# A constructive proposal for an operator approach to the crossing property

dedicated to Jacques Bros on the occasion of his 70th birthday

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**Abstract**

In trying to generalize constructive ideas underlying the bootstrap-formfactor program beyond the limitation of d=1+1 factorizing models, the main obstacle is the lack of an operator interpretation of the crossing property. Here we propose a hypothesis involving a "masterfield" whose connected formfactors define an auxiliary thermal QFT, in which the KMS cyclicity equation for the thermal expectation values of the auxiliary field is identical to the crossing property of the formfactors of the master field. The speculative nature of the idea is somewhat mitigated by its successfull check in factorizing models.

## 1 History of the crossing property

The so-called crossing property of the S-matrix and formfactors[1] is a deep and important but at the same time incompletely understood structure in particle physics. As a result of its inexorable link with analyticity properties in scattering theory within the setting of QFT, it is not a symmetry in the standard sense (of Wigner) even though it is often referred to as "crossing symmetry".

In contrast to the underlying causality principles which are "off-shell", i.e. are formulated in terms of local observables or fields with unrestricted Fourier transforms, the crossing property is "on-shell", that is to say it refers to particle states which are described by wave functions on the forward mass hyperboloid $p^2 = m^2$. Particle properties are intrinsic to a theory whereas fields are (point-like [1] or string-like[2]) "coordinatizations" of local algebras i.e. only local

[1] In the setting of this paper the S-matrix as a transition amplitude between incoming and outgoing scattering states is a special case of a (generalized) formfactor associated with the identity operator.

equivalence classes of fields or the local algebras generated by fields are intrinsic[2]. The use of the word "intrinsic" in local quantum physics (LQP) is reminiscent of the use "invariant" (as opposite to coordinate-dependent) in geometry; in this analogy the coordinates in geometry correspond to the coordinatization of spacetime-indexed algebras by pointlike field generators. In the Lagrangian quantization approach to quantum field theory as well as in the more intrinsic algebraic approach to local quantum physics, crossing plays no important role. Only in formulations of particle physics which start with on-shell quantities and aim at the construction of spacetime-indexed local algebras (or local equivalence classes of fields), the crossing becomes an important structural tool.

Examples par excellence of pure on-shell approaches are the various attempts at S-matrix theories which aim at direct constructions of scattering data without the use of local fields and local observables. The motivation behind such attempts was already (before renormalization theory) clearly spelled out by Heisenberg [3]: by limiting oneself to particles and their mass-shells one avoids (integration over) fluctuation over arbitrarily small spacelike distances which are the cause of ultraviolet divergencies.

This idea is radically different, and in a way more conservative, than the attempts at improving shortdistance properties by introducing non-local interactions in a field theoretic framework (for a review see [4]). Since it does not cast doubts on the existence of pointlike fields, but unlike Lagrangian quantization, it only tries to avoid such delicate inherently singular objects in calculational steps. Heisenberg's S-matrix setting paid attention to unitarity and certain analytic properties, but his Ansatz already had problems with implementing cluster factorization properties for the multiparticle scattering[3].

Since the early 1950s, in the aftermath of renormalization theory, the relation between particles and fields received significant clarification through the derivation of time dependent scattering theory. It also became clear that Heisenberg's proposal had to be amended by adding the crossing property i. e. a prescription of how to analytically continue particle momenta on the complex mass shell in order to relate matrix elements of local operators (or the identity in case of the S-matrix) between incoming ket and outgoing bra states.

The LSZ time-dependent scattering theory and the associated reduction formalism relates such a matrix element[4] (generalized formfactor) to one in which an incoming particle becomes "crossed" into an antiparticle on the backward real mass shell; it is at this point where the analytic continuation enters. The important point here is that the use of particle states requires the restriction of the analytic continuation to the complex mass shell ("on-shell"). If one were

[2]The individuality of classical fields is lost in QFT where e.g. a meson field is any local (relative local with respect to the local obervables of the theory) covariant object with a nonvanishing matrix element between the vacuum and a one-meson state ("interpolating field").

[3]There is a more recent scheme of "direct particle interaction" (a kind of relativistic multiparticle quantum mechanics without vacuum polarization) which leads to Poincaré invariant unitary clustering S-matrices [5][4] and in this way completes Heisenberg's program.

[4]More precisely the crossing property involves the connected part of (generalized) formfactors (see next section).

to allow interplations via "off-shell" field states, the derivation of the crossing would be much easier since it would follow from off-shell spectral representations of the Jost-Lehmann-Dyson kind or perturbatively from Feynman diagrams and time-ordered functions. In this paper crossing will only used in the for the more restrictive on-shell analytic continuation as it is needed for formfactors.

A rigorous on-shell derivation for for two particle scattering amplitude i.e. for the two-particle formfactor of the identity operator where in order to maintain energy-momentum conservation the above in-out crossing has to compensated by a simultaneous out-in crossing (i.e. a true crossing of two particles) has been given by Bros[5], Epstein and Glaser [6]. A derivation of crossing from QFT for general multi-particle configurations as it is used in many applications (see later) does not exist, and it is not clear to me whether the present state of QFT would permit to go significantly beyond the old and still impressive results in [6].

The crossing property became the cornerstone of the so-called bootstrap S-matrix program and many ad hoc representations of analytic scattering amplitudes were invented (Mandelstam, Regge..) in order to accomodate crossing into a managable constructive S-matrix bootstrap formalism. An interesting early historical chance to approach QFT from a different direction by using on-shell global objects without short distance singularities was wasted when the S-matrix bootstrap approach ended in a verbal cleansing rage against QFT[6].

Some of the ideas were later used by Veneziano [7] in the construction of the dual model. But there is an essential difference in the way crossing was implemented. Whereas the field theoretic crossing involves a finite number of particles with the scattering continuum participating in an essential way, the dual model crossing does not need the continuum which is substituted by a discrete infinite "particle tower" with ever increasing masses (the origin of what is called "stringyness") which later were interpreted as the various particle exitations of a relativistic string. It is important to note that Veneziano's successful experiment with properties of Gamma functions did satisfy the phenomenological desires in those days (in this sense it was more than a mathematical invention) which was calling for a one-particle "saturation" (suggested by fast converging dispersion relations in connection with Regge poles) of crossing in the setting of Mandelstam's representation of the 2-particle scattering amplitude. It is important to mention these ideas which separate the dual model from the field-theoretic ideas about crossing in order to counteract the wrong impression (partially spread by textbooks on string theory) that the dual model is the legacy of a QFT related S-matrix bootstrap approach.

It is somewhat ironical that Chew, who desperately wanted a clear separation of his S-matrix ideas from QFT did not succeed in these attempts (the S-matrix bootstrap returned many years later enriched with the formfactor program in

[5] This is the reason the dedication of this particular work to Jacques Bros on the occasion of his $70^{th}$ birthday is particularly appropriate.

