LANDAU-94-TMP-4
hep-th/9406118
June 1994

SECTORS OF MUTUALLY LOCAL FIELDS IN INTEGRABLE MODELS OF QUANTUM FIELD THEORY*

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It is known that 2D field theories admit several sectors of mutually local fields so as two fields from different sectors are generally mutually nonlocal. We show that any one-particle integrable model with $\mathbf{Z_2}$ symmetry contains three sectors: bosonic, fermionic and 'disorder' one. We generalize the form factor axioms to fermionic and 'disorder' sectors. For the particular case of the sinh-Gordon model we obtain several form factors in these sectors.

* The work was supported in part by Landau Institute Foundation, International Science Foundation, and Russian Foundation for Fundamental Investigations.

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1. Introduction

Any quantum field theory consists of two structures: quantum mechanics (state space and Hamiltonian) and a set of mutually local fields. Any two-dimensional field theory with given quantum-mechanical structure admits different local field realizations.[1] The most wellknown examples are the following. The Ising model can be performed either as a free fermion or as an interacting boson.[2,3] The symplectic model can be repesented either as a free neutral boson or as an interacting fermion.[3] The sin-Gordon model and the Thirring model coincide quantum-mechanically,[4] but their local field contents are different: the bosonic kink fields create the same asymptotic states as the Thirring fermions.[5]

Recall some properties of the Ising and symplectic models.[3] The Ising model in the scaling limit above the critical point can be considered as a free Majorana fermion $\psi(x)$ with the action

$$\mathcal{A} = \frac{1}{2}\int d^2x \, \left(i\overline{\psi}\gamma^\mu\partial_\mu\psi - M\overline{\psi}\psi \right).$$

On the other hand there are two bosonic fields, $\varphi(x)$ (order parameter) and $\mu(x)$ (disorder parameter), connected with the fermion $\psi(x)$ nonlinearly and nonlocally. Their simulaneous commutation relations are

$$\begin{aligned} \varphi(x)\psi(y) &= \epsilon(x^1 - y^1)\psi(y)\varphi(x), \\ \mu(x)\psi(y) &= -\epsilon(x^1 - y^1)\psi(y)\mu(x), \\ \mu(x)\varphi(y) &= -\epsilon(x^1 - y^1)\varphi(y)\mu(x) \quad \text{for} \quad x^1 \neq y^1, \end{aligned} \tag{1.1}$$

where (x^0, x^1) are Minkowski coordinates and $\epsilon(\xi) \equiv \text{sign}\,\xi$. This means that these fields are mutually nonlocal. It is important to note that the fermion alone creates the whole state space, and the order parameter acts in it, but does not create any new space. The order parameter $\varphi(x)$ creates the same asymptotic states as fermions $\psi(x)$, but the S-matrix of the bosonic field is nontrivial. The S-matrix of N bosons is equal to $(-1)^{\frac{1}{2}N(N-1)}$. The disorder parameter $\mu(x)$ does not create any asymptotic states.

So there are at least three sectors in the Ising model: 'bosonic' sector containing fields commuting with the order parameter, 'fermionic' sector containing fields commuting or anticommuting with the fermion, and 'disorder' sector of fields commuting with the disorder parameter.

Similar situation is observed in the symplectic model. Bosonic sector contains a free neutral boson with the action

$$\mathcal{A} = \frac{1}{2}\int d^2x \, \left(\partial_\mu\varphi\,\partial^\mu\varphi - M^2\varphi^2 \right).$$

Fermionic and 'disorder' sectors contain interacting fermion $\psi(x)$ and bosonic 'disorder' parameter $\mu(x)$ respectively. The S-matrix of N fermions is equal to

$(-1)^{\frac{1}{2}N(N-1)}$. The fields $\psi(x)$, $\varphi(x)$ and $\mu(x)$ possess the same simultaneous commutation relations (1.1).

On the basis of these two examples and Ref. 1 we can formulate the

Conjecture. *Any integrable 2D field theory with $\mathbf{Z_2}$ symmetry of changing the sign at the 'elementary' field $\varphi(x)$ ($\varphi(x) \longrightarrow -\varphi(x)$) contains at least three sectors of local fields. Two 'dual' sectors contain n-component fields $\varphi(x)$ and $\psi(x)$ respectively, and a 'disorder' sector contains an n-component bosonic field $\mu(x)$. The following conditions hold:*

1) the fields φ_i, ψ_j and μ_k are pairwise mutually nonlocal;

2) if $\varphi_i(x)$ is a boson, then $\psi_i(x)$ is a fermion and the commutation relations (1.2) hold for φ_i, ψ_i and μ_i; if $\varphi_i(x)$ is a fermion, $\psi_i(x)$ is a boson;

3) the fields $\varphi(x)$ and $\psi(x)$ create the same assymptotic states, their S-matrix is factorizable and corresponding two-particle S-matrices $S_\varphi(\theta)$ and $S_\psi(\theta)$ (θ is the difference of rapidities) are connected:

$$S_\psi(\theta) = -S_\varphi(\theta); \tag{1.2}$$

3) the 'disorder' field does not produce asymptotic states.

In this paper we substantiate this conjecture in the simplest case of the one-component interacting fields $\varphi(x)$ and $\psi(x)$, and illustrate it by the sinh-Gordon model.

In Sec. 2 we recall some fundamental facts about form factors. We use the formulation through the Zamolodchikov algebra,[6] because it clarifies some aspects of the form factor axioms. In Sec. 3 we formulate Karowski–Weisz–Smirnov axioms for form factors[7,8] in slightly generalized form and obtain the sectors of mutually local fields. Sec. 4 reviews trivial examples: the Ising and the symplectic models. A more complicated example — sinh-Gordon model — is analysed in Sec. 5. Conclusions and unsolved problems are listed in Sec. 6.

2. Zamolodchikov algebra and form factors

Let $S(\theta)$ be an analitical single-valued function without poles in the strip $0 < \mathrm{Im}\,\theta \le \pi$ and satisfying the equations

$$S(-\theta) = S(\theta + i\pi) = S^{-1}(\theta). \tag{2.1}$$

It is necessary to stress that we do not identify the function $S(\theta)$ with any two-particle S-matrix. The connection between $S(\theta)$ and S-matrix will be established in the following section.

