ADDITIONAL CONSERVATION LAWS IN THE THIRRING MODEL?

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We computed pair-production and nontrivial three-particle-scattering amplitudes of the massive Thirring model in tree approximation. They are found to be zero.

Recently interesting relations between the Sine-Gordon equation and the massive Thirring model have been established [1]. On a classical level the Sine-Gordon equation can be treated by means of the inverse scattering method [2]. There exist infinitely many conversation laws [3], which apparently imply the conservation of particle number and of the set of their momenta [4]. This should carry over to the massive Thirring model, provided there are no anomalies in the currents. We compute the pair-production $(2 \to 4)$ in tree approximation, this is by a simple substitution law related to the scattering $(3 \to 3)$, which can be easier calculated. The former vanishes identically and the latter has only trivial contributions.

For further use we first consider the connected part of the matrix element for two-particle-scattering

$$M_{22}^{\text{conn.}} = {}^{\text{out}}\langle p_{2'}, p_{1'} | p_1, p_2\rangle_{\text{conn}}^{\text{in}}$$
$$= (2\pi)^2\, \delta^{(2)}(p_1 + p_2 - p_{1'} - p_{2'})\, (2\pi)^{-2} A_{22}.$$

Using Klaiber's [5] γ-matrices and light-cone variables

$$p^+ = p^0 + p^1 = ma, \qquad p^- = p^0 - p^1 = ma^{-1},$$

we choose the spinors to be of the form

$$u(p) \equiv u(a) = \sqrt{m} \begin{pmatrix} a^{-1/2} \\ a^{1/2} \end{pmatrix},$$

$$v(p) \equiv v(a) = \exp(-\mathrm{i}\,\pi/2)\sqrt{m} \begin{pmatrix} a^{-1/2} \\ -a^{1/2} \end{pmatrix},$$

normalized such that

$$u(p)\bar{u}(p) = \not{p} + m, \qquad v(p)\bar{v}(p) = \not{p} - m. \tag{1}$$

The relative phases of the spinors u, v are choosen such that the replacement of an ingoing particle by an outgoing antiparticle corresponds to the simple substitution law

$$a \to \exp(\mathrm{i}\pi)\bar{a}, \qquad u(a) \to u(\exp(\mathrm{i}\pi)\bar{a}) = v(\bar{a}). \tag{2}$$

In the tree approximation (cf. fig. 1) we have

$$A_{22}^{\text{tr}} = -\mathrm{i}g\,2m^2 \frac{(a_{1'} - a_{2'})}{(a_1 a_2)^{1/2}} \cdot \frac{(a_1 - a_2)}{(a_1 a_2)^{1/2}}. \tag{3}$$

In two space-time dimensions energy-momentum conservation admits only two discrete solutions

$$a_{1'} = a_1, \quad a_{2'} = a_2, \qquad \text{or} \quad a_{1'} = a_2, \quad a_{2'} = a_1. \tag{4}$$

Secondly we compute the connected part of the matrix element for three-particle-scattering

$$M_{33}^{\text{conn.}} = (2\pi)^2\, \delta^{(2)}(p_1 + p_2 + p_3 - p_{1'} - p_{2'} - p_{3'})$$
$$\times (2\pi)^{-3} A_{33}.$$

In the tree approximation A_{33} is determined by essentially one graph (cf. fig. 2)

$$A_{33}^{\text{tr.}} = \sum_{\alpha', \alpha} B(\alpha', \alpha),$$

where the summation corresponds to different assignments of the labels to the external lines $\alpha = (1, 2, 3)$, $(2, 3, 1)$ and $(3, 1, 2)$ and similarly for α'.

Using (1) the principal value part of the internal propagator is seen to be

$$\frac{\mathrm{i}}{\not{p}_1 + \not{p}_3 - \not{p}_{3'} - m}$$

$$= \frac{\mathrm{i}}{(p_1 + p_2 - p_{3'})^2 - m^2} \sum_{i=1,2,3'} \epsilon_i u(p_i)\, \bar{u}(p_i)$$

$$\epsilon_1 = \epsilon_2 = -\epsilon_{3'} = 1$$

where in terms of a's:

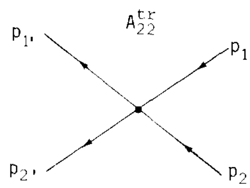

Fig. 1.