[6] One glance at the old conference proceedings and review articles of the Chew S-matrix school reveals what is meant here. The ideological fervor is hard to understand, in particular since almost all the concepts came from QFT.

the limited context of d=1+1 factorizing models [8]), whereas Veneziano, who had no such ideological aims, layed the seeds of string theory whose structural relation to particle physics, despite its phenomenological beginnings (and its later paradigmatic change into an ultraviolet-finite candidate for quantum gravity), became increasingly mysterious. Its endurance up to the present can not be separated from the fact that led to significant mathematical enrichments in a manner going far beyond its humble mathematical origins.

The main reason why the old bootstrap approach ended in the dustbin of history (besides suffering the merited consequences of its derogatory view of QFT shortly before the discovery of the physical relevance of gauge theories) is its lack of success in taking the crossing property out of its purely analytic setting and to convert it into an operational tool. It is precisely this point which we will address in this article with the help of a new idea. The reader who expects a deep mooring of the crossing property with the principles underlying QFT may be disappointed by the content of this article; what we offer is rather a highly speculative idea about the existence of an auxiliary non-local thermal theory which describes the formfactors of a distinguihed quantum field (the "Masterfield") in the local original theory. We feel that the fact that crossing, ever since its discovery in the early 50s, has (despite innumerous attempts) remained the most mysterious and at the same time enigmatic among the incompletely understood properties of local quantum physics, more than justifies speculative attempts as the present one. Even though the support for this new operational hypothesis about crossing comes from d=1+1 factorizing models, the new "masterfield" proposal is not a priory limited as that based on the Zamolodchikov-Faddeev [9] algebra structure.

# 2 The bootstrap-formfactor program in d=1+1 factorizing QFT

As already mentioned in the introduction, a more modest, but in its own right extremely sucessful version of the S-matrix bootstrap approach with strong field theoretic roots emerged in the second half of the 70s from some quasiclassical findings [10]. These seemingly exact quasiclassical observations on two-dimensional models of QFT required an explanation beyond quasiclassical approximations [11] and from this emerged a constructive program of a bootstrap-formfactor construction of so-called d=1+1 factorizable models [8][12][13]. This new nonperturbative way of constructing a particular class of field theories produced a steady flux of new models and continues to be an important innovative area of research.

In contrast what was expected from the ambitious original bootstrap approach, this program uses the S-matrix bootstrap idea in the limited context of S-matrices which factorize into 2-particle elastic components. The advantage is that the classification and calculation of factorizing S-matrices [14] can be treated as a separate problem from that of formfactors of nontrivial fields[7] in order to classify and construct associated quantum field theories via their generalized "formfactors" (matrixelements of localized operators in the incoming-ket/outgoing-bra particle basis). The motivating idea in favor of an on-shell approach, namely the total avoidance of ultraviolet divergencies, is brilliantly vindicated in this approach.

The pointlike fields (which in the present stage of development are only known in form of their multi-particle formfactors) in these models have a quite interesting structure; despite their lack of real particle creation through scattering processes they nevertheless possess the full vacuum polarization structure i.e. the setting is significantly different from the "direct particle interaction" approach of relativistic quantum mechanics. Their short distance dimensions go beyond canonical behavior of those fields (example: superrenormalizable power couplings) whose existence can be mathematically controlled by the functional analytic methods of so-called "constructive QFT" [15]. The fact that this approach following from general principles and does not need ad hoc prescriptions (as one still encounters in its first systematic formulation [13]) becomes particularily transparent if placed into the Tomita-Takesaki modular theory of operator algebras as adapted to the local quantum physics setting (also referred to as the method of modular localization) [16][17][18] which will be briefly sketched below.

This setting also highlights the "existence problem of QFT" in a new and very promising fashion [19]. Here we remind the reader that among all areas of theoretical physics QFT has in an essential remained outside of mathematical control. Even after almost 8 decades after its discovery and despite its perturbative and asymptotic (in the short distance regime) successes, the description of interacting particles by interacting covariant pointlike fields in 4-dimensional Minkowski spacetime is still part of mathematical uncharted territory, i.e. we cannot be sure that we are dealing with something which really exists in this form (even the perturbative results which exist in their own right have an unknown relation to the expected true content).

The algebraic basis of the bootstrap-formfactor program for the special family of d=1+1 factorizable theories is the validity of a momentum space Zamolodchikov-Faddeev algebra [9]. The operators of this algebra are close to free fields in the on-shell sense, but they are non-local in the specific sense of being localizable in wedge algebras [16][18]. From this follows that the wedge algebras generated via their Fourier transforms by these on-shell Z-F field operators possess vacuum-**p**olarization-**f**ree **g**enerators (shortly called PFGs). These are on-shell operators; the only property which distinguishes them from standard creation/annihilation operators is the fact that the product of two such operators in opposite orders is different by rapidity-dependent structure functions (matrix-valued in case of multi-component creation/annihilation operators). In fact the existence of "temperate" (Fourier transformable) wedge localized PFGs which implies the absence of real particle creation through scattering processes [17] turns out to be the prerequisite for the working of the present bootstrap-

[7]The matrixelements of S are the formfactors of the identity operator.

formfactor program which is a calculational scheme in which one uses only formfactors and no (short-distance singular) correlation functions. According to an old structural theorem which uses analytic properties of a field theoretic S-matrix [20] virtual particle creation without real particle creation is only possible in d=1+1 theories. According to an argument by Karowski[8] it even rules out direct 3- or higher- particle elastic processes so that PFGs describing interacting d=1+1 QFT appear to be limited to Z-F algebra structures.

It is important to note that the bootstrap-formfactor construction for d=1+1 factorizing models contains a valuable information about the uniqueness of the inverse scattering problem. As a result of the insensitivity of the S-matrix against local changes within the same QFT, the unicity of the inverse problem is defined to mean that the S-matrix determines the local equivalence classes of fields (rather than individual fields). This unique relation of a crossing symmetric S-matrix with a local equivalence class of fields is indeed what the bootstrap-formfactor program within the limited context of the factorizing models[9]. The crucial property which links scattering data with off-shell operators spaces is the crossing property. It relates the multiparticle component of vectors obtained by one-time application of fields to the vacuum with the (connected) formfactors of those fields. In other words, the crossing serves to construct from the vacuum-polarization components of wedge-localized operators their general (connected) matrixelements in the multi-particle basis between incoming ket- and outgoing bra- states. If a given S-matrix would lead to two different wedge algebras then the modular subspace i.e. the space which the Tomita-Takesaki modular theory associates with this algebra (relativ to the cyclic and separating vacuum) must already have been different. But this is impossible since the position of this subspace is determined by the Tomita operator which in turn is fixed in terms of the S-matrix [22]. This unicity property which results from this general crossing argument (without using the factorizability of the model) is based on a formulation of the crossing property[10] which is stronger than what had been derived in the work of Bros and Epstein [6] (see below). Even assuming this stronger form of crossing which is sufficient for unicity of the inverse scattering problem, it has not been able to prove the existence of a local field system (or the exitence of associated formfactor spaces) for given scattering data.