The Zamolodchikov algebra[6] is generated by symbols $V(\theta)$, $V^+(\theta)$ ($\theta \in \mathbf{R}$) satisfying the equations

$$\begin{aligned} V(\theta_1)V(\theta_2) &= S(\theta_1-\theta_2)V(\theta_2)V(\theta_1), \\ V^+(\theta_1)V^+(\theta_2) &= S(\theta_1-\theta_2)V^+(\theta_2)V^+(\theta_1), \\ V(\theta_1)V^+(\theta_2) &= S^{-1}(\theta_1-\theta_2)V^+(\theta_2)V(\theta_1) + 2\pi\delta(\theta_1-\theta_2). \end{aligned} \tag{2.2}$$

We can introduce normal ordering, $:\cdots:$, by the rules

$$
\begin{aligned}
&: V^+(\theta_1)\cdots V^+(\theta_m)V(\vartheta_1)\cdots V(\vartheta_n) := V^+(\theta_1)\cdots V^+(\theta_m)V(\vartheta_1)\cdots V(\vartheta_n),\\
&:\cdots V(\theta_1)V^+(\theta_2)\cdots := S^{-1}(\theta_1-\theta_2) :\cdots V^+(\theta_2)V(\theta_1)\cdots : .
\end{aligned}
\tag{2.3}
$$

Now we shall describe a quantum-mechanical system. The vacuum $|0\rangle$ is defined by the equation

$$
V(\theta)|0\rangle = 0, \quad \theta\in\mathbf{R}. \tag{2.4}
$$

The state space is spanned on the basis

$$
V^+(\theta_1)\cdots V^+(\theta_n)|0\rangle, \quad n=0,1,2,\cdots, \quad \theta_i\in\mathbf{R}. \tag{2.5}
$$

Evidently

$$
\begin{aligned}
&\langle V(\theta_1)V(\theta_2)\rangle = \langle V^+(\theta_1)V^+(\theta_2)\rangle = \langle V^+(\theta_1)V(\theta_2)\rangle = 0,\\
&\langle V(\theta_1)V^+(\theta_2)\rangle = 2\pi\delta(\theta_1-\theta_2)
\end{aligned}
\tag{2.6}
$$

($\langle\cdots\rangle \equiv \langle 0|\cdots|0\rangle$), and the Wick theorem holds.

The Hamiltonian is given by

$$
\mathcal{H} = \int_{-\infty}^{\infty}\frac{d\theta}{2\pi}\,P_0(\theta)V^+(\theta)V(\theta), \tag{2.7}
$$

where $P_0(\theta)$ is the time component of the covector

$$
P(\theta) = (M\cosh\theta, -M\sinh\theta), \tag{2.8}
$$

M is a parameter. Also we shall use the designation

$$
P(\theta_1,\cdots,\theta_n) = M\sum_{i=1}^{n}(\cosh\theta_i, -\sinh\theta_i). \tag{2.9}
$$

The system described above is linear and, therefore, its solution is evident. The only problem is to construct local fields in this theory. If $S(\theta)\equiv 1$ (boson) or $S(\theta)\equiv -1$ (fermion), it is easy to introduce local fields

$$
\varphi(x) = \int_C \frac{d\theta}{2\pi}e^{-iP(\theta)x}V(\theta) \quad \text{for} \quad S(\theta)=1 \tag{2.10}
$$

and

$$
\psi_\pm(x) = \sqrt{M}\int_C \frac{d\theta}{2\pi}e^{\mp\frac{1}{2}(\frac{i\pi}{2}+\theta)}e^{-iP(\theta)x}V(\theta) \quad \text{for} \quad S(\theta)=-1. \tag{2.11}
$$

The contour C is a sum of two straight lines: $C=(-\infty-i0,+\infty-i0)+(-\infty-i\pi+i0,+\infty-i\pi+i0)$; the lines are directed to the right. The sense of the infinitesimal shifts will be clarified later. Here we used the following designation

$$V(\theta-i\pi)=V^{+}(\theta),\quad \theta\in\mathbf{R}. \tag{2.12}$$

The fields $\varphi(x)$ and $\psi_{\pm}(x)$ are usual bosonic and fermionic local fields of mass M. For general $S(\theta)$ we consider a field

$$\phi(x)=\sum_{n=0}^{\infty}\frac{1}{n!}\int_C\frac{d\theta_1}{2\pi}\cdots\int_C\frac{d\theta_n}{2\pi}e^{-iP(\theta_1,\cdots,\theta_n)x}F_\phi^{(n)}(\theta_1,\cdots,\theta_n):V(\theta_n)\cdots V(\theta_1):. \tag{2.13}$$

Here $F_\phi^{(n)}(\theta_1,\cdots,\theta_n)$ are some analitical single-valued functions satisfying the equation

$$F_\phi^{(n)}(\cdots,\theta_i,\theta_{i+1},\cdots)=S(\theta_i-\theta_{i+1})F_\phi^{(n)}(\cdots,\theta_{i+1},\theta_i,\cdots). \tag{2.14}$$

Eq. (2.13) gives nearly general form of a Heisenberg operator

$$i\frac{\partial\phi(x)}{\partial x^0}=[\phi(x),\mathcal{H}\,]. \tag{2.15}$$

The operator $\phi(x)$ is Hermitain if and only if

$$F_\phi^{(n)}(\theta_1,\cdots,\theta_n)=F_\phi^{(n)*}(\theta_n+i\pi,\cdots,\theta_1+i\pi)\text{ for }\theta_1,\cdots,\theta_n\in\mathbf{R}\cup(\mathbf{R}-i\pi). \tag{2.16}$$

Functions $F_\phi^{(n)}(\theta_1,\cdots,\theta_n)$ are referred to as form factors of the operator $\phi(x)$. It is easy to represent correlation functions in form of infinite series using the Wick theorem. For example,

$$\begin{aligned}\langle\phi_1(x)\phi_2(y)\rangle=&\sum_{n=0}^{\infty}\frac{1}{n!}\int_{-\infty}^{\infty}\frac{d\theta_1}{2\pi}\cdots\int_{-\infty}^{\infty}\frac{d\theta_n}{2\pi}e^{-iP(\theta_1,\cdots,\theta_n)(x-y)}\\&\times F_{\phi_1}^{(n)}(\theta_1,\cdots,\theta_n)F_{\phi_2}^{(n)}(\theta_n-i\pi,\cdots,\theta_1-i\pi).\end{aligned}$$

There are two important problems to be solved. Firstly, what is the condition of mutual locality of two fields? Secondly, what asymtotical states do the fields (2.13) create? The Karowski–Weicz–Smirnov axioms answer these questions.

