=0}^{N} B(n) = \sum_{n=0}^{N} \binom{N}{n} p^{n} (1-p)^{N-n} = \left[p+(1-p)\right]^{N} = 1^{N} = 1$$

Distribution Parameters

The values of a distribution model or the data of an observed or given distribution give information on the distribution. Hence statistical functions have been defined with which the more important properties can be quantitatively represented.

(8)

	Discrete distribution $F(x_i)$	Distribution function $\rho(x)$
	Mean value $\bar{x} = \sum x_i\, F(x_i)$	Expectation value $\mu = \int x\,\rho(x)\,dx$
	Mean quadratic deviation $s^2 = \sum (x_i - \bar{x})^2\, F(x_i)$	Variance $\sigma^2 = \int (x-\mu)^2\, \rho(x)\,dx$

The quadratic deviation takes into consideration the different signs of the deviation (the mean deviation would vanish in a symmetric distribution). The *standard deviation* is the square root of the mean quadratic deviation or variance.

Calculating the mean value and variance of the binomial distribution is time consuming using algebraic methods, but much more simple using the (plausible) summation rules:

(9)

$$\overline{(x_i + y_i)} = \overline{x_i} + \overline{y_i} \quad \text{und} \quad s^2(x_i + y_i) = s^2(x_i) + s^2(y_i)$$

The two-valued *Bernoulli distribution* for the limiting case of the binomial distribution with n = 1 has the values n = 0 with probability (1-p) and n = 1 with probability p. The mean value and variance then follow from (8):

$$\bar{n} = 0\,(1-p) + 1\,p = p \quad \text{and}$$

$$s^2 = (0-p)^2\,(1-p) + (1-p)^2\,p = (1-p)\,p$$

And with (9) one gets the binomial distribution for N events:

(10) $\bar{n} = p\,N$ and $s^2 = (1-p)\,p\,N = (1-p)\,\bar{n}$

Poisson Distribution

The *Bernoulli distribution* is for practical purposes unwieldy because of the binomial coefficients which can very quickly take on large values. For "seldom" events with small mean values and small probabilities, which occur frequently, and for values around the mean value a more comfortable approximation distribution can be developed. The assumptions are:

(11) $\bar{n}, n << N$ and $p << 1$

(N-n)! is eliminated from the binomial coefficients and the remaining factors in the numerator from (N-n+1) to N are set to N as an approximation:

(12) $\dfrac{N!}{(N-n)!} \approx N^n$ and $N^n\, p^n = \bar{n}^n$

The power $(1-p)^{N-n}$ is written as an exponential term, and the required logarithm in the vicinity of 1 is approached linearly:

(13) $(1-p)^{N-n} = e^{\ln(1-p)^{N-n}} = e^{(N-n)\ln(1-p)} \approx e^{-p\,N} = e^{-\bar{n}}$

since

(14) $\ln(1-p) \approx -p$.

The Bernoulli distribution thus passes over into the *Poisson distribution* which is only determined by one paramer $\bar{n}$:

(15) $$P(n) = \frac{\bar{n}^n}{n!} e^{-\bar{n}}$$

The *Poisson distribution is also normalized* :

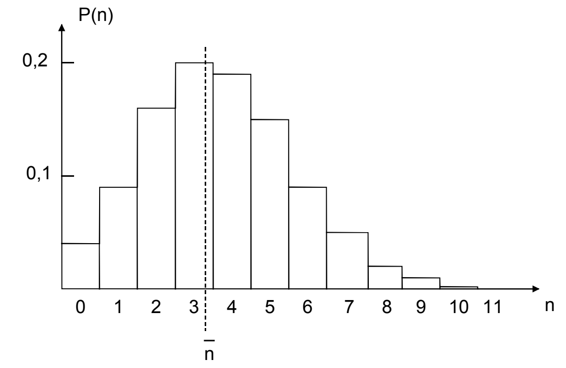
(16) $\sum_{n=0}^{\infty} P(n) = 1$ because $\sum_{n=0}^{\infty} \dfrac{\bar{n}^n}{n!} = e^{\bar{n}}$

Here the validity of (15) is accepted notwithstanding the restriction for n → ∞ which is justifiable because of the strong decline of the function P(n) for large values.

The mean value is given by:

(17) $\sum_{n=0}^{\infty} n\,P(n) = \bar{n}$

The figure below shows a Poisson distribution for $\bar{n} = 3.8$.



The *Poisson distribution* is asymmetric. With increasing $\bar{n}$ it goes over into a distribution which is symmetric around $\bar{n}$.

Gauß Distribution

The *Bernoulli-* and *Poisson distribution* are discrete distributions and in many cases it is of practical advantage to work with a continuous distribution function.

To develop a function from the *Poisson distribution*, the assumption (11) is supplemented by requiring that the mean value is small with respect to N, but large with respect to 1:

(18) $\bar{n} >> 1$

and the considerations are to be restricted around $\bar{n}$ so that:

$$|n-\bar{n}| << \bar{n} \tag{19}$$

The transition to a continuous distribution function takes place by setting up a difference quotient for the discrete Poisson distribution, which with the approximation assumptions can be interpreted as a differential equation:

$$\frac{P(n)-P(n-1)}{\Delta n = 1} = P(n) - P(n-1) = -\frac{n-\bar{n}}{\bar{n}} P(n) \tag{20}$$

Because of (18), the right-hand term in (20) represents with *n* a slowly varying quantity so that *n* may be considered as a continuous variable (and thus written as *x* below), and (20) interpreted as a differential equation of a distribution function G(n):

$$\frac{\mathrm{d}G(x)}{\mathrm{d}x} = -\frac{x-\bar{x}}{\bar{x}} G(x) \tag{21}$$

It has the solution:

$$G(x) = C\ e^{-\frac{(x-\bar{x})^2}{2\bar{x}}} \tag{22}$$

This is the *Gauß-* or *Normal distribution*. The integration constant is so determined that the function is normalized. With the integral:

$$\int_{-\infty}^{+\infty} e^{-t^2} dt = \sqrt{\pi} \quad \text{it follows that} \quad C = \frac{1}{\sqrt{2\pi\,\bar{x}}} \tag{23}$$

and

$$G(n) = \frac{1}{\sqrt{2\pi\,\bar{x}}} e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sqrt{\bar{x}}}\right)^2} \tag{24}$$

In the transition from a discrete to a continuous random variable, the individual (x-) values become infinitesimal, and their probabilities approach zero. Actual events are now formed by intervals of the random variables (with a "volume" determined by the width), whose probability is described by the area of the distribution across the interval. The function (24) represents the *probability density* of random variables.

General Normal Distribution

The form (24) is a special normal distribution with only one parameter, where $\bar{X}$ the expectation value (trivially because of the symmetry of the distribution) is simultaneously the variance. The general form of the normal distribution with independent values of the expectation value μ and variance σ^2 is:

$$N(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{25}$$

The figure below shows two normal distribution with the same expectation value (μ = 5.0), but different variances or standard deviations (σ = 0.5 and S = 1.5).