This raises the question of whether there exists a stronger constructive on-shell approach in terms of formfactors which is not limited by the existence of tempered PFGs and which could work also in d=1+3. It is clear that *any approach which uses in its computations only formfactors* (the S-matrix is always included as the formfactor of the identity operator between incoming ket and outgoing bra states) and avoids correlation functions of pointlike field coordi-

[8] private communication by M. Karowski.

[9] See a recent review [21] in which the minimal formfactor contributions, which are a joint property of the local equivalence class, have been separated from the polynomial contributions (the "p-functions") which distinguish between the vacuum polarization contributions of individual fields.

[10] Without the crossing property hypothesis it is presently not possible to exclude the existence of several local field systems (or local nets) with one S-matrix [23].

nates *will be free of ultraviolet problems.*

From the previous remarks one expects that such an approach requires a much better operational understanding of crossing which has remained the most mysterious property of particle physics. Time and again has shown that analyticity properties on their own may be sufficient to derive certain sum rules, but are insufficient as the a of calculations and model constructions.

In the remainder of this paper an operational setting of crossing is presented which in principle does not suffer from the above limitations of temperate PFGs, although one is presently only able to test it by explicit model examples in the d=1+1 factorizing setting. It is based on the working hypothesis that each quantum field theory possesses a distinguished field called a "Masterfield" whose connected parts of its formfactors defines a global (i.e. no local substructure) quantum field theory in the on-shell momentum space variables in a thermal state at the KMS Hawking temperature in such a way that the cyclic KMS property (the thermal aspect of modular theory) is identical with the cyclic crossing property. By construction this theory obeys momentum space cluster decomposition properties in the rapidity variable. The simplicity of d=1+1 factorizing models finds its expression in the fact that the auxiliary operator, whose KMS correlation functions are identified with the connected formfactors of the masterfield, is an exponential of a bilinear expression in free creation/annihilation operators. There is a good chance that this structure is characteristic for factorizing models.

The subsequent content of the paper is organized as follows. In the next section we recall the formal aspects of the crossing equations from the LSZ scattering theory. The fourth section reviews the modular approach to the problem of factorizing theory with special emphasis on the rigorous vacuum polarization free particle-based (PFG) Zamolodchikov-Faddeev algebra structure of the wedge-localized operator algebras and some formal as well as mathematical aspects of the construction of double cone localized operator algebras which exhibit the full vacuum polarization structure. In the fifth section we formulate the working hypothesis of the existence of a "master field" by which we hope to go beyond the confines of tempered PFGs. This is a speculative proposal for an operational formulation of crossing in terms of an auxiliary global thermal QFT which we then check in some models. These model illustrations make contact with some relations to observations about clustering properties of formfactors in the rapidity variables [21] and even more surprisingly with recent "free field representations" of formfactors [24].

# 3 Crossing recipe from LSZ scattering theory

Crossing has been first observed in Feynman perturbation before a formal derivation in the setting of LSZ scattering theory was given. Its formal aspects are easily obtained from the LSZ asymptotic convergence

$$lim_{t\to\mp\infty}A^{\#}(f_t)\Phi = A^{\#}(f)_{in,out}\Phi,\ A^{\#}=A\ or\ A^* \tag{1}$$
$$A(f_t)=\int f_t(x)U(x)AU^*(x)d^4x,\ A\in\mathcal{A}(\mathcal{O})$$
$$f_t(x)=\frac{1}{(2\pi)^2}\int e^{i(p_0-\omega(p))t-ipx}f(\vec{p})d^4x,\ \omega(p)=\sqrt{\vec{p}^2+m^2}$$

which can be derived on a dense set of states. This is known to lead to the well-known LSZ reduction formulas (for a rigorous derivation see [25] [26]) which in terms of connected matrix elements (leaving out the smearing with one-particle wavefunctions) read

$$^{out}\langle q_1,q_2,...q_m\,|F|\,p_n,...p_2,p_1\rangle^{in}_{conn} = \tag{2}$$
$$-i\int {}^{out}\langle q_2,...q_m\,|K_yTFA^*(y)|\,p_1,p_2...p_n\rangle^{in}_{conn}\,d^4ye^{-iq_1y} =$$
$$-i\int {}^{out}\langle q_1,q_2,...q_m\,|K_yTFA(y)|\,p_2...p_n\rangle^{in}_{conn}\,d^4ye^{ip_1y} =$$

Here the time-ordering $T$ involving the original operator $F\in\mathcal{A}(\mathcal{O})$ and the interpolating Heisenberg field[11] $A(x)\equiv U(x)AU^*(x)$ or $A^*(x)$ with $A\in\mathcal{A}(\mathcal{O})$ for some $\mathcal{O}$ appears in the reduction of a particle from the bra- or ket state. For the definition of the time ordering of a fixed finitely localized operator $F$ and a field with variable localization $y$ we may use $TFA(y)=\theta(-y)FA(y)+\theta(y)A(y)F$, however as we place the momenta on-shell, the definition of time ordering for $y$ near $locF$ is irrelevant[12]. Each such reduction is accompanied by another disconnected contribution in which the creation operator of an outgoing particle say $a^*_{out}(q_1)$ changes to an incoming annihilation $a_{in}(q_1)$ acting on the incoming configuration (and the opposite situation i.e. $a^*_{in}(p_1)\to a_{out}(p_1)$). These terms (which contain formfactors with two particle less in the bra- and ket- vektors) have been omitted since they do not contribute to generic nonoverlapping momentum contributions and to the analytic continuations and hence do not enter the connected part. Under the assumption that there is an analytic path from $p\to -p$ (or $\theta\to\theta-i\pi, p_\perp\to -p_\perp$ in the wedge adapted rapidity parametrization) the comparison between the two expressions gives the desired crossing property: a particle of momentum p in the ket state within the connected part of a formfactor is indistinguishable from a bra antiparticle at momentum -p (here denoted as -p̄).

In order to obtain that required analytic path on the complex mass-shell of the $2\to 2$ scattering amplitude it is convenient to pass from time ordering to retardation

$$TFA(y)=RFA(y)+\{F,A(y)\} \tag{3}$$

[11] The operators may be bounded or unbounded with Wightman like domain peoprties. The asymptotes are always local linear combinations of (necessarily unbounded) creation/annihilation operators.

[12] These on-shell reduction formulas remain valid if one used as interpolating pointlike fields $A(x)$ the translates of bounded localized operators [26].

