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Polynomial form factors in the O(3) nonlinear σ -model

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Abstract

We study the general structure of Smirnov's axioms on form factors of local operators in integrable models. We find various consistency conditions that the form factor functions have to satisfy. For the special case of the O(3) σ -model we construct simple polynomial solutions for the operators of the spin-field, current, energy-momentum tensor and topological charge density.

In integrable quantum field theories physical quantities can be determined exactly in the bootstrap approach. Making general assumptions about the properties of the S-matrix has led to the complete determination of the factorized scattering matrix of many well-known models [1–4]. The bootstrap method has also been applied to the determination of matrix elements of local operators in these theories. This form factor bootstrap was initiated in [5,6] and further developed in Refs. [7-10]. In addition to the explicit solutions for the Sine-Gordon, SU(2) Gross-Neveu and O(3) σ -models, the authors of Refs. [7–10] also gave a complete set of axioms that the form factors of an integrable model have to satisfy. Using these axioms (which we will call the Smirnov axioms), the form factor functions in other integrable models could also be calculated [11-13].

In this Letter we study the general structure of Smirnov's axioms and recast them into a form that is very convenient for the recursive determination of the form factor functions in the O(3) NLS model. We shall show that apart from a simple multiplicative factor, which is explicitly given, the form factor functions in the O(3) model are polynomials in the rapidity variables. These polynomials are determined by a simple recursive formula. Although we do not claim originality on the solution of the form factor functions we think that our approach will turn out to be useful for the practical study of the structure of operators in the O(3)NLS model. (Note that while the solution for the form factor functions of the O(3) NLS model is completely explicit in Refs. [9,10], the complicated definitions make them almost impossible to use in practical calculations.)

For simplicity, we consider integrable models describing charge-selfconjugate bosons of equal, nonzero mass m and no bound states, as in the O(3) NLS model. (In case of bound states existing the analytical properties of the form factors are more complicated and model-dependent, and the existence of antiparticles makes the formulae hard to read because of upper and lower indices and charge conjugation matrices. We think, nevertheless, that it is possible to generalize the following arguments also for these cases.)

We parametrize the asymptotic states by the rapidities and the internal indices of the particles. These states are created from the physical vacuum by the Zamolodchikov-Fadeev operators, whose exchange relation is governed by the (two-particle) S-matrix:

$$Z_A^+(\theta)Z_B^+(\theta') = S_{AB,YX}(\theta - \theta')Z_X^+(\theta')Z_Y^+(\theta) . \quad (1)$$

Here the capital indices belong to the internal degrees of freedom.

First we recall the list of requirements that in the bootstrap approach the two-particle S-matrix is assumed to satisfy. First, it is meromorphic in the complex θ -plane and analytic in the $0 \le \Im \theta \le \pi$ physical strip (no bound states). Bose-symmetry and T-reversal invariance imply:

$$S_{AB,CD}(\theta) = S_{BA,DC}(\theta) , \qquad (2)$$

$$S_{AB,CD}(\theta) = S_{CD,AB}(\theta) .$$
(3)

The most important assumptions in the S-matrix bootstrap are unitarity, crossing symmetry and Yang-Baxter equation, respectively:

$$S_{AB,CD}(\theta)S_{CD,A'B'}(-\theta) = \delta_{AA'}\delta_{BB'}, \qquad (4)$$

$$S_{AB,CD}(i\pi - \theta) = S_{AD,CB}(\theta) , \qquad (5)$$

 $S_{AB,VW}(\theta)S_{VC,A'Z}(\theta+\theta')S_{WZ,B'C'}(\theta')$ = $S_{BC,VW}(\theta')S_{AW,ZC'}(\theta+\theta')S_{ZV,A'B'}(\theta)$. (6)

Finally, we require that all the eigenvalues of the Smatrix at zero momentum transfer are -1 (which is satisfied in most integrable models, including NLS):

$$S_{AB,CD}(0) = -\delta_{AD}\delta_{BC}.$$
(7)

Let us now take a local field operator X of spin s. We restrict our attention to its matrix elements between the vacuum and an n-particle state. (Other matrix elements can be derived from these ones [10].) The definition of the n-particle form factor is:

$$\langle 0 | X(0) | \theta_1 A_1, ..., \theta_n A_n \rangle = f_{A_1...A_n}^{(n)}(\theta_1, ..., \theta_n) .$$
(8)