3. Form factor axioms and sectors of local fields

In this section we consider Karowski–Weisz–Smirnov axioms[7,8] as conditions of locality. We follow Smirnov,[8] but slightly weaken the axioms. It will allow us to consider fermionic and 'disorder' fields.

Namely, let us impose the following conditions on form factors:

0. Definite Lorentz spin s_ϕ

$$F_\phi^{(n)}(\theta_1+\vartheta,\cdots,\theta_n+\vartheta)=e^{-s_\phi\vartheta}F_\phi^{(n)}(\theta_1,\cdots,\theta_n). \tag{3.1}$$

1. Consistency with the Zamolodchikov algebra (Eq. (2.14)):

$$F_\phi^{(n)}(\cdots,\theta_i,\theta_{i+1},\cdots)=S(\theta_i-\theta_{i+1})F_\phi^{(n)}(\cdots,\theta_{i+1},\theta_i,\cdots). \tag{3.2}$$

2. Cyclic eqation

$$F_\phi^{(n)}(\theta_1+2\pi i,\theta_2,\cdots,\theta_n)=\lambda_\phi F_\phi^{(n)}(\theta_2,\cdots,\theta_n,\theta_1),\quad \lambda_\phi=\pm 1. \tag{3.3}$$

3. The only poles in the strip $0\le \mathrm{Im}\,(\theta_i-\theta_j)\le\pi$, $i<j$ are $\theta_i=\theta_j+i\pi$ and

$$\begin{gathered}F_\phi^{(n+2)}(\vartheta+i\pi+\varepsilon,\vartheta,\theta_1,\cdots,\theta_n)\simeq\frac{i}{\varepsilon}\left[1-\zeta_\phi\prod_{i=1}^{n}S(\vartheta-\theta_i)\right]F_\phi^{(n)}(\theta_1,\cdots,\theta_n),\\ \varepsilon\longrightarrow 0,\quad \zeta_\phi=\pm 1.\end{gathered} \tag{3.4}$$

Later we shall see that

$$\lambda_\phi=\zeta_\phi, \tag{3.3$'$}$$

but we shall ignore it for a while.

Notice that (in absence of bound states) the axioms do not connect the form factors $F_\phi^{(2n)}$ and $F_\phi^{(2n-1)}$. Therefore, it is sufficient to consider such fields ϕ that contain nonzero form factors of either even or odd degree only. It is natural to call these fields even or odd respectively.

First of all let us calculate the physical S-matrices. It can be shown that even fields do not create asymptotic states. Consider $t\longrightarrow\pm\infty$ asymptotics of odd fields:[3,8]

$$\begin{aligned}\phi_{out/in}(\theta)&=\lim_{x^0\to\pm\infty}\frac{i\epsilon(e^\theta)}{2}\int_{-\infty}^{\infty}dx^1\left(e^{iP(\theta)x}\partial_0\phi(x)-\phi(x)\,\partial_0 e^{iP(\theta)x}\right)\\ &=\lim_{t\to\pm\infty}\frac{\epsilon(e^\theta)}{2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)!}\int_C\frac{d\theta_1}{2\pi}\cdots\int_C\frac{d\theta_{2n+1}}{2\pi}\\ &\times F_\phi^{(2n+1)}(\theta_1,\cdots,\theta_{2n+1}):V(\theta_{2n+1})\cdots V(\theta_1):\\ &\times\left[\sum_i\left(e^{\theta_i}+e^{-\theta_i}\right)+e^\theta+e^{-\theta}\right]\cdot 2\pi\delta\left[e^\theta-e^{-\theta}-\sum_i\left(e^{\theta_i}-e^{-\theta_i}\right)\right]\\ &\times\exp\left\{iM\frac{t}{2}\left[e^\theta+e^{-\theta}-\sum_i\left(e^{\theta_i}+e^{-\theta_i}\right)\right]\right\}.\end{aligned} \tag{3.5}$$

Only the poles contribute to asymptotic fields according to the formula

$$\lim_{t\to\pm\infty} \frac{-i}{x-i0} e^{-ixat} = \Theta(\mp a)\cdot 2\pi\delta(x), \tag{3.6}$$

with $\Theta(x)$ being the Heaviside theta-function. Consider the pole $\theta_{2i-1} \longrightarrow \theta_{2i}+i\pi$, $i=1,2,\cdots,n$. Eq. (3.4) leads to

$$F_\phi^{(2n+1)}(\theta_1,\cdots,\theta_{2n+1}) \simeq i^n F_\phi^{(1)}(\theta_{2n+1}) \prod_{i=1}^{n} \frac{1-\zeta_\phi S(\theta_{2i-1}-\theta_{2n+1}}{\theta_{2i-1}-\theta_{2i}-i\pi}. \tag{3.7}$$

Because of the δ-function in Eq. (3.5)

$$e^\theta - e^{\theta_{2n+1}} = \sum_{i=1}^{n} \left(e^{\theta_{2i-1}} + e^{\theta_{2i}}\right) \frac{1-e^{-\theta_{2i-1}-\theta_{2i}}}{1-e^{-\theta-\theta_{2i+1}}}. \tag{3.8}$$

Substituting Eq. (3.8) into the exponential in Eq. (3.5), and Eq. (3.7) into Eq. (3.5), using Eq. (3.6) [infenitesimal shifts in the contour C produce $-i0$ for Eq. (3.6)], and taking into account a combinatorial factor $(2n+1)!/n!$ from permutations of θ_i in Eq. (3.7), we get asymptotic fields

$$\phi_{out/in}(\theta) = F_\phi^{(1)}(\theta) : V(\theta) e^{\int_{-\infty}^{\infty} \frac{d\theta'}{2\pi} \Theta(\pm\ \mathrm{Re}\ (\theta-\theta'))\left[\zeta_\phi S(\theta-\theta')-1\right] V^+(\theta')V(\theta')} :. \tag{3.9}$$