The unordered (anticommutator) term does not have the pole structure on which the Klein-Gordon operator $K_y$ can have a nontrivial on-shell action and therefore drops out. The application of the JLD spectral representation puts the p-dependence into the denominator of the integrand of an integral representation where the construction of the analytic path for formfactors proceeds in an analog fashion to the derivation of crossing for the S-matrix [27][26]. Whereas it is fairly easy to find an off-shell analytic path, the construction of an on-shell path which remains in the complex mass shell is a significantly more difficult matter [6]. The LSZ reduction formalism is suggestive of crossing but far to weak for securing the existence of paths on the complex mass shell which link real forward and backward mass shells.

The simplifications of the LSZ formalism resulting from factorizability of models can be found in an appendix of [28]

The result of the comparision between the reduction 2 applied to outgoing and incoming configurations may be written in the following suggestive way (for spinless particles)

$$\begin{aligned} &{}^{out}\left\langle p_1, p_2, ...p_l \left| F \right| q_{k,k-1}...q_2, q_1 \right\rangle^{in} = \\ &\underset{q_c \to -q_1}{a.c.} {}^{out}\left\langle q_c, p_1, p_2, ...p_l \left| F \right| q_k, q_{k-1}...q_2 \right\rangle^{in} + c.t = \\ &:^{out}\left\langle -q_1, p_1, p_2, ...p_l \left| F \right| q_k, q_{k-1}...q_2 \right\rangle^{in} + c.t. \end{aligned} \tag{4}$$

where the contraction terms c.t. involve momentum space $\delta$-functions (which are part of the LSZ reduction theory) and the last line denotes a shorthand notation for the analytic continuation to the real negative mass shell. Instead of crossing from incoming ket to outgoing bras one may also cross from bras to kets. The important physical role of the crossing property is to relate the vacuum polarization components of an operator to the connected part of the transition it causes between in and out scattering states via iterated crossing

$${}^{out}\left\langle p_1, p_2, ...p_n \left| F \right| \Omega \right\rangle \overset{iteration}{\longrightarrow} {}^{out}\left\langle p_k, p_{k+1}, ...p_n \left| F \right| -\bar{p}_{k-1} ... -\bar{p}_2, -\bar{p}_1 \right\rangle_{conn}^{in} \tag{5}$$

Note that the vacuum polarization components are already connected. It is very important to realize that the simplicity of the crossing property occurs only for the connected part of the matrix elements; in order to write down the relation for the full matrix elements one must keep track of all the momentum space contraction terms in the iterative application of the LSZ formalism. It is the connected part which is described by one analytic "master function" whose different boundary values correspond to the connected part of the different matrix elements. This fact indicates that one should expect problems if one wants to understand crossing as an operational property in the original theory of operators. Indeed the first naive attempts to relate crossing to the cyclicity property of thermal expectation values in KMS states failed[13].

[13] The paper [29] contains an irreparable mistake and even an earlier paper of the present author contains some incorrect remarks about this point [30].

In one aspect the case $F=1$ (the S-matrix) is a degenerate case of a localized operator, namely in that case the momentum conservation leads to a vanishing vacuum polarization. The lowest non-vanishing contribution comes from the elastic two particle S-matrix and in order the crossing property requires the simultaneous crossing of pairs of particles (an incoming particle from the ket is interchanged with an outgoing bra particle, in fact this is the historical origin of the terminology "crossing"). S-matrix crossing played an important role in the proof of the impossibility of having only vacuum polarization (virtual creation) without creation of real particles (on-shell creation) [20] for QFTs of Bosons/Fermions in d >1+1. Only in d=1+1there exist interacting models with a rich vacuum polarization structure caused by sub-wedge localization[14] without on-shell particle creation.

# 4 The Zamolodchikov-Faddeev algebra and its relation to modular localization

In this section we recall some details about how the modular localization formalism leads to a profound understanding of the bootstrap-formfactor construction of factorizable models. It turns out that the best insight into the working of this method is obtained if one first looks at this new formalism in the simpler case of the net of algebras associated to free fields from the modular structure of the Wigner representation theory of unitary irreducible positive energy representations of the Poincaré group. The mathematical implementation of this idea was known for a long time: the CCR-(Weyl)- or CAR functor applied to modular localized real subspaces of the complex Wigner representation space of integer or halfinteger spin representations maps these objects directly into local von Neumann algebras [32]. What was not known at that time however was how to explicitely compute those subspaces by intrinsic (avoiding pointlike field coordinatizations) modular methods. The guiding idea was found only quite late, although its basic ingredience namely the geometric nature of the wedge localization spaces should have been known, since the work of Bisognano-Wichmann [33] dealing with the geometric aspects of the modular objects associated with wedge algebras is quite old.

There were two independent settings in which constructions based on modular localization emerged. On the one hand it was shown by the present author that the prescriptions of the d=1+1 bootstrap-formfactor program (the Smirnov postulates and the Zamolodchikov-Faddeev algebraization) permitted a natural and profound explanation in the setting of modular localization and the modular subspaces of the simplest spinless representation were constructed as a pedagogical exercise as well as for obtaining additional support for the central role of wedge localization in QFT constructions without Lagrangian quantization.

On the other hand Brunetti Guido and Longo systematically constructed the family of wedge-localized real subspaces for all positive energy Wigner repre-

[14] This possibility is related to the existence of wedge-localized PFGs [16].

sentations by combining purely group-theoretical ideas with aspects of modular theory. Although spaces corresponding to causally complete subwedge regions (e.g. spacelike cones, double cones) have no geometric modular characterization, they can easily be constructed by intersecting wedge localized spaces. The work of these authors culminated in a surprising theorem stating that positive energy representations of the Poincare group always have nontrivial spacelike cone localized subspaces (for arbitrary small opening angles) and that the wedge like localization can be re-obtained by additivity from small spacelike cone localized subspaces which approximate the given region from the inside.

In case of (half)integer spin representations one can show that the compact spacelike cone intersections are nontrivial by constructing a dense set of double cone localized wave functions. However there were two class of representations which did not allow compact localization (i.e. the double cone localization spaces were trivial) namely the d=1+2 representations with "anyonic" spin (s$\neq$ (half)integer) and the famous Wigner family of zero mass infinite helicity representations. Both cases lead to nontrivial spacelike cone localized subspaces and hence it was suspected that these representations may be associated with string-localized field theories [2][4]. This turned out to be true; but only in the infinite spin case the string-localized fields are genuine free fields whereas in the anyonic case the string localized fields applied to the vacuum are always accompanied by vacuum polarization which throws them off mass shell [34].

The modular approach to the d=1+1 factorizing models is similar in that the principle structure is always associated with wedge-localized algebras and the subwedge-localized objects are formed by intersections. The main difference is that in all the situations of positive energy Wigner representations (with the exception of the case of d=1+2 anyons) there also exist subwedge-localized (e.g. double cone-localized) PFGs and the algebras are obtained from a spatial modular theory in a functorial way (the CCR or CAR functor). This had led me already in a very early stage of my investigations to view the whole constructive program based on modular theory as an extension of the functorial relation between spatially and algebraically localized objects to the realm of interactions [30].