$$(p_1+p_2-p_{3'})^2-m^2 = -m^2\,\frac{a_1+a_2}{(a_1a_2)^{1/2}}\,\frac{a_1-a_{3'}}{(a_1a_{3'})^{1/2}}\,\frac{a_2-a_{3'}}{(a_2a_{3'})^{1/2}}\,. \tag{6}$$

From (3), (5) and (6) we have

$$B(1',2',3';3,1,2) = \mathrm{i}g^2\,4m^2(a_1a_2a_3a_{1'}a_{2'}a_{3'})^{-1/2} \times \frac{(a_{1'}-a_{2'})(a_1-a_2)a_{3'}}{a_1+a_2}\left\{a_2\,\frac{a_3-a_1}{a_2-a_{3'}}+a_1\,\frac{a_3-a_2}{a_1-a_{3'}}\right\}.$$

After some algebra we find

$$A^{\mathrm{tr.}}_{33} = \mathrm{i}g^2\,8m^2(a_1a_2a_3a_{1'}a_{2'}a_{3'})^{-1/2} \times (a_{1'}-a_{2'})(a_{2'}-a_{3'})(a_{3'}-a_{1'})\cdot Z$$

$$Z = \frac{(a_3a_1-a_2^2)(a_3-a_1)}{N_2} + \frac{(a_1a_2-a_3^2)(a_1-a_2)}{N_3} + \frac{(a_2a_3-a_1^2)(a_2-a_3)}{N_1}$$

where

$$N_1 = \frac{1}{a_1^2}(a_2+a_3)(a_3+a_1)\cdot(a_1-a_{1'})(a_1-a_{2'})(a_1-a_{3'}),$$

etc. At this stage it is convenient to make use of energy-momentum conservation:

$$a_1+a_2+a_3 = a_{1'}+a_{2'}+a_{3'},$$
$$a_1^{-1}+a_2^{-1}+a_3^{-1} = a_{1'}^{-1}+a_{2'}^{-1}+a_{3'}^{-1}. \tag{7}$$

The easily derived identity

$$(a_1-a_{1'})(a_1-a_{2'})(a_1-a_{3'}) = \frac{a_1(a_2+a_3)}{a_2a_3} \times (a_{1'}a_{2'}a_{3'}-a_1a_2a_3)$$

implies

$$N_1 = N_2 = N_3 = N.$$

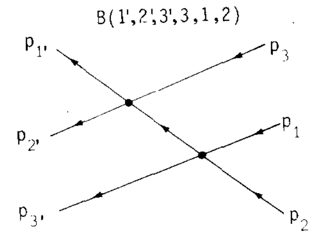

Fig. 2.

We obtain for all momenta subject to the energy-momentum restrictions (7)

$$Z\cdot N = [(a_3a_1-a_2^2)(a_3-a_1) + (a_1a_2-a_3^2)(a_1-a_2)+(a_2a_3-a_1^2)(a_2-a_3)] = 0.$$

Therefore the matrix element for three-particle-scattering vanishes identically in tree approximation for generic momenta ($\{p_{i'}\} \neq \{p_j\}$):

$$M^{\mathrm{conn}}_{33} \equiv 0. \tag{8}$$

The δ-function term in the propagator (5) which has been neglected so far gives a contribution to M_{33} when the sets of incoming and outgoing momenta are equal. It is a sum of products of two particle scattering terms given by (3).

$$M^{\mathrm{conn}}_{33} = \sum_{\alpha',\alpha} M(\alpha',\alpha)$$

$$M(1',2',3';3,1,2) = \frac{1}{2}\int\frac{\mathrm{d}p_0^1}{2p_0^0} \times M(1',2';3,0)M(0,3';1,2).$$

The matrix element for pair production (2→4)

$$M_{42} = {}^{\mathrm{out}}\langle p_{3'},p_{2'},p_{1'};\bar{p}|p_1,p_2\rangle^{\mathrm{in}}$$

is obtained from M^{conn}_{33} by the substitution law (2). Since there are no contributions from the δ-function part:

$$M_{42} \equiv 0. \tag{9}$$

As already mentioned recent progress on the Sine-Gordon theory and on its connection with the massive Thirring model leads to the conjecture that particle number and set of their momenta are conserved even in the Thirring model. The results of this short note show the tree approximation to be in agreement with this conjecture. From (9) we see no pair-production in the considered case and (8) together with (4) implies the conservation of the set of particle momenta in three particle scattering. Even more we suspect the vanishing of the connected part of $(n \to n)$-particle-scattering, implying no pair production at all in this approximation.

In a forthcoming paper we will consider the one loop approximation. This is interesting because of the possibility of anomalies [6].

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