The form factors are originally defined for ordered sets of real rapidities corresponding to the asymptotic states, but they can be analytically continued to the complex plane in all variables. The Smirnov axioms postulate the properties of this analytically extended f function:

$$f_{A_1...A_n}^{(n)}(\theta_1, ..., \theta_n) = e^{\lambda s} f_{A_1...A_n}^{(n)}(\theta_1 - \lambda, ..., \theta_n - \lambda), \qquad (9)$$

$$= S_{AB,YX}(\theta - \theta') f^{(n)}_{\dots XY\dots}(\dots, \theta', \theta, \dots) , \qquad (10)$$

$$= f_{A_{1}A_{2}...A_{n}}^{(n)}(\theta_{1}, \theta_{2}, ..., \theta_{n})$$

$$= f_{A_{2}...A_{n}A_{1}}^{(n)}(\theta_{2}, ..., \theta_{n}, \theta_{1} - 2i\pi), \qquad (11)$$

$$\operatorname{Res}(f_{ABU_1...U_n}^{(n+2)}(\beta+i\pi,\beta,\theta_1,...,\theta_n))$$

$$= \frac{i}{2\pi} \left[\delta_{AB} f_{U_1...U_n}^{(n)}(\theta_1, ..., \theta_n) - S_{BU_1...U_n,V_1...V_nA}(\theta_1, ..., \theta_n \mid \beta) \right]$$
(12)
× $f_{V_1...V_n}^{(n)}(\theta_1, ..., \theta_n) \left] .$

The matrix $S(\theta_1, ..., \theta_n | \beta)$ entering (12) is a product of two-particle S-matrices corresponding to the scattering of particles (βB , $\theta_1 U_1$, ..., $\theta_n U_n$) into the set ($\theta_1 V_1$, ..., $\theta_n V_n$, βA). One should also postulate that the functions f are meromorphic in all variables and they are analytic in the physical strip except for poles explicitly given in (12). (In case of bound states giving extra (nonkinematical) poles a fifth equation applies connecting the (n+1)-particle form factor to the nparticle one.)

We can simplify Eq. (9) if we write f as a product of a "scalar" form factor F and an overall factor that carries the Lorentz-transformation character and cancels the $e^{s\lambda}$ in Eq. (9):

$$f_{A_{1...A_{n}}}^{(n)}(\theta_{1},...,\theta_{n}) = \left(\sum_{i=1}^{n} e^{\theta_{i}}\right)^{s} F_{A_{1...A_{n}}}^{(n)}(\theta_{1},...,\theta_{n}) .$$
(13)

Eqs. (10), (11) and (12) are of the same form (with F instead of f), except for (12) in the two-particle case, which changes to

$$(\theta_1 - \theta_2 - i\pi)^s F_{A_1A_2}^{(2)}(\theta_1, \theta_2) = \text{finite}$$

$$(\theta_1 \to \theta_2 + i\pi) . \tag{14}$$

The first three axioms together describe an invariance property of the form factor function F, while (12) governs the structure of the poles. To identify the symmetry group described by the first three axioms, we summarize the way it is realized. Let us consider a

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group \mathcal{G} , a manifold \mathcal{M} , and the set of functions mapping the manifold into a linear space: $\mathcal{F} = \{V: \mathcal{M} \to \mathcal{L}\}$. Let φ be an action of the group \mathcal{G} on \mathcal{M} :

$$\varphi(\varphi(p, g_1), g_2) = \varphi(p, g_1 g_2) ,$$

$$\forall g_1, g_2 \in \mathscr{G}, \quad p \in \mathscr{M}$$
(15)

and assume that for all $p \in \mathcal{M}$, $g \in \mathcal{G}$ there corresponds a nonsingular linear operator $M_g(p)$ acting on \mathcal{L} satisfying

$$M_{g_1}(p)M_{g_2}(\varphi_{g_2}(p)) = M_{g_1g_2}(p) .$$
(16)