Consider asymptotic states:

$$|\theta\rangle_{out/in} = \phi^+_{out/in}(\theta)|0\rangle = F_\phi^{(1)}(\theta)V^+(\theta)|0\rangle,$$

because only the 0th term in the decomposition of the exponential acts on the vacuum nontrivially. Similarly, for $\theta_1 > \theta_2$

$$\begin{aligned} |\theta_1,\theta_2\rangle_{out} &= \zeta_\phi S(\theta_1-\theta_2) F_\phi^{(1)}(\theta_1) F_\phi^{(1)}(\theta_2) V^+(\theta_1)V^+(\theta_2)|0\rangle, \\ |\theta_1,\theta_2\rangle_{in} &= F_\phi^{(1)}(\theta_1) F_\phi^{(1)}(\theta_2) V^+(\theta_1)V^+(\theta_2)|0\rangle, \end{aligned}$$

and we see that the physical S-matrix is given by

$$S_\phi(\theta) = \zeta_\phi S(\theta). \tag{3.10}$$

It is easy to cheque that Eq. (3.10) holds also for $\theta_1 < \theta_2$, and that for n particles the physical S-matrix is

$$S^{(n)}_{\phi,\mathrm{phys}}(\theta_1,\cdots,\theta_n) = \prod_{i<j} S_\phi(\theta_i-\theta_j). \tag{3.11}$$

It may seem strange that the S-matrix depends on the field $\phi(x)$ while the asymptotic states are independent of it up to normalization factors. However, the S-matrix depends on the choice of the 'unperturbed' Hamiltonian (or, in other words, of the interaction representation). Fixing a field ϕ we fix the 'unperturbed' Hamiltonian with respect to which the field ϕ is free.

Now we shall cheque locality.[8] Let $x^0 = y^0$, $x^1 > y^1$. Then in an integral $\int_{-\infty}^{\infty} \frac{d\xi}{2\pi} e^{-iP(\xi)(x-y)}(\cdots)$ one can shift the contour upward till $i\pi$ preserving convergence of the integral. Consider the product

$$\begin{aligned}
\phi_1(x)\phi_2(y) = & \sum_{m,n,k=0}^{\infty} \frac{1}{m!n!k!} \int_{-\infty}^{\infty} \frac{d^k\xi}{(2\pi)^k} \int_C \frac{d^m\theta}{(2\pi)^m} \int_C \frac{d^n\vartheta}{(2\pi)^n} \\
& \times \exp\left[-iP(\xi)(x-y) - iP(\theta)x - iP(\vartheta)y\right] \\
& \times F_{\phi_1}^{(m+k)}(\xi_1 - i0, \cdots, \xi_k - i0, \theta_1, \cdots \theta_m) \\
& \times F_{\phi_2}^{(n+k)}(\vartheta_n, \cdots, \vartheta_1, \xi_k - i\pi + i0, \cdots, \xi_1 - i\pi + i0) \\
& \times :V(\theta_m)\cdots V(\theta_1)V(\vartheta_1)\cdots V(\vartheta_n): .
\end{aligned}$$

We used the usual rule for calculating the normal form of two normally ordering multipliers. Now we shift the contours over ξ_i up: $\int_{-\infty}^{\infty} \frac{d^k\xi}{(2\pi)^k} \longrightarrow \int_{-\infty+i\pi}^{\infty+i\pi} \frac{d^k\xi}{(2\pi)^k}$. The contour catch on the poles (3.4). Calculating residues of the poles and regrouping the terms we get

$$\begin{aligned}
\phi_1(x)\phi_2(y) = & \sum_{m,n,k=0}^{\infty} \frac{1}{m!n!k!} \int_{-\infty}^{\infty} \frac{d^k\xi}{(2\pi)^k} \int_C \frac{d^m\theta}{(2\pi)^m} \int_C \frac{d^n\vartheta}{(2\pi)^n} \\
& \times \exp\left[iP(\xi)(x-y) - iP(\theta)x - iP(\vartheta)y\right] \\
& \times F_{\phi_1}^{(m+k)}(\xi_1 + i\pi + i0, \cdots, \xi_k + i\pi + i0, \theta_1, \cdots \theta_m) \\
& \times F_{\phi_2}^{(n+k)}(\vartheta_n, \cdots, \vartheta_1, \xi_k - i0, \cdots, \xi_1 - i0) \\
& \times \sum_{l=1}^{n}(-)^{l-1} \sum_{j_1<\cdots<j_l}^{n} \prod_{r=1}^{l} \left[1 - \zeta_\phi \prod_{i=1}^{k} S(\vartheta_{j_r} - \xi_i - i\pi) \prod_{i=1}^{m} S(\vartheta_{j_r} - \theta_i)\right] \\
& \times :V(\theta_m)\cdots V(\theta_1)V(\vartheta_1)\cdots V(\vartheta_n): .
\end{aligned}$$

The double sum here can be calculated:

$$\sum_{l=1}^{n}(-)^{l-1} \sum_{j_1<\cdots<j_l}^{n} \prod_{l=1}^{l}(1-\alpha_{j_r}) = \prod_{j=1}^{n} \alpha_j .$$

Doing cyclic transposition in $F_{\phi_2}^{(n+k)}$ according to Eq. (3.3), pulling ξ's through θ's according to Eq. (3.2) in $F_{\phi_1}^{(m+k)}$, and reordering Zamolodchikov operators, we obtain

$$\begin{aligned}\phi_1(x)\phi_2(y) = \sum_{m,n,k=0}^{\infty} \frac{\lambda_{\phi_1}^k \zeta_{\phi_1}^n}{m!n!k!} \int_{-\infty}^{\infty} \frac{d^k\xi}{(2\pi)^k} \int_C \frac{d^m\theta}{(2\pi)^m} \int_C \frac{d^n\vartheta}{(2\pi)^n} \\ \times \exp\left[-iP(\xi)(y-x) - iP(\theta)x - iP(\vartheta)y\right] \\ \times F_{\phi_2}^{(n+k)}(\xi_1 - i0, \cdots, \xi_k - i0, \vartheta_1, \cdots, \vartheta_n) \\ \times F_{\phi_1}^{(m+k)}(\theta_m, \cdots, \theta_1, \xi_k - i\pi + i0, \cdots, \xi_1 - i\pi + i0) \\ \times :V(\vartheta_n)\cdots V(\vartheta_1)V(\theta_1)\cdots V(\theta_m): .\end{aligned} \tag{3.12}$$

We see that

$$\phi_1(x)\phi_2(y) = \phi_2(y)\phi_1(x), \quad x^0 = y^0,\ x^1 > y^1, \quad \text{if } \lambda_{\phi_1}^k \zeta_{\phi_1}^n = 1, \tag{3.13a}$$
$$\phi_1(x)\phi_2(y) = -\phi_2(y)\phi_1(x), \quad x^0 = y^0,\ x^1 > y^1, \quad \text{if } \lambda_{\phi_1}^k \zeta_{\phi_1}^n = -1. \tag{3.13b}$$

For definite commutation relations one of the conditions must hold for all nonzero terms in Eq. (3.12). Because k is an arbitrary number and $(-1)^{k+n}$ is fixed by the evenness of the field $\phi_2(x)$, we get

$$\lambda_\phi = \zeta_\phi. \tag{3.3$'$}$$

It is easy to cheque from consistency of the axioms [and Eq. (3.3′)] that $s_\phi \in \mathbf{Z}$ for an even field $\phi(x)$ and

$$\zeta_\phi = (-1)^{2s_\phi} \quad \text{(odd)} \tag{3.14}$$

for odd one.