Here I am using equivalent intrinsic definitiones of the meaning of "interacting". On the one hand one can take the absence of PFGs associated to subwedge-localized algebras as the definition of absense of interactions (this assumption excludes "infraparticles" [31] which are always surrounded by a infrared polarization cloud). It follows from this definition that such PFG can be reduced to free fields. The second more standard definition relates absence of interactions with the scattering matrix being trivial S=1. Under the standard assumptions for the derivation of scattering theory as well (as the for the setting of the previously formulated crossing property) one can show that this definition also implies that there exist free fields (only different from the PFGs by normalizations) which generate the local algebras. The main nontrivial step in which the above formulated property of formfactor crossing enters in an essential way is the uniqueness of the inverse scattering problem in QFT, which for a crossing symmetric situation with S=1 amonts to the equivalence class of the free field as the only field system with the trivial S-matrix. For the S-matrix of the d=1+1 factorizing models (whose S-matrix is given in terms of a combinatorial formula in terms of the elastic two-particle scattering) the unicity is implicit in the construction of a unique system of formfactors (without being forced to invoke an additional selection principle) of an associated composite field system.

There are two observations about factorizing models which turned out be make a perfect match. On the one hand it has been noted [16] that the Fourier transforms of the Zamolodchikov-Faddeev algebra creation and annihilation operators in the case of absence of boundstates[15] are generators of wedge-localized Weyl algebras. In the simplest case we have (in the absence of bound state poles of $S(\theta)$)

$$\begin{aligned}
\phi(x) &= \frac{1}{\sqrt{2\pi}}\int(e^{ip(\theta)x(\chi)}Z(\theta)+h.c.)d\theta \qquad (6)\\
Z(\theta)Z^*(\theta') &= S^{(2)}(\theta-\theta')Z^*(\theta')Z(\theta)+\delta(\theta-\theta')\\
Z(\theta)Z(\theta') &= S^{(2)}(\theta'-\theta)Z(\theta')Z(\theta)
\end{aligned}$$

Here $p(\theta)=m(ch\theta,sh\theta)$, is the rapidity parametrizations of the d=1+1 mass-shell and $x=r(sh\chi,ch\chi)$ parametrizes the right hand wedge in Minkowski spacetime; $S(|\theta|)$ is the physical S-matrix of elastic two particle scattering and $S(z)$ its analytic continuation. It fulfills unitarity $S^2(z)^*=S^{(2)}(-z)$ and crossing [14]. The $Z$ operators are deformation of standard creation/annihilation operators; the definition of $Z$ on a standard Fock space vector created by an ordered product $(\theta_1>\theta_2>...>\theta_n)$ of incoming $a^*(p)$ reads

$$Z^*(\theta)a^*(\theta_1)a^*(\theta_2)...a^*(\theta_n)\Omega=\prod_{i=1}^{k}S(\theta-\theta_i)a^*(\theta_1)a^*(\theta_2)..a^*(\theta).a^*(\theta_n)\Omega \qquad (7)$$

where $\theta<\theta_i$ $i=1..k$, $\theta>\theta_i$ $i=k+1,..n$. The Z-F commutation relation follows from this defintion. In the presence of bound states it is better to work with the appropriately modified state vector definition than with the Z-F algebra. The general Zamolodchikov-Faddeev algebra is a matrix generalization of this structure. As a result of the rapidity dependent commutation relations of the $Z's$, the field is Poincare-covariant but not local in the sense of the standard usage of this terminology[16]. It is however wedge-localized in the sense that the generating family of operator for the righthand wedge $W$ algebra $alg\left\{A(f),suppf\subset W\right\}$ commute with the TCP transformed algebra $alg\left\{JA(g)J,suppg\subset W\right\}$ which is the left wedge algebra [18]

$$\begin{aligned}
[A(f),JA(g)J] &= 0 \qquad (8)\\
J &= J_0S_{scat}
\end{aligned}$$

[15] A situation which in case of factorizing models with variable coupling (as e.g. the Sine-Gordon theory) can always be obtained by choosing a sufficiently small coupling.

[16] The world local is reserved for "commuting for spacelike distances". In this work we are dealing with non-local fields which are nevertheless localized in causally complete subregions (wedges, double cones) of Minkowski spacetime.

where $J_0$ is the TCP symmetry of the free field theory associated with $a^{\#}(\theta)$ and $S_{scat}$ is the factorizing S-matrix which has the form

$$S_{scat}a^*(\theta_1)a^*(\theta_2)...a^*(\theta_n)\Omega = \prod_{i<j} S^{(2)}(\theta_i - \theta_j)a^*(\theta_2)...a^*(\theta_n)\Omega \tag{9}$$

if we identify the $a^{\#}(\theta)$ with the incoming creation/annihilation operators. It is then possible to give a rigorous proof [18] that the Weyl-like algebra generated by exponential unitaries is really wedge-localized and fulfills the Bisognano-Wichmann property

$$\begin{aligned} \mathcal{A}(W) &= alg\left\{e^{iA(f)} | \, suppf \subset W\right\} \\ \mathcal{A}(W)' &= J\mathcal{A}(W)J = \mathcal{A}(W') \end{aligned} \tag{10}$$

where the upper dash on operator algebras denotes their von Neumann commutant whereas the dash on spacetime regions it stands for the causal complement. The operator TCP operator $J$ is the (antiunitary) angular part of the polar decomposition of Tomita's algebraically defined unbounded antilinear S-operator with the following characterization

$$\begin{aligned} SA\Omega &= A\Omega, \ A \in \mathcal{A}(W) \\ S &= J\Delta^{\frac{1}{2}}, \ \Delta^{it} = U(\Lambda(-2\pi t)) \end{aligned} \tag{11}$$

with $\Lambda(\chi)$ being the Lorentz boost at the rapidity $\chi$. At this point the setup looks like relativistic QM since the $A(f)$ (similar to genuine free fields if applied to the vacuum) do not generate vacuum polarization clouds. The advantage of the algebraic modular localization setting is that vacuum polarization is generated by algebraic intersections which is in agreement with the intrinsic definition of the notion of interaction presented in terms of PFGs in the previous section

$$\begin{aligned} \mathcal{A}(D) &\equiv \mathcal{A}(W) \cap \mathcal{A}(W_a') = \mathcal{A}(W) \cap \mathcal{A}(W_a)' \\ D &= W \cap W_a' \end{aligned} \tag{12}$$

This is the operator algebra associated with a double cone $D$ (which is chosen symmetric around the origin by intersecting suitably translated wedges and their causal complements). The problem of computing intersected von Neumann algebras algebras is difficult since there are yet no general computational techniques.