Now we can define an action of the group on the set \mathscr{F} of linear functions, $\Phi: \mathscr{G} \times \mathscr{F} \to \mathscr{F}$ as:

$$[\Phi(g, V)](p) \equiv M_g(p)V(\varphi(p, g)), \qquad (17)$$

 $\Phi(g_1, \, \Phi(g_2, \, V)) = \Phi(g_1g_2, \, V),$

$$\forall g_1, g_2 \in \mathcal{G}, \quad p \in \mathcal{M} . \tag{18}$$

Introducing a compact notation for the action of φ and Φ , this can be summarized as:

$$\varphi(p,g) \equiv p^{g}, \qquad (19)$$

$$\Phi(g, V) \equiv_g V, \tag{20}$$

$$M_{g_1}(p)M_{g_2}(p^{g_1}) = M_{g_1g_2}(p) , \qquad (21)$$

$$_{g}V(p) = M_{g}(p)V(p^{g})$$
. (22)

The invariance group of the form factor functions can now be identified by studying the action on the arguments. (The elements of the manifold \mathcal{M} are *n*component rapidity vectors.) It is easy to see that the transformations form a direct product group with a continuous and a discrete component. The continuous component is the (1+1)-dimensional Lorentz group L acting through shifting all the rapidities by a common value as in Eq. (9). The discrete component contains all the permutations of the n variables generated by the transpositions (10). In addition, any of the n rapidities can be shifted by $2ki\pi$, where k is an integer. (One can construct these transformations by repeatedly using (10) and (11).) Since the permutations act on the generators of this latter group as well, the discrete component is a half-direct product. (Note that this discrete group can be generated by the two elements corresponding to (10) and (11).) Thus the symmetry group of the first three axioms is:

$$\mathscr{G} = \mathbf{L} \otimes (\mathbf{S}_n \wedge \mathbf{Z}^{\otimes n}) \,. \tag{23}$$

It is easy to find the matrices corresponding to the linear transformations in Eqs. (9)-(11) explicitly and verify that as a consequence of the S-matrix properties (2)-(6) they satisfy the consistency relation (16). This means that the first three axioms can be summarized as:

$${}_{\mathscr{G}}F = F \,. \tag{24}$$

Now we turn to the fourth Smirnov axiom which determines the singularity structure of the form factors. We write the form factor F as a product of two terms, an overall factor which carries all the singularities and a regular term with the tensorial structure of F:

$$F^{(n)} = \left(\frac{\pi}{2}\right)^{n-1} \left(\prod_{i < j} \Psi(\theta_i - \theta_j)\right) G^{(n)} .$$
⁽²⁵⁾

Here the function Ψ is chosen to satisfy

Res
$$(\Psi(z); z=i\pi) = -\frac{4}{\pi^2},$$
 (26)

$$\Psi(i\pi+\theta) = -\Psi(i\pi-\theta) . \qquad (27)$$

The advantage of using the reduced form factor $G^{(n)}$ is that while it is still invariant under the group \mathscr{G} , it is analytic in the physical strip and the fourth axiom directly gives its value at the point $\theta_1 - \theta_2 = i\pi$. More precisely, if we define

$$E(\theta) \equiv \frac{1}{\Psi(\theta)\,\Psi(\theta + i\pi)}\,,\tag{28}$$

$$R_{AB,CD}(\theta) \equiv -S_{AB,CD}(\theta)E(\theta) , \qquad (29)$$

$$\tilde{S}_{AB,CD}(\theta) \equiv R_{AB,CD}(\theta) / E(-\theta) , \qquad (30)$$

then \tilde{S} plays the role of the "reduced" S-matrix and G satisfies (10) with \tilde{S} instead of S and (11) with an extra $(-1)^{n-1}$ factor inserted at the rhs. These modified equations, together with (9), define transformation matrices also satisfying the relations (16) and they can be compactly written as

$${}^{\mathscr{G}}G=G\,,\tag{31}$$

where the upper left index notation now indicates this new, modified group action. In the rest of the paper we will use this modified action only.

