PARTICLE NUMBER CONSERVATION IN THE MASSIVE THIRRING MODEL

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We computed pair-production and nontrivial three-particle scattering amplitudes of the massive Thirring model in one loop approximation. They are found to be zero.

In contrast to the massless Thirring model in the massive Thirring model an explicit solution is unknown. Methods [1] useful in the massless case cannot be applied because the axial current is not conserved. Moreover the S-matrix derived by Berezin and Sushko [2] is in contradiction with explicit calculation in perturbation theory [3]. New insights may come from the existence of additional conservation laws for which there is now very strong evidence.

In the last few years there has been considerable progress in the treatment of nonlinear wave equations [4]. This provided the basis for further developments of interesting quantization methods [5], which start from classical solutions. The Sine-Gordon equation is a well known example where these methods turn out to be useful. In the classical Sine-Gordon theory infinitely many conserved currents [6] imply the conservation of a) the number of particles and b) the set of their momenta. Araf'eva and Korepin [7] claim these properties to hold also for the quantized Sine-Gordon field in all orders of perturbation theory. Flume [8], however, has shown that there are anomalies in the Ward identities of the currents.

In the classical Sine-Gordon theory, properties a) and b) correspond to the so-called soliton-property [9]. It has been suggested [10] that they are also true for the quantized soliton solutions. Since the latter are connected [11] with the fermion field of the massive Thirring model, properties a), and b) can be conjectured to hold in the Thirring model, too. We believe that explicit Thirring model perturbative calculations can help to clarify the present indefinite situation. In particular if the conjecture were true a necessary consequence would be that the pair production amplitude ($2 \to 4$) vanishes and that the three-particle scattering amplitude ($3 \to 3$) is nonzero only if the sets of incoming and outgoing momenta are equal.

Previous results in the tree approximation were reported recently [12, 13]. In this letter we present the results up to the one loop approximation and verify that the properties in question continue to hold. For the explicit calculation [14] we made use of Veltman's algebraic program SCHOONSHIP [15].

In the tree approximation [12] the matrix element for three-particle scattering vanished identically for generic momenta ($\{p_{\text{in}}\} \neq \{p_{\text{out}}\}$) and the contribution for equal sets of incoming and outgoing momenta is

$$
\begin{aligned}
&{}^{\text{out}}\langle p_6, p_5, p_4 | p_1, p_2, p_3\rangle^{\text{in}} \\
&\quad = {}^{\text{in}}\langle p_6, p_5, p_4 | p_1, p_2, p_3\rangle^{\text{in}} \exp[\mathrm{i}\delta_{33}(p_1, p_2, p_3)] \qquad (1)
\end{aligned}
$$

with

$$
\begin{aligned}
&\delta_{33}(p_1, p_2, p_3) \\
&\quad = \delta_{22}(p_1, p_2) + \delta_{22}(p_2, p_3) + \delta_{22}(p_3, p_1)
\end{aligned}
$$

and the two-particle scattering phase is given by

$$\exp[\mathrm{i}\delta_{22}(p_1, p_2)] = 1 - \mathrm{i}g(-t_{12}/s_{12})^{1/2} + \mathrm{O}(g^2) \qquad (2)$$

with

$$s_{12} = (p_1 + p_2)^2, \qquad t_{12} = (p_1 - p_2)^2 .$$

It is a consequence of this result that the matrix element for pair production ($2 \to 4$) is identically zero.

We now consider the connected part of the three-particle scattering amplitude in third order of perturbation theory. In addition to the genuine third order contributions which give rise to one loop graphs there are tree graphs with coupling constant renormalization counterterms. The latter, however, are proportional to the tree amplitude. Therefore their contributions vanish separately and we are left with the one loop graphs.

In the one loop approximation there exist six types of graphs. One observes that the (logarithmically) divergent terms cancel. This is a reflection of the well known fact that the coupling constant renormalization of the massive Thirring model is finite or equivalently that the Callan-Symanzik function β vanishes identically [16]. It is a pecularity of two space-time dimensions that all one loop integrals can be reduced to a single typical integral:

$$I(p^2) = \frac{1}{(2\pi)^2} \int \mathrm{d}^2 k \, \frac{1}{k^2 - m^2 + \mathrm{i}\epsilon} \, \frac{1}{(k+p)^2 - m^2 + \mathrm{i}\epsilon}.$$

Therefore we obtain for the three-particle scattering amplitude the following decomposition:

$$A_{33}^{\text{loop}}(p_6, p_5, p_4; p_1, p_2, p_3)$$

$$= c_0 I(0) + \sum_{\substack{i<j \\ i,j \in (1,2,3) \\ i,j \in (4,5,6)}} c_{ij} I(s_{ij}) + \sum_{\substack{i \in (1,2,3) \\ j \in (4,5,6)}} c_{ij} I(t_{ij}).$$

We can prove [14] that the coefficients in eq. (3) vanish separately, unless the sets of incoming and outgoing momenta are equal. Furthermore formula (1) remains valid in the loop approximation. Again the result implies that there is no pair production ($2 \to 4$) in the considered approximation.

We conclude that there is so far no direct evidence against the existence of infinitely many conservation laws in the massive Thirring model. We are optimistic that this question will be settled in the near future [17].

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[17] I.Ya. Araf'eva informed us of a paper of hers to be published in Theoreticheskaja i mathematicheskaja Physika. She claims to have found "conservation laws which imply the impossibility of particle production in the massive Thirring model".