Absence of spontaneous chiral symmetry breaking and the $1/N$ expansion in two dimensions in an exactly soluble model

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In an exactly soluble two-dimensional model we show how mass generation for a fermion field can be reconciled with the absence of chiral symmetry breaking.

Owing to the absence of particle production a large number of two-dimensional models[1] have been solved exactly for their S matrices. Recently an S matrix has been proposed for the chiral Gross-Neveu model[2]

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \tfrac{1}{2} g^2 [(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma^5\psi)^2] . \tag{1}$$

The $1/N$ expansion, which one uses to check whether the proposed S matrix[3] belongs to the Lagrangian (1) or not, shows spontaneous symmetry breaking of the U(1) chiral symmetry and an associated Goldstone boson. These features cannot exist in two space-time dimensions.[4]

In an interesting paper,[5] Witten has shown how one could reconcile spontaneous mass generation with the absence of spontaneous symmetry breaking in the chiral Gross-Neveu model. Since no explicit computations verifying Witten's proposal have been carried out yet, we feel it instructive to exhibit the problems involved, in an exactly soluble model described by the Lagrangian

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \tfrac{1}{2}(\partial_\mu\phi)^2 + g\bar{\psi}\gamma^\mu\gamma^5\psi\,\partial_\mu\phi$$
$$- M\bar{\psi}\exp(-i2g\gamma^5\phi)\psi , \tag{2}$$

where ψ is an N-plet of fermion fields, ϕ is an isoscalar field, and g^2 plays the role of $1/N$.

On the classical level we have a $U(1) \times U(1)$ symmetry, the chiral part being

$$\psi \to e^{i\beta\gamma^5}\psi ,$$
$$\phi \to \phi + \beta/g \tag{3}$$

with associated Noether current

$$j_{\mu 5} = \frac{\beta}{g}\partial_\mu\phi . \tag{4}$$

From the equations of motion

$$i\partial\!\!\!/\psi + g\gamma^\mu\gamma^5\psi\,\partial_\mu\phi - Me^{-i2g\gamma^5\phi}\psi = 0 ,$$
$$\partial^2\phi + g\partial_\mu(\bar{\psi}\gamma^\mu\gamma^5\psi) + 2igM\bar{\psi}\gamma^5 e^{-i2g\gamma^5\phi}\psi = 0 , \tag{5}$$

it follows that the chiral current is conserved,

since ϕ is a free massless field

$$\partial^\mu j_{\mu 5} = \frac{\beta}{g}\partial^2\phi = 0 . \tag{6}$$

Notice that to zeroth order in g we have a massive fermion field ψ breaking chiral invariance, whereas a mass term $m^2\phi^2$ for the ϕ field would be compatible with that invariance. On the other hand, for $g \neq 0$ the equations of motion (5) show that chiral symmetry is exact and requires ϕ to have zero mass like a bona fide Goldstone boson. After quantization a mild anomaly appears[6] which can be eliminated by rescaling ϕ, and one easily obtains the operator solution

$$\psi = {:}\exp(i\tilde{g}\gamma^5\phi){:}\psi_M , \tag{7}$$

where

$$\tilde{g}^2 = g^2/(1 - g^2 N/\pi)$$

and ψ_M is a free massive fermion field.

Although the zero-mass field ϕ does not exist in two dimensions, it is well known[7] that exponentials $:\exp(ia\phi):$ can be defined by introducing a mass m for ϕ and taking the zero-mass limit after a multiplicative renormalization

$${:}\exp(ia\phi){:} \to \left(\frac{me^\gamma}{2\mu}\right)^{a^2/4\pi} {:}\exp(ia\phi){:} , \tag{8}$$

where γ is Euler's constant and μ plays the role of an infrared regulator. This motivates the definition

$$\left\langle T\prod_{j=1}^{N} e^{ia_j\phi(x_j)}\right\rangle \equiv \exp\left[-\sum_{i<j} a_i a_j \Delta_F(x_i - x_j)\right]\delta_{(\Sigma_j a_j;0)} , \tag{9}$$

where

$$\Delta_F(x) = -\frac{1}{4\pi}\ln(-\mu^2 x^2 + i0) .$$

With respect to the exact solution (7) we now note several facts:

(a) The Lagrangian describes a free massive fermion field ψ_M and a field ϕ which can be decoupled by using Eq. (7). We expect this type of procedure to be useful also in less trivial models.

(b) In spite of its mass ψ_M does not break the chiral symmetry generated by j_{μ_5} because it carries no chiral charge

$$[\psi_M, j_{05}] = \left[\psi_M, \frac{\beta}{g}\partial_0\phi\right] = 0 \qquad (10)$$

since ψ_M and ϕ are independent fields.

(c) In contrast to ψ_M ψ does carry chiral charge and consequently cannot develop a mass. This can easily be seen calculating its two-point function

$$\langle T\psi(x)\bar{\psi}(y)\rangle = \langle T: \exp[i\bar{g}\gamma_x{}^5\phi(x)]:: \exp[i\bar{g}\gamma_y{}^5\phi(y)]:\rangle\langle T\psi_M(x)\bar{\psi}_M(y)\rangle$$

$$= C\int \frac{d^2p}{(2\pi)^2} e^{-ip(x-y)} \int \frac{d^2k}{(2\pi)^2} \frac{\not{k}}{k^2 - M^2 + i0}[(p-k)^2 + i0]^{\bar{g}^2/4\pi - 1}, \qquad (11)$$

where C depends only on μ^2 and g^2. In obtaining this result it was essential to respect the selection rule represented by the Kronecker δ in Eq. (9); the contribution of the mass term of $\langle T\psi_M(x)\bar{\psi}_M(y)\rangle$ to the numerator of the propagator (11) was scaled to zero by the multiplicative renormalization (8). Notice that this propagator satisfies the correct equation of motion (5) in spite of the absence of the mass in the numerator, if one defines the normal product

$$N[\exp(-i2\bar{g}\gamma^5\phi)\psi]$$

occurring in Eq. (5) as

$$N\{\exp[-i2\bar{g}\gamma^5\phi(x)]\psi(x)\} = : \exp[-i\bar{g}\gamma^5\phi(x)]: \psi_M(x).$$

$$(12)$$

(d) Although our model shows no spontaneous generation of mass, the existence of a mass term in (2) evading chiral symmetry breaking is made possible because the two-point function of :exp $\times (-i2\bar{g}\gamma^5\phi)$: behaves for large distances as

$$\langle T: e^{-i2\bar{g}\gamma^5\phi(x)}:: e^{-i2\bar{g}\gamma^5\phi(y)}:\rangle \cong [\mu^2(x-y)^2]^{-\bar{g}^2/\pi} \qquad (13)$$

exactly as in less trivial situations such as the XY model[8] and in Witten's proposal for the chiral Gross-Neveu model.

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