(12) now reduces to

$$G_{ABU_{3...U_{n}}}^{(n)}(\beta + i\pi, \beta, \theta_{3}, ..., \theta_{n})$$

= $T_{ABU_{3...U_{n}},V_{3...V_{n}}}(\beta | \theta_{3}, ..., \theta_{n})$
× $G_{V_{3...,V_{n}}}^{(n-2)}(\theta_{3}, ..., \theta_{n})$ (32)

except for the n = 2 case which is modified to

$$(\theta_1 - \theta_2 - i\pi)^{|s| - 1} G^{(2)}_{A_1 A_2}(\theta_1, \theta_2) = \text{finite}$$

$$(\theta_1 \to \theta_2 + i\pi) . \tag{33}$$

The matrix appearing in (32) can be explicitly given in terms of the matrix R and the function E:

$$T_{ABU_3...U_n,V_3...V_n}(\beta | \theta, ..., \theta_n)$$

$$= \frac{1}{2i\pi} \left[E(\beta - \theta_3) \dots E(\beta - \theta_n) \delta_{AB} \delta_{U_3V_3} \dots \delta_{U_nV_n} + (-1)^{n-1} R_{BU_3X_4V_3} \times (\beta - \theta_3) \dots R_{X_nU_nAV_n}(\beta - \theta_n) \right].$$
(34)

Note that Eq. (32) gives the value of the *n*-particle reduced form factor in terms of the (n-2)-particle one at the specified point (and not its residue as (12)). Now (after choosing a suitable function Ψ) our task is to determine a series of *analytic* functions $G^{(n)}$ that solves Eqs. (31)-(33).

If we know the (n-2)-particle reduced form factor, Eq. (32) gives the value of the *n*-particle one at the special point. Assuming that the n-particle form factor also solves the equations, one can apply the elements of the group \mathcal{G} and compute the values of this function at other special points as well. We can compute the value of $G^{(n)}$ at all points where any two of its arguments differ by an odd multiple of $i\pi$. This opens the possibility of determining the form factors completely. If the form factors belong to some special class of functions, the knowledge of their values at some specially chosen points could be sufficient for their determination. This is the case for the O(3) NLS model form factors, since the reduced form factors in this model, as we shall see, are polynomials.

However, a question of consistency arises here. It is easy to see that the determination of the value of $G^{(n)}$ at a special point $(\theta_1, ..., \theta_n)$ defined above is not necessarily well-defined, since in general there are many different points $(\theta'_2 + i\pi, \theta'_2, ..., \theta'_n)$ from which one can get there by applying the elements of \mathcal{G} . If the system of the equations is consistent, the different ways determining $G^{(n)}$ at a given special point should give the same result. This condition gives consistency equations.

Now we present some equations of this kind, which will play an important role in the study of the O(3)model form factors. To make the formulae easier to read we introduce a compact notation: dot over (under) a variable means a shift by $i\pi$ ($-i\pi$), and we denote the kth θ_k rapidity simply by its index k. We define the following functions built from the (n-2)-particle form factor:

$$H_{ABU_{3...U_{n}}}^{(2)}(12...n) = T_{ABU_{3...U_{n}},V_{1...V_{3}}}(2|3...n)G_{V_{3...V_{n}}}^{(n-2)}(3...n),$$
(35)

$$H^{(k)} = {}^{P_{k...P_3}} H^{(2)}, \quad k = 3, ..., n.$$
 (36)

Here the P_i s are those group elements that represent the transposition of the (i-1)th and the *i*th component of the rapidity vector. The $H^{(k)}$ s are nothing but the values of $G^{(n)}$ at special points:

$$G^{(n)}(k23...n) = H^{(k)}(12...n)$$
 (37)

We will use the following consistency equations satisfied by the $H^{(k)}$ s:

$$H^{(k)}(\dots \ \theta_k \dots \ \theta_l \dots) = H^{(l)}(\dots \ \theta_k \dots \ \theta_l \dots) ,$$

$$\theta_k = \theta_l , \qquad (38)$$

$$P_{s}H^{(k)} = H^{(k)}, \quad s \neq k, \, k+1$$
, (39)

$${}^{P_k}H^{(k)} = H^{(k-1)}, (40)$$

$${}^{P_{k+1}}H^{(k)} = H^{(k+1)}, \qquad (41)$$

$$H_{A_{1}...A_{n}}^{(2)}(12...n) = (-1)^{n-1} H_{A_{2}...A_{n}A_{1}}^{(n)}(13...n2), \qquad (42)$$

$$H_{A_{1}...A_{n}}^{(k)}(1l3...n) = (-1)^{n-1} H_{A_{2}...A_{n}A_{1}}^{(l-1)}(13...nk).$$
(43)

Using the identification (37) and assuming that $G^{(n)}$ satisfies the Smirnov axioms it is easy to understand the meaning of these consistency equations. However, what we need is to prove them directly by using the definitions (35), (36) and the fact that the (n-2)particle form factors $G^{(n-2)}$ entering these definitions satisfy the Smirnov axioms. Indeed, one can show that if the two-particle S-matrix possesses all the properties (2)-(6) and all the (n-2) and the (n-4)-particle reduced form factors (from which the H's are generated) satisfy the four Smirnov's axioms, then (38)-(43) are satisfied.