The problem becomes more amenable if one considers instead of operators their formfactors i.e. their matrix elements between incoming ket and outgoing bra state vectors. In the spirit of the old LSZ formalism one can then make an Ansatz in form of a power series in $Z(\theta)$ and $Z^*(\theta) \equiv Z(\theta - i\pi)$ (corresponding to the power series in the incoming free field in LSZ theory)

$$A = \sum \frac{1}{n!} \int_C ... \int_C a_n(\theta_1, ...\theta_n) : Z(\theta_1)...Z(\theta_n) : \tag{13}$$

Each integration path $C$ extends over the upper and lower part of the rim of the $(0,-i\pi)$ strip in the complex $\theta$-plane. The strip-analyticity of the coefficient functions $a_n$ expresses the wedge-localization of $A$. It is easy to see that these coefficients are identical to the vacuum polarization form factors of $A$

$$\langle\Omega\left|A\right|p_n,..p_1\rangle^{in} = a_n(\theta_1,...\theta_n) \tag{14}$$

whereas the crossing of some of the particles into the left hand bra state leads to the connected part of the formfactors

$$^{out}\langle p_1,..p_l\left|A\right|p_n,..p_{l+1}\rangle^{in}_{conn} = a_n(\theta_1+i\pi,...\theta_l+i\pi,\theta_{l+1},..\theta_n) \tag{15}$$

In terms of the formfactors the relative commutant (12) results from restricting the series (13) by requiring that the $A's$ commute with the generators of the shifted algebra $\mathcal{A}(W_a)$

$$[A,U(a)A(f)U(a)^*]=0$$

Using the Z-F algebra relation this leads to a recursive equation for the coefficient function which relates the $a_n$ coefficients whose n differs by 2. This is nothing else than the famous "kinematical pole condition" (the only restriction in the absence of bound states) first introduced as one of the construction recipes by Smirnov [13]. This together with the Payley-Wiener Schwartz analytic properties and the crossing property (which links the crossed formfactor to the analytic continuation between the two rims of the $\theta$-strip $\mathbb{R}+i(0,\pi)$) characterizes the space of formfactors associated with the algebra $\mathcal{A}(D)$.

The multiplicative structure is outside of mathematical control of the formfactor formalism as long as one is unable to control the convergence of the infinite sums. The linear space of formfactors can be parametrized in terms of a covariant basis which corresponds to the formfactors of a basis of would be composite fields. It turns out that the dependence on the individual composite field in the Borchersclass of relatively local fields can be encodes into a polynomial factor [28] after splitting of a "kinematical factor" which only depends on the superselected charge of the class of fields. This tells us that if we knew that those operator algebras are nontrivial then the associated quantum field theory exists and we have a nonperturbative formalism to compute formfactors of pointlike fields or of more general operators in $\mathcal{A}(D)$. Since the formalism unlike the Lagrangian quantization formalism totally avoids correlation functions of pointlike fields it is nowhere threatened by ultraviolet problems.

There has been extensive work on the calculation of formfactors of composite fields. Similar to Wick polynomials there is a basis of (composite) fields in the same superselection sector. The formfactors of this family contain one factor which is common to all of them (the so-called minimal formfactor); as in the case of Wick monomials the individual fields differ by polynomial factors which obey rather complicated rules. The polynomial factors actually complicate the calculation of correlation functions as convergent series over formfactors. In fact apart from two-pointfunctions in very spacial cases this program has had no success despite serious attempts over many years.

The suggestion from the modular approach would be to look at formfactors of bounded operators in $\mathcal{A}(D)$ which would have better asymptotic behaviour for large rapidities than pointlike fields. Recently Buchholz and Lechner [19] proposed a very elegant idea which may have a good chance to show the nontriviality of $\mathcal{A}(D)$ for factorizing models without entering a case to case study. These authors proved that the nontriviality of this algebra is equivalent to a nuclearity property of a certain set of vectors defined in terms of the modular operators associated with wedge algebras. This opens the possibility that the existence problem of quantum field theory can be solved in terms of algebraic properties of wedge algebras (which have simpler properties than subwedge algebras).

# 5 Going beyond factorizable models, the hypothesis of a Masterfield

For factorizable models the crossing relation of the analytic coefficient functions in the series representation (13) is a consequence of the algebraic properties of the $Z's$. Since there are no Z-F operators for models with non factorizing S-matrices, one must look for a more general operational formulation of crossing. To obtain an idea in what direction to look for, let us first recall the precise conceptual position of factoring models within the general setting of massive models to which scattering theory applies [17].

As was mentioned before, wedge-localized PFG always exist but their pathological properties with respect to Fourier transformation (as a result of a rather singular behavior near the edge of the wedge) forbids their iterative application to the vacuum as one is used to in Wightman theory. It turns out that the requirement of temperedness which attributes such Wightman-like properties to the PFG's is only consistent with an elastic S-matrix [17]. As was mentioned before the only way one can avoid in a QFT of Bosons/Fermions that elastic processes via crossing generate inelastic ones is to go to d=1+1 and to have the elastic S-matrix factorize into 2-particle contributions $S^{(2)}$[17]. This is a very peculiar situation in which the two contributions in $S^{(2)} = 1 + T^{(2)}$ have the same $\delta$-function which means that the cluster argument does not allow to separate the identity from the T-matrix in a physical way. In this sense the requirement of having computationally useful generators of wedge algebras becomes synonymous with d=1+1 factorizability.

So the crucial question is how can one get a operational formulation of crossing in formfactors[18] without having a Zamolodchikov-Faddeev structure? One tempting idea is of course to interpret the crossing as a KMS property, but KMS of what operator theory if, as mentioned before, it cannot be the QFT

[17] Direct (connected) elastic 3-particle S-matrices would also lead to particle production via crossing [35].