If $\phi_2(x)$ is even, the condition (3.13a) holds. If $\phi_2(x)$ is odd, the condition (3.13a) holds for $\zeta_{\phi_1} = 1$, and the condition (3.13b) holds for $\zeta_{\phi_1} = -1$. We reproduce the spin-statistics correspondence.[a]

Let $\tau(x)$ be an even field with $\zeta_\tau = 1$, $\varphi(x)$ an odd field with $\zeta_\varphi = 1$, $\mu(x)$ an even field with $\zeta_\mu = -1$, and $\psi(x)$ an odd field with $\zeta_\psi = -1$. We get the

[a] To avoid confusion, note that 'statistics' only means here the commutation relations, but not the statistics of the gas which is governed by $S(0) = \pm 1$.

simulaneous commutation relations

$$\begin{aligned}
\tau(x)\tau(y) &= \tau(y)\tau(x), \\
\tau(x)\varphi(y) &= \varphi(y)\tau(x), \\
\tau(x)\mu(y) &= \mu(y)\tau(x), \\
\tau(x)\psi(y) &= \psi(y)\tau(x), \\
\varphi(x)\varphi(y) &= \varphi(y)\varphi(x), \\
\varphi(x)\mu(y) &= \epsilon(x^1 - y^1)\mu(y)\varphi(x), \\
\varphi(x)\psi(y) &= \epsilon(x^1 - y^1)\psi(y)\varphi(x), \\
\mu(x)\mu(y) &= \mu(y)\mu(x), \\
\psi(x)\mu(y) &= \epsilon(x^1 - y^1)\mu(y)\psi(x), \\
\psi(x)\psi(y) &= -\psi(y)\psi(x).
\end{aligned} \tag{3.15}$$

Let $[\tau]$, $[\varphi]$, $[\mu]$, $[\psi]$ be spaces of fields with the same evenness and ζ_ϕ as τ, φ, μ, ψ respectively. From Eq. (3.15) we obtain three sectors of mutually local fields:

'bosonic' sector $\{[\tau],[\phi]\}$;

'fermionic' sector $\{[\tau],[\psi]\}$;

'disorder' sector $\{[\tau],[\mu]\}$.

Now we shall consider examples.

4. Simple examples: Ising and symplectic models

Here we recall some formulas concerning the Ising and symplectic models[3] and cheque that their form factors satisfy Eqs. (3.1)–(3.4).

We begin with the Ising model. The Zamolodchikov algebra of this model is the Klifford one $[S(\theta) = -1]$. Local fermion, disorder and spin operators are given by

$$\begin{aligned}
\psi_\pm(x) &= \sqrt{M}\int_C \frac{d\theta}{2\pi} e^{\mp\frac{1}{2}(\theta + \frac{i\pi}{2})} e^{-iP(\theta)x} V(\theta), \\
\mu(x) &=: \exp\frac{1}{2}\rho_F(x) :, \\
\varphi(x) &=: \psi_0(x) \exp\frac{1}{2}\rho_F(x) :,
\end{aligned}$$

where

$$\begin{aligned}
\rho_F(x) &= -i\int_C \frac{d\theta_1}{2\pi}\int_C \frac{d\theta_2}{2\pi} \tanh\frac{1}{2}(\theta_1 - \theta_2)\cdot e^{-iP(\theta_1,\theta_2)x} V(\theta_1)V(\theta_2), \\
\psi_0(x) &= \int_C \frac{d\theta}{2\pi} e^{-iP(\theta)x} V(\theta).
\end{aligned}$$

It means that a unique nonzero form factor of the fermion is given by

$$F^{(1)}_{\psi_\pm}(\theta) = \sqrt{M} e^{\mp\frac{1}{2}(\theta + \frac{i\pi}{2})}, \quad s_{\psi_\pm} = \pm\frac{1}{2}. \tag{4.1}$$

Nonzero form factors of the disorder operators are

$$
\begin{aligned}
F_{\mu}^{(2n)}(\theta_1,\cdots,\theta_{2n}) &= (-)^n \frac{(2n)!}{n!} \text{ Alt } \prod_{i=1}^{n} \tanh\frac{1}{2}(\theta_{2i-1}-\theta_{2i}) \\
&= (-)^n \prod_{i<j}^{2n} \tanh\frac{1}{2}(\theta_i-\theta_j), \quad n=0,1,2,\cdots,
\end{aligned}
\tag{4.2}
$$

where Alt means antisymmetrization in θ's. Nonzero form factors of the spin operator are given by[3,9]

$$
\begin{aligned}
F_{\varphi}^{(2n+1)}(\theta_1,\cdots,\theta_{2n+1}) &= (-)^n \frac{(2n+1)!}{n!} \text{ Alt } \prod_{i=1}^{n} \tanh\frac{1}{2}(\theta_{2i}-\theta_{2i+1}) \\
&= (-)^n \prod_{i<j}^{2n+1} \tanh\frac{1}{2}(\theta_i-\theta_j), \quad n=0,1,2,\cdots,
\end{aligned}
\tag{4.3}
$$

The energy-momentum tensor gives an example of a field from $[\tau]$. In light-cone coordinates $x^{\pm}=\frac{1}{2}(x^0 \pm x^1)$ it is given by

$$
\begin{aligned}
T_{+-}(x) &= iM\psi_+(x)\psi_-(x), \\
T_{++}(x) &= -i :\psi_+(x)\partial_+\psi_+(x):, \\
T_{--}(x) &= -i :\psi_-(x)\partial_-\psi_-(x): .
\end{aligned}
$$