Before turning to the special case of the O(3) model we note that the Smirnov axioms alone cannot determine the form factors completely. Indeed, it is easy to see that from a set of solutions $G^{(n)}$ we can generate new ones if we multiply the form factors by scalar functions which are \mathscr{G} -symmetric

$$\Omega^{(n)}((\theta_1, ..., \theta_n)^g) = \Omega^{(n)}(\theta_1, ..., \theta_n) ,$$

$$\forall g \in \mathcal{G}$$
(44)

and satisfy

$$\Omega^{(n+2)}(\beta + i\pi, \beta, \theta_1, ..., \theta_n) = \Omega^{(n)}(\theta_1, ..., \theta_n) .$$
(45)

An important example of such functions is provided by the invariant squared mass of the *n*-particle state:

$$\mu^{2}(\theta_{1}, ..., \theta_{n}) \equiv \left(\sum \cosh(\theta_{i})\right)^{2} - \left(\sum \sinh(\theta_{i})\right)^{2}.$$
 (46)

From now on we consider the O(3) σ -model, where our considerations lead to the complete determination of the form factor functions. The S-matrix of the model is given by [2]:

$$S_{AB; CD}(\theta) = S_1(\theta) \,\delta_{AB} \,\delta_{CD} + S_2(\theta) \,\delta_{AC} \,\delta_{BD} + S_3(\theta) \,\delta_{AD} \,\delta_{BC} , \qquad (47)$$

where

$$S_{1}(\theta) = \frac{2i\pi\theta}{(\theta + i\pi)(\theta - 2i\pi)},$$

$$S_{2}(\theta) = \frac{\theta(\theta - i\pi)}{(\theta + i\pi)(\theta - 2i\pi)},$$

$$S_{3}(\theta) = \frac{2i\pi(i\pi - \theta)}{(\theta + i\pi)(\theta - 2i\pi)}$$
(48)

and we choose the well-known two-particle form factor for Ψ :

$$\Psi(\theta) = \frac{\theta - i\pi}{\theta(2i\pi - \theta)} \tanh^2 \frac{\theta}{2}, \qquad (49)$$

$$E(\theta) = (\theta + i\pi)(\theta - 2i\pi) .$$
 (50)

The fact that makes the form factors explicitly calculable in this case is that the function E is identical to the denominator of the S-matrix (48). From this it follows that the *R*-matrix is (-1x) the numerator of S, thus it is a polynomial of θ . Since both E and R are polynomials, the matrix T entering (32) is also a polynomial of the rapidities. One can easily check that its degree is (2n-5) in β and that it is quadratic in the other rapidities. From this it follows that if $G^{(n-2)}$ is a polynomial, the function $H^{(2)}$ is also a polynomial and it is straightforward to prove that all the $H^{(k)}$ s are polynomials.

More precisely the following statement holds: if $G^{(n-2)}$ satisfies the axioms and it is a polynomial in all its variables of maximum degree (n-4), then the $H^{(k)}$ s are polynomials of maximum degree (2n-5) in θ_k and (n-2) in the other arguments.

Now it is a natural assumption that $G^{(n)}$ is also a polynomial and we can write the following Ansatz:

$$G^{(n)}(12...n) = \sum_{k=2}^{n} H^{(k)}(12...n) \prod_{\substack{l=2\\l\neq k}}^{n} \frac{(1\dot{l})}{(kl)},$$
(51)

where

$$(kl) \equiv (\theta_k - \theta_l) . \tag{52}$$

(51) is the main result of this paper. Together with the definitions (35) and (36) it provides a recursive polynomial solution for all the form factors of the model. In writing down (51) we have made use of the fact that a polynomial of degree (n-2) (which we assume $G^{(n)}$ is in the variable θ_1) is determined by its values at (n-1) different points. This is precisely the information contained in (37).