[18] We always mean the connected part of the formfactors which is what one gets by starting with the outgong components of the one field (or operator from a local algebra) state and crossing from outgoing bras to incoming kets.

which we want to construct? One alternative idea would consist in declaring simply the formfactors to be correlation functions of (nonlocal) operators $R^{(A)}$ in rapidity momentum space (the auxiliary $R's$ will be referred to as "Rindler operators")

$$\langle \theta_1, ...\theta_n \left| A \right| \Omega \rangle \overset{?}{=} \left\langle R^{(A)}(\theta_1)...R^{(A)}(\theta_n) \right\rangle \tag{16}$$

but this would only work if we find a special operator $A$ whose formfactors define a system of positive correlation functions so that the GNS reconstruction would lead to an auxiliary operator field theory. Since positivity can hardly be checked directly, let us first look for a property which is much easier to check namely the cluster separation property. It is known that this property holds also in global operator algebras (i.e. without a local net substructure) as long as the operator algebra happens to be a von Neumann factor in which case it is related to the property of asymptotic abelieness [31]. For all factorizing models for which the search for such a field with clustering formfactors has been carried out, precisely one such field was identified [36]. We will make the following slightly stronger hypothesis about the existence of a quantum field which we call a "masterfield"

**Definition 1** *A masterfield M(x) associated to a QFT is a distinguished scalar neutral Boson field within the Borchers class of locally equivalent fields whose connected formfactors defines a thermal auxiliary "Rindler QFT" at the Hawking temperature $\beta = 2\pi$ in terms of a nonlocal field $R(\theta, p_\perp)$ in the sense of the above formula (16) with A being the masterfield M(x) at x=0.*

The KMS relation in $\theta$ reads

$$\begin{aligned}
&\left\langle R^{(M)}(\theta_1, p_{1\perp})...R^{(M)}(\theta_{n-1}, p_{n-1\perp})R^{(M)}(\theta_n, p_{n\perp}) \right\rangle_{\beta=2\pi} \\
&= \left\langle R^{(M)}(\theta_n - 2\pi i, p_{n\perp})R^{(M)}(\theta_1, p_{1\perp})...R^{(M)}(\theta_{n-1}, p_{n-1\perp}) \right\rangle_{\beta=2\pi} \\
&= (R^{(M)}(\theta_n - 2\pi i, p_{n\perp})^* \Omega_{\beta=2\pi}, R^{(M)}(\theta_1, p_{1\perp})...R^{(M)}(\theta_{n-1}, p_{n-1\perp})\Omega_{\beta=2\pi}) \\
&= (JR^{(M)}(\theta_n - \pi i, p_{n\perp})\Omega_{\beta=2\pi}, R^{(M)}(\theta_1, p_{1\perp})...R^{(M)}(\theta_{n-1}, p_{n-1\perp})\Omega_{\beta=2\pi})
\end{aligned} \tag{17}$$

where in the last two lines we used the more convenient state vector notation for the thermal expectation values and in the last line we used modular theory in order to convert $\Delta^{\frac{1}{2}}$ into $J$. The identification of this expression with the crossing property of the formfactor of $M(0)$

$$\begin{aligned}
&\langle 0 \left| M(0) \right| p_1, ...p_n \rangle = \langle -\bar{p}_n \left| M(0) \right| p_1, ...p_{n-1} \rangle \\
&= (JR^{(M)}(\theta_n - \pi i, p_{n\perp})\Omega_{\beta=2\pi}, R^{(M)}(\theta_1, p_{1\perp})...R^{(M)}(\theta_{n-1}, p_{n-1\perp})\Omega_{\beta=2\pi})
\end{aligned} \tag{18}$$

is consistent with the action of $J$ as $JR^{(M)}(\theta_n - \pi i, p_{n\perp})\Omega_{\beta=2\pi} = R^{(M)}(\theta_n - 2\pi i, -p_{n\perp})^* \Omega_{\beta=2\pi}$.

In d=1+1 the interpretation of crossing in terms of KMS of an auxiliary theory simplifies since there in this case there is no transverse momentum $p_\perp$.

It is important to notice that the auxiliary field theory associated with the formfactors of the master field is not subject to the restriction of PFGs which led to factorizable models. In fact being a global (without a local net structure) KMS theory, the concept of particles and in particular the concept of PFG becomes meaningless.

Let us first look at the trivial illustration of a master field namely

$$\begin{aligned} &M(x) \equiv :e^{\gamma A(x)}:, \ A(x) = free\, field \\ &(\Omega\, |M(0)|\, p(\theta_1, p_{1,\perp})...p(\theta_n, p_{n,\perp})) = e^{c\gamma} \end{aligned} \tag{19}$$

where the positive constant c is related to the vacuum-oneparticle normalization of $A$. Clearly the resulting auxiliary R-theory is abelian ($\theta$-independence and hence no fluctuations) with trivial clustering (as a result of the exponential structure). The free field case is the only one with an abelian auxiliary masterfield description.

The two following illustrations are two-dimensional. For their presentation we need some formalism of KMS states on free field algebras.

For bosonic quasifree KMS states at the KMS temperature $\beta$ one obtains

$$\begin{aligned} &\langle c(q)c(q')\rangle_\beta = e^{\beta q}\left\{\langle c(q)c(q')\rangle_\beta - [c(q), c(q')]\right\} \\ &\curvearrowright \langle c(q)c(q')\rangle_\beta = \frac{e^{\beta q}}{e^{\beta q}-1}\varepsilon(q)\delta(q+q') \end{aligned} \tag{20}$$

For the first illustration we take the Sinh-Gordon theory. The field which leads to formfactors which have the cluster factorizsation property in the rapidity variable is again an exponential $M(x) = e^{\varphi}$ operator in terms of the basic Sinh-Gordon field $\varphi$ [36]. They have the following structure

$$\begin{aligned} &(\Omega\, |M(0)|\, p(\theta_1)...p(\theta_n)) = K_n(\underline{\theta}) \prod_{i<j\leq n} F(\theta_{ij}) \\ &K_n(\underline{\theta}) = \sum_{l_1=0,1} ... \sum_{l_n=0,1} (-1)^{\Sigma l_i} \prod_{i<j} (1 + (l_i - l_j)\frac{i\sin \pi\nu}{\sinh \theta_{ij}}) \prod_k C e^{i\pi\frac{\gamma}{\beta}(-1)^{l_k}} \end{aligned} \tag{21}$$

where the coupling strength $\beta$ and $\nu$ are related by $\frac{1}{\nu} = \frac{8\pi}{\beta^2}$. The product factor involving the 2-particle formfactor $F$ has the combinatorics of an exponential which is bilinear in $c(q)$ free Boson operators. This suggests to start from the complex exponential

$$\begin{aligned} C(\theta) &= e^{ia(\theta)} \\ a(\theta) &= \int dq w(q) c(q) e^{iq(\theta - i\frac{\pi}{2})} \end{aligned} \tag{22}$$

and look for a Rindler operator $R$[19] as a Hermitian combination of the form

$$R(\theta) = N\left\{ e^{i\gamma} C(\theta - i\frac{\pi}{2}) + h.a. \right\} \tag{23}$$

$$C(\theta)C(\theta') = S(\theta-\theta')C(\theta')C(\theta),\ S(\theta) = exp\int_0^\infty dqf(q)sinhq\frac{\theta}{i\pi}$$

$$\left\langle C(\theta - i\frac{\pi}{2})C(\theta' - i\frac{\pi}{2})\right\rangle_{2\pi} = exp\int_0^\infty dqf(q)\frac{1-chq(\pi + i(\theta-\theta'))}{2sh\pi q} \equiv F_{min}(\theta-\theta')$$

The function $F_{min}(\theta)$ is the so-called minimal 2-particle formfactor of the model i.e. the unique function which obeys $F(\theta) = S(\theta)F(-\theta)$ and is holomorphic in the closed strip.