Its nonzero form factors are

$$
\begin{aligned}
F_{T_{+-}}^{(2)}(\theta_1,\theta_2) &= iM^2 \sinh\frac{1}{2}(\theta_1-\theta_2), \\
F_{T_{++}}^{(2)}(\theta_1,\theta_2) &= -iM^2 e^{-\theta_1-\theta_2} \sinh\frac{1}{2}(\theta_1-\theta_2), \\
F_{T_{--}}^{(2)}(\theta_1,\theta_2) &= -iM^2 e^{\theta_1+\theta_2} \sinh\frac{1}{2}(\theta_1-\theta_2).
\end{aligned}
\tag{4.4}
$$

For the symplectic model the Zamolodchikov algebra coincides with the Heisenberg algebra $[S(\theta)=1]$. Boson, disorder and fermionic order fields are given by

$$
\begin{aligned}
\varphi(x) &= \int_C \frac{d\theta}{2\pi} e^{-iP(\theta)x} V(\theta), \\
\mu(x) &=: \exp\frac{1}{2}\rho_B(x):, \\
\psi_{\pm}(x) &=: \varphi_{\pm}(x) \exp\frac{1}{2}\rho_B(x):,
\end{aligned}
$$

where

$$\rho_B(x) = -2\int_C \frac{d\theta_1}{2\pi}\int_C \frac{d\theta_2}{2\pi}\frac{e^{-iP(\theta_1,\theta_2)x}}{\cosh\frac{1}{2}(\theta_1-\theta_2)}V(\theta_1)V(\theta_2),$$

$$\varphi_\pm(x) = \sqrt{M}\int_C \frac{d\theta}{2\pi}e^{\mp\frac{1}{2}(\theta+\frac{i\pi}{2})}e^{-iP(\theta)x}V(\theta).$$

Corresponding nonzero form factors are

$$\begin{aligned}
F_\varphi^{(1)}(\theta) &= 1,\\
F_\mu^{(2n)}(\theta_1,\cdots,\theta_{2n}) &= \frac{(-2)^n(2n)!}{n!}\ \mathrm{Sym}\prod_{i=1}^n \cosh^{-1}\frac{1}{2}(\theta_{2i-1}-\theta_{2i}),\\
F_{\psi_\pm}^{(2n+1)}&(\theta_1,\cdots,\theta_{2n+1})\\
&= \sqrt{M}\frac{(-2)^n(2n+1)!}{n!}\ \mathrm{Sym}\ e^{\mp\frac{1}{2}(\theta_1+\frac{i\pi}{2})}\prod_{i=1}^n \cosh^{-1}\frac{1}{2}(\theta_{2i}-\theta_{2i+1}),
\end{aligned} \tag{4.5}$$

where Sym means symmetrization in θ's.

The energy-momentum tensor

$$\begin{aligned}
T_{+-}(x) &= M^2 :\varphi^2: (x),\\
T_{++}(x) &=: (\partial_+\varphi)^2 : (x),\\
T_{--}(x) &=: (\partial_-\varphi)^2 : (x)
\end{aligned}$$

belongs to the space $[\tau]$ and has the form factors

$$\begin{aligned}
F_{T_{+-}}^{(2)}(\theta_1,\theta_2) &= M^2,\\
F_{T_{++}}^{(2)}(\theta_1,\theta_2) &= -M^2 e^{-\theta_1-\theta_2},\\
F_{T_{--}}^{(2)}(\theta_1,\theta_2) &= -M^2 e^{\theta_1+\theta_2},
\end{aligned} \tag{4.6}$$

It is easy to cheque directly that all form factors (4.1)–(4.6) satisfy the generalized form factor axioms.

5. sinh-Gordon model

Consider a more complicated example: the sinh-Gordon model with the action

$$\mathcal{A} = \int d^2x\left(\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\alpha}{\beta^2}\cosh\beta\varphi\right). \tag{5.1}$$

The parameter α depends on cutoff and is not essential. Really the model depends on two physical parameters: mass M of the elementary exitation and

$$\kappa = \frac{\pi\beta^2}{8\pi+\beta^2}. \tag{5.2}$$

The S-matrix of the boson $\varphi(x)$ is given by[10]

$$S(\theta) = \frac{\tanh\frac{1}{2}(\theta - i\kappa)}{\tanh\frac{1}{2}(\theta + i\kappa)} = \frac{\sinh\theta - i\sin\kappa}{\sinh\theta + i\sin\kappa}. \tag{5.3}$$

The bosonic sector of the sinh-Gordon model has been well investigated.[11] Every form factor $F_\phi^{(n)}(\theta_1, \cdots, \theta_n)$, $\phi \in [\tau] \oplus [\varphi]$ takes the form

$$F_\phi^{(n)}(\theta_1, \cdots, \theta_n) = P_\phi^{(n)}(e^{\theta_1}, \cdots, e^{\theta_n}) \prod_{i<j} \frac{\tilde{F}(\theta_i - \theta_j)}{e^{\theta_i} + e^{\theta_j}}, \tag{5.4}$$

where $P_\phi^{(n)}(x_1, \cdots, x_n)$ is a symmetric polynomial, $\tilde{F}(\theta)$ is so called minimal two-point form factor

$$\begin{aligned} \tilde{F}(\theta) &= \tilde{F}(-\theta) S(\theta), \\ \tilde{F}(\theta) &= \tilde{F}(2\pi i - \theta), \\ \lim_{\theta\to\infty} \tilde{F}(\theta) &= 1, \end{aligned} \tag{5.5}$$

and $\tilde{F}(\theta)$ has no poles in the strip $0 < \ \mathrm{Im}\ \theta < \pi$. Explicit form of the minimal form factor for the sinh-Gordon model is[11]

$$\begin{aligned} \tilde{F}(\theta) &= \mathcal{N} \exp\left[8\int_0^\infty \frac{dx}{x} \frac{\sinh\frac{\kappa x}{2\pi}\ \sinh\frac{1}{2}\left(1 - \frac{\kappa}{\pi}\right)x\ \sinh\frac{x}{2}}{\sinh^2 x} \sinh^2\frac{(i\pi - \theta)x}{2\pi}\right], \\ \mathcal{N} &= \exp\left[-4\int_0^\infty \frac{dx}{x} \frac{\sinh\frac{\kappa x}{2\pi}\ \sinh\frac{1}{2}\left(1 - \frac{\kappa}{\pi}\right)x\ \sinh\frac{x}{2}}{\sinh^2 x}\right]. \end{aligned} \tag{5.6}$$

There is an additional property of $\tilde{F}(\theta)$ specific for the sinh-Gordon model

$$\tilde{F}(\theta + i\pi)\tilde{F}(\theta) = \frac{\sinh\theta}{\sinh\theta + i\sin\kappa}. \tag{5.7}$$

Polynomials $P_\phi^{(n)}(x_1, \cdots, x_n)$ satisfy the equation

$$\begin{aligned} &(-)^n P_\phi^{(n+2)}(-x, x, x_1, \cdots, x_n) = x D_n^-(x, x_1, \cdots, x_n) P_\phi^{(n)}(x_1, \cdots, x_n). \\ &D_n^- = \frac{-i}{\tilde{F}(i\pi)} \left[\prod_{i=1}^n (x + e^{i\kappa}x_i)(x - e^{-i\kappa}x_i) - \prod_{i=1}^n (x - e^{i\kappa}x_i)(x + e^{-i\kappa}x_i)\right]. \end{aligned} \tag{5.8}$$

It follows from Eq. (3.4).