It can be proven that (51) indeed defines a polynomial solution of the Smirnov axioms. First, one can show that if the (n-2)-particle form factor satisfies Smirnov's axioms and it is a polynomial of maximal degree (n-4) in all variables, then the function $G^{(n)}$ constructed by (51) is a polynomial in all its variables of maximal degree (n-2). Here one has to use (38) and the counting of the degrees is straightforward. One can then check that Eq. (10) is satisfied by (51) by verifying it at (n-1) points (which is sufficient since both sides are polynomials of degree (n-2)). Here one uses (39)-(41). (11) is also satisfied at these

points as a consequence of (42) and (43). Finally, the last Smirnov axiom is satisfied by (51) by construction.

To summarize, (51) provides us with a whole family of form factor functions corresponding to the matrix elements of the local field operator between the vacuum and an increasing number of particles, provided a starting element of this family (corresponding to *n* particles) is known, it is a polynomial satisfying Smirnov's axioms and its degree is not higher than (n-2).

Studying the one- and two-particle form factors of the model, we have found that the four basic operators (spin-field, current, energy-momenum tensor, topological charge) all have matrix elements of the type described above.

We define the reduced form factors of these four operators, respectively, as:

$$f_{A_1...A_n}^a(1 ... n) = \Psi_0 G_{A_1...A_n}^{(\text{spin})a}(1 ... n) , \qquad (53)$$

$$= \left(\sum_{i=1}^{n} e^{\pm \theta_i}\right) \Psi_0 G^{(\operatorname{curr.})a}_{A_1 \dots A_n}(1 \dots n) , \qquad (54)$$

$$f_{A_1\dots A_n}^{\pm \pm} (1 \dots n)$$

$$= \left(\sum_{i=1}^n e^{\pm \theta_i}\right) \left(\sum_{i=1}^n e^{\pm \theta_i}\right) \Psi_0 G_{A_1\dots A_n}^{(\text{E-M})} (1 \dots n) ,$$
(55)

$$f_{A_1...A_n}(1 ... n) = (\mu^2 - 1) \Psi_0 G^{(\text{top.})}_{A_1...A_n}(1 ... n) , \qquad (56)$$

where

 $f_{A_{1}...A_{n}}^{\pm; a}(1 ... n)$

$$\Psi_0 = \left(\frac{\pi}{2}\right)^{n-1} \prod_{i < j} \Psi(\theta_i - \theta_j) .$$
(57)

Note that the operators are defined by their Lorentzand isospin transformation character. In case of the Lorentz-vector and (1, 1)-tensor the conservation is included in the definition. The current and E-M tensor have non-vanishing form factors for an even number of particles only, whereas the spin and topological charge operators have odd particle form factors only. Since there is no invariant isovector, the family of form factors of the topological charge begins with the threeparticle case and that is why the $\mu^2 - 1$ factor has been introduced here to cancel the unwanted poles of the three-particle matrix element.

The starting elements of the respective families are:

$$G_A^{(\text{spin})a}(\theta) \equiv \delta_{aA} , \qquad (58)$$

$$G_{A_1A_2}^{(\text{curr.})a}(\theta_1, \theta_2) \equiv \epsilon_{aA_1A_2}, \qquad (59)$$

$$G_{A_1A_2}^{(\text{E-M})}(\theta_1, \theta_2) \equiv \frac{1}{i\pi - (\theta_1 - \theta_2)} \,\delta_{A_1A_2},\tag{60}$$

$$G_{A_1A_2A_3}^{(\text{top.})}(\theta_1, \theta_2, \theta_3) \equiv \epsilon_{A_1A_2A_3}.$$
(61)

It is easy to see that these functions satisfy the axioms. The form factors of the current and topological charge are really polynomials of the right degree and the recursion works automatically. In the case of the spin and E-M tensor this condition does not hold, but one can show that the Ansatz (51) does give the next element of the family (i.e. the function we get satisfies the axioms as well) and the three-particle spin and fourparticle E-M tensor form factors are polynomials of degree 1 and 2, respectively. This means that we can use our recursive construction also for these operators.

We think that the recursive formula (51) will prove to be useful in the problem of studying the correlation functions in the O(3) NLS model. The problem of constructing the correlation functions by summing over an infinite number of possible intermediate states is discussed in Refs. [11,13] for the case of integrable models with no internal quantum numbers. Some preliminary results in the case of the more difficult problem of the O(3) model are discussed in [14].

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