The Sinh-Gordon S-matrix

$$S_{sh}(\theta) = \frac{th\frac{1}{2}(\theta - i\kappa)}{th\frac{1}{2}(\theta + i\kappa)},\ \kappa = \frac{\pi\beta^2}{8\pi+\beta^2} = \pi B \leq \pi \tag{24}$$

fixes the quasifree commutation relation of the Rindler operators $R(\theta)$ with

$$f(q) = \frac{2sh\frac{\kappa q}{2}sh\frac{\kappa q 8\pi}{2\beta^2}}{qch\frac{\pi q}{2}} = \frac{2sh\frac{q\pi}{2}Bsh\frac{q\pi}{2}(2-B)}{qch\frac{\pi q}{2}} \tag{25}$$

In the last line in (23) we used the fact that the KMS state at the inverse temperature $2\pi$ fixes the quasi-free state on the Rindler creation/annihilation operator algebra which in turn determines the thermal expectations of the $C$-operators. The n-point function

$$\left\langle C(\theta_1 - i\frac{\pi}{2})....C(\theta_n - i\frac{\pi}{2})\right\rangle_{2\pi} \sim \prod_{i<k} F_{min}(\theta_{ik}) \tag{26}$$

fulfills the commutation relation of the $R$-algebra (which is identical to that of the $C$-algebra as well as the KMS condition. Our interest lies in the Hermitian field operator $R$ For convenience we have adjusted our notation to the resulting combinatorics for the thermal $Z$-expectation which are sums of terms with different $C_l(\theta) := C(\theta - il\frac{\pi}{2}),\ l = \pm$

$$\left\langle C_{l_n}...C_{l_1}\right\rangle_{2\pi} = \prod_{i<k} F_{min}(\theta_{ik})\left\{1 - (l_k - l_i)\frac{isin\kappa}{sh\theta_{ki}}\right\},\ l_i = \pm 1 \tag{27}$$

$$\left\langle R(\theta_n)...R(\theta_1)\right\rangle_{\beta=2\pi} \sim \sum \left\langle C_{l_n}...C_{l_1}\right\rangle_{2\pi} expi\pi\alpha(l_1 + l_2 + ...l_n)$$

where $\alpha$ depends on the numerical prefactors.

With this construction of an auxiliary global Rindler QFT for the formfactors of the masterfield we have reproduced a curious observation by Lukyanov [24]

[19] We use the letter $Z$ for the particle physics representation of the Zamolodchikov algebra (the Minkowski soaetime operators which are related to the PFG wedge generators) whereas $R$ is used for the thermal Rindler representation.

which is known in the literature on factorizing models under the name of "free field representations". Besides our use of this model as an illustration of a new idea of a general operational approach to crossing and the fact that some of our physical concepts and mathematical formalism is somewhat different from that of Lukyanov, the core of the actual calculation is identical. The difference is that for us it is important to emphasize that the states to be used are not Gibbs states but rather KMS states as one expects from their Rindler aspect. The underlying thermality is not that of a heat bath but is rather related to vacuum polarization phenomenon caused by localization and in such a situation there is no natural analog of the thermal quantization box which in the thermodynamic limit leads to KMS states.

Since fields whose formfactors cluster have been constructed in many similar factorizing models[20], one would expect that the idea of an auxiliary Rindler theory works in all of them. Moreover it would be tempting to conjecture that the simplifying feature of factorizing models consists that the auxiliary formfactor theory is bilinear exponential in $c(q)$ operators. This conjecture draws support from a recent observation by Babujian and Karowski who observed that a suitably adjusted form of clustering also holds in statistics changing $Z_n$-models [37] of which the lowest one is the Ising field theory. In that case a combination of disorder/order field formfactors leads to clustering [21].

In the following we briefly show that a "twisted" KMS state does the job. The twisting consists in changing the KMS formula by a -sign.

$$\begin{aligned} \langle c(q)c(q')\rangle_\beta &= -e^{\beta q}\left\{\langle c(q)c(q')\rangle_\beta - [c(q),c(q')]\right\} \\ \langle c(q)c(q')\rangle_\beta &= \frac{e^{\beta q}}{1+e^{\beta q}}\varepsilon(q)\delta(q+q') \\ \langle c(q)c(q')\rangle_\pi &= \frac{e^{\frac{\pi}{2}q}}{2cosh\frac{\pi}{2}q}\varepsilon(q)\delta(q+q') \end{aligned} \tag{28}$$

Where the third line contains the KMS two-point function at $\beta=\pi$ which we are going to use together with the following definition of $a(\theta)$

$$\begin{aligned} a(\theta) &= \int c(q)dq \\ \langle a(\theta)a(\theta')\rangle_{\beta=\pi} &= \int_0^\infty \frac{\sinh q(i\theta - i\theta' + \frac{\pi}{2})}{\cosh\frac{\pi}{2}q}\frac{dq}{q} = \ln i\tanh\frac{\theta-\theta'}{2} \end{aligned} \tag{29}$$

which finally leads to the well-known disorder/order Ising formfactors which is given by a combinatorial expression in the two-particle formfactor of the disorder operator (which correspond to an even number of particles)

$$\left\langle e^{a(\theta)}e^{a(\theta')}\right\rangle_\pi = \tanh\frac{\theta-\theta'}{2} \tag{30}$$

[20] In [37] it was shown that distinguished fields with clustering formfactors exist for all $A_{n-1}$ affine Toda field theories of which the Sinh-Gordon is the lowest member.

As the Sinh-Gordon model is the simplest representative of the class of $A_{n-1}$ affine Toda models [37], the Ising field theory is the beginning of the family of $Z_n$ models. These models are more difficult as a consequence of their $Z_n$ braid group statistics and a candidate for a masterfield is not immediately visible. The suggestion from the Ising case would be that a suitable combination of all disorder/order operators would be a candidate for a field which fulfills some generalized clustering (i.e. adjusted to the exotic statistics). It is encouraging that Babujian and Karowski (private communication) made some observations which support this. This lets one hope that there exists suitably twisted KMS theory on an exponential expression which is non-local bilinear in creation/annihilation operators.

# 6 Concluding remarks

The present masterfield hypothesis leaves one with the problem of what to expect in non-factorizing models and in particular in higher spacetime dimensions. In that case one cannot expect a simple description in terms of free creation/annihilation operators but one could hope for a perturbative finite approach to formfactors. Such an approach must contain the standard results of renormalized Lagrangian perturbation theory restricted to formfactors.

The fact that we had to distinguish a masterfield in order to be able to formulate an operational hypothesis about crossing does not mean that there is no operator interpretation of formfactors of other fields of the theory; it only expresses our inability to think of a more general KMS setting.

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