For the basic field $\varphi(x)$ and the trace of the energy-momentum tensor $\tau(x) = T_\mu^\mu(x) = T_{+-}(x)$ the polynomials take the form[11]

$$\begin{aligned} &P_\varphi^{(1)}(x) = 1, \\ &P_\varphi^{(2n+1)}(x_1, \cdots, x_{2n+1}) = \left(\frac{4\sin\kappa}{\tilde{F}(i\pi)}\right)^n \sigma_{2n+1}^{(2n+1)} p_{2n+1}(x_1, \cdots, x_{2n+1}), \quad n \geq 1, \end{aligned} \tag{5.9a}$$

$$P_\tau^{(2)}(x_1,x_2) = \frac{M^2}{\tilde F(i\pi)}(x_1+x_2),$$

$$P_\tau^{(2n)}(x_1,\cdots,x_{2n}) = \frac{M^2}{\tilde F(i\pi)}\left(\frac{4\sin\kappa}{\tilde F(i\pi)}\right)^{n-1}\sigma_1^{(2n)}\sigma_{2n-1}^{(2n)}p_{2n}(x_1,\cdots,x_{2n}),\quad n\geq 2, \tag{5.9b}$$

where $\sigma_i^{(n)} = \sigma_i^{(n)}(x_1,\cdots,x_n)$ are elementary symmetric polynomials

$$\begin{aligned}
\sigma_1^{(n)} &= x_1+x_2+\cdots+x_n,\\
\sigma_2^{(n)} &= x_1x_2+x_1x_3+\cdots+x_{n-1}x_n,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
\sigma_n^{(n)} &= x_1x_2\cdots x_n.
\end{aligned}$$

It is easy to cheque using Eq. (5.9b) that the Hamiltonian from Eq. (2.7) can be performed as

$$\mathcal{H} = \int dx^1 T_0^0(x). \tag{5.10}$$

Polynomials $p_n(x_1,\cdots,x_n)$ for several first n's are[11]

$$\begin{aligned}
p_3(x_1,\cdots,x_3) &= 1,\\
p_4(x_1,\cdots,x_4) &= \sigma_2,\\
p_5(x_1,\cdots,x_5) &= \sigma_2\sigma_3 - 4\cos^2\kappa\cdot\sigma_5,\\
p_6(x_1,\cdots,x_6) &= \sigma_2\sigma_3\sigma_4 - 4\cos^2\kappa\cdot(\sigma_4\sigma_5+\sigma_1\sigma_2\sigma_6) - (1-4\cos^2\kappa)\sigma_3\sigma_6,\\
p_7(x_1,\cdots,x_7) &= \sigma_2\sigma_3\sigma_4\sigma_5 - 4\cos^2\kappa\cdot(\sigma_4\sigma_5^2+\sigma_1\sigma_2\sigma_6+\sigma_2^2\sigma_3\sigma_7)\\
&\quad + 16\cos^4\kappa\cdot\sigma_2\sigma_5\sigma_7 - (1-4\cos^2\kappa)(\sigma_1\sigma_2\sigma_4\sigma_7+\sigma_3\sigma_5\sigma_6)\\
&\quad - (1-4\cos^2\kappa)^2\sigma_1\sigma_6\sigma_7 - 4\cos^2\kappa\cdot(1-4\cos^2\kappa)\sigma_7^2,
\end{aligned} \tag{5.11}$$

where obvious superscripts of $\sigma_i^{(n)}$ are omitted.

Consider now fields with $\zeta_\phi = -1$. We shall search form factors in the form

$$F_\phi^{(n)}(\theta_1,\cdots,\theta_n) = e^{\frac12\sum\theta_i}Q_\phi^{(n)}\left(e^{\theta_1},\cdots,e^{\theta_n}\right)\prod_{i<j}\frac{\tilde F(\theta_i-\theta_j)}{e^{\theta_i}+e^{\theta_j}}, \tag{5.12}$$

where $Q_\phi^{(n)}(x_1,\cdots,x_n)$ is a rational symmetric function that has no poles out of zero. The first multiplier in the right hand side provides $\lambda_\phi = -1$ in Eq. (3.3). Eq. (3.4) leads to the equation

$$(-)^n Q_\phi^{(n+2)}(-x,x,x_1,\cdots,x_n) = D_n^+(x,x_1,\cdots,x_n)Q_\phi^{(n)}(x_1,\cdots,x_n).$$

$$D_n^+ = \frac{-1}{\tilde F(i\pi)}\left[\prod_{i=1}^n(x+e^{i\kappa}x_i)(x-e^{-i\kappa}x_i)+\prod_{i=1}^n(x-e^{i\kappa}x_i)(x+e^{-i\kappa}x_i)\right]. \tag{5.13}$$

Consider a spinor field $\psi(x) = (\psi_+(x), \psi_-(x))$ with the Lorentz spin $s_{\psi_\pm} = \pm\frac{1}{2}$. It is convenient to slightly modify Eq. (5.12). Namely, let

$$F^{(2n+1)}_{\psi_\pm}(\theta_1, \cdots, \theta_{2n+1}) = \sqrt{M} e^{\mp\frac{1}{2}(\frac{i\pi}{2}+\sum\theta_i)} Q^{(2n+1)}(e^{\mp\theta_1}, \cdots, e^{\mp\theta_{2n+1}}) \prod_{i<j} \frac{\tilde{F}(\theta_i - \theta_j)}{e^{\mp\theta_i} + e^{\mp\theta_j}}, \tag{5.14}$$

The total degree of $Q^{(2n+1)}(x_1, \cdots, x_n)$ is determined by the spin and is equal to $2n^2$. Using Eq. (2.16) we get that $\psi(x)$ is a Majorana fermion

$$\psi^+_\alpha(x) = \psi_\alpha(x). \tag{5.15}$$

Suppose that the form factors of $\psi_\pm$ behave as $\exp(\pm\frac{1}{2}\theta_i)$ as $\theta_i \longrightarrow \pm\infty$ (as for a free fermion). Then $Q^{(2n+1)}$ must be a polynomial. The last condition determines the first few Q's uniquely:

$$\begin{aligned}
Q^{(1)}(x) &= 1, \\
Q^{(3)}(x_1, \cdots, x_3) &= \left(-\frac{2}{\tilde{F}(i\pi)}\right)(\sigma_1^2 + \sigma_2), \\
Q^{(5)}(x_1, \cdots, x_5) &= \left(-\frac{2}{\tilde{F}(i\pi)}\right)^2 [(\sigma_1^2 + \sigma_2)(\sigma_3^2 + \sigma_2\sigma_4) \\
&\quad + (1 - 4\cos^2\kappa)\sigma_1\sigma_3\sigma_4 - 2(1+\cos^2\kappa)\sigma_4^2 - 4\cos^2\kappa \cdot \sigma_1^3\sigma_5 \\
&\quad - \sigma_1\sigma_2\sigma_5 - \sigma_3\sigma_5].
\end{aligned} \tag{5.16}$$

This Majorana fermion has the pair S-matrix

$$S_\psi(\theta) = -\frac{\tanh\frac{1}{2}(\theta - i\kappa)}{\tanh\frac{1}{2}(\theta + i\kappa)}. \tag{5.17}$$

Similarly we can find the form factors of a field from the 'disorder' sector. We shall search for a scalar field, $\mu(x)$, whose form factors are finite at the infinity. Let

$$F^{(2n)}_\mu(\theta_1, \cdots, \theta_{2n}) = e^{\pm\frac{1}{2}\sum\theta_i} Q^{(2n)}(e^{\pm\theta_1}, \cdots, e^{\pm\theta_{2n}}) \prod_{i<j} \frac{\tilde{F}(\theta_i - \theta_j)}{e^{\pm\theta_i} + e^{\pm\theta_j}} \tag{5.18}$$

with some polynomials $Q^{(2n)}(x_1, \cdots, x_{2n})$ of the total degree $2n(n-1)$. First polinomials are

$$\begin{aligned}
Q^{(0)} &= 1, \\
Q^{(2)}(x_1, x_2) &= \left(-\frac{2}{\tilde{F}(i\pi)}\right), \\
Q^{(4)}(x_1, \cdots, x_4) &= \left(-\frac{2}{\tilde{F}(i\pi)}\right)^2 (-4\cos^2\kappa \cdot \sigma_4 + \sigma_1\sigma_3 + \sigma_2^2).
\end{aligned} \tag{5.19}$$

We see that at least for the first form factors the choice of the sign in Eq. (5.18) is not essential.

6. Conclusion

We see that at least in one-particle models with $\mathbf{Z_2}$ symmetry there are three sectors of mutually local fields: bosonic, fermionic and 'disorder' ones. Fields from different sectors are generally mutually nonlocal. The bosonic sector contains one neutral boson with the S-matrix $S(\theta)$. The fermionic sector contains one Majorana fermion with the S-matrix $-S(\theta)$. We stress that for constructing a full basis in the state space it is enough to act on the vacuum by either bosons or fermions. Both bosons and fermions discribe the same dynamical system. They create the same asymptotic states. The 'disorder' fields do not create any asymptotic states. The role of 'disorder' fields seems somewhat mysterious.

The situation is different in the theories without $\mathbf{Z_2}$ symmetry. For example, the reduced sin-Gordon model with the S-matrix

$$S(\theta) = \frac{\tanh \frac{1}{2}\left(\theta + \frac{2\pi i}{3}\right)}{\tanh \frac{1}{2}\left(\theta - \frac{2\pi i}{3}\right)}$$

contains only one particle b and allows the virtual fusion $b + b \longrightarrow b$. This fusion imposes a connection between even and odd form factors of the particle b.[12] It means that fermions are forbidden. But there are at least two bosonic sectors in these theories. Particle fields from different sectors are mutually nonlocal.[13]

In Sec. 5 we discussed the sinh-Gordon theory. We cited several leading form factors in each sector. But for this model explicit integral formulas for form factors in the bosonic sector are known.[13] It would be important to construct integral representations for form factors in other sectors. It would proove the solubility of the form factor axioms in fermionic and 'disorder' sectors.

We deformed the form factor axioms very slightly introducing one sign factor. It would be interesting to find more essential deformations preserving locality of fields.

Acknoledgments

The author is grateful to A. V. Antonov, A. A. Belov, A. A. Kadeishvili, S. E. Parkhomenko and V. V. Postnikov for useful discussions.

References

1. E. C. Marino, B. Schroer, J. A. Swieca, *Nucl. Phys.* **B200 [FS4]**, 473 (1982)
2. Vl. S. Dotsenko, A. M. Polyakov, *Adv. Stud. Pure Math.* **16**, 171 (1988)
3. M. Sato, M. Jimbo, T. Miwa, *Publ. RIMS* **14**, 223 (1978); **15**, 577; 871 (1979); **16**, 531 (1980)
4. S. Colemann, *Phys. Rev.* **D11**, 2088 (1975)
5. T. R. Klassen, E. Melzer, *Int. J. Mod. Phys.* **A8**, 4131 (1993)
6. A. B. Zamolodcikov, Al. B. Zamolodchikov, *Annals Phys.* **120**, 253 (1979)

7. M. Karowski, P. Weisz, *Nucl. Phys.* **B139**, 455 (1978)
8. F. A. Smirnov, *in* Introduction to quantum group and integrable massive models of quantum field theory, Nakai Lectures on Mathematical Physics (World Scientific, Singapore 1990)
9. B. Berg, M. Karowski, P. Weisz, *Phys. Rev.* **D19**, 2477, 1979
10. A. E. Arinshtein, V. A. Fateev, A. A. Zamolodchikov, Phys. Lett. **B87**, 389 (1979)
11. A. Fring, G. Mussardo, P. Simonetti, *Nucl. Phys.* **B393**, 413 (1993)
12. F. A. Smirnov, *Int. J. Mod. Phys.* **A4**, 4213 (1989)
13. F. A. Smirnov, *Nucl. Phys.* **B337**, 156 (1990)