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CHIRAL SYMMETRY, THE $1/N$ EXPANSION AND THE SU(N) THIRRING MODEL

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In the two-dimensional SU(N) Thirring model, the $1/N$ expansion seems to predict spontaneous breaking of the continuous chiral symmetry. This is impossible in two-dimensions. Reasoning along the lines of Berezinski, Kosterlitz and Thouless for the two-dimensional X-Y model, we argue that, in fact, rather than showing long-range order, $\langle \bar{\psi}\psi(x)\ \bar{\psi}\psi(0)\rangle$ vanishes in this model as $|x|^{-1/N}$ at large $|x|$. The $1/N$ expansion is, in fact, a rather good guide to the properties of this model.

1. Introduction

The purpose of this paper is to resolve a problem that arises in the SU(N) Thirring model in two space-time dimensions. This model possesses a U(1) chiral symmetry which, apparently, should prevent the fermions from acquiring mass. The model, however, has been solved in the $1/N$ expansion [1], which is usually reliable, and in this expansion it is found that the chiral symmetry is spontaneously broken, the fermions acquire mass and a Goldstone boson appears.

The problem is that in two space-time dimensions, spontaneous breaking of a continuous symmetry is not possible, and there are no Goldstone bosons. This might seem to show that the $1/N$ expansion must be seriously wrong in this theory. However, we will see that, if treated carefully, the $1/N$ expansion is a good guide to the properties of the SU(N) Thirring model. We will argue that the properties of this model are as follows: the symmetry is not spontaneously broken; there is a massless particle but it is not a Goldstone boson; the physical fermions have mass.

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2. A soluble model

As a preliminary, let us consider a scalar field theory with U(1) invariance:

$$\mathcal{L} = \int d^2x \, [\partial_\mu \phi^* \partial_\mu \phi - g^2(\phi^*\phi - a^2)^2] \, . \tag{1}$$

In more than two dimensions this theory can exhibit a phase with spontaneous symmetry breaking and long-range order:

$$\lim_{|x| \to \infty} \langle \phi^*(x) \, \phi(0) \rangle \neq 0 \, . \tag{2}$$

In addition there is a symmetric phase with masses and exponential fall-off of correlations

$$\langle \phi^*(x) \, \phi(0) \rangle \underset{|x| \to \infty}{\sim} e^{-mx} \, , \qquad m > 0 \, . \tag{3}$$

In two space-time dimensions, spontaneous breaking of a continuous symmetry is, according to Coleman's theorem [2], not possible, and one might expect that only a phase analogous to (3) would be present. However, it has become clear in recent years [3] that in addition to the high-temperature phase (3), this theory possesses in two space-time dimensions an additional phase in which the symmetry is "almost" spontaneously broken and the correlation functions fall like powers

$$\langle \phi^*(x) \, \phi(0) \rangle \sim |x|^{-\alpha} \, . \tag{4}$$

Our main claim in this paper is that theories with U(1) chiral symmetry in two space-time dimensions possess a low-temperature phase of the Berezinski-Kosterlitz-Thouless type, similar to (4). In this phase, the fermions have mass, just as if there were true spontaneous symmetry breaking.

In this section we will consider a soluble example; in sect. 2 we turn to the SU(N) Thirring model. (The model that follows has been discussed by Kogut and Sinclair [4] and many of the points that follow have been treated in their paper.) The soluble model with U(1) chiral symmetry is described by the Lagrangian

$$\mathcal{L} = \int d^2x \, [\bar{\psi} i \partial\!\!\!/ \psi + \tfrac{1}{2}(\partial_\mu \sigma)^2$$
$$- \tfrac{1}{2}\lambda [\bar{\psi}(1 + \gamma_5) \, \psi \, e^{i\sigma/a} + \bar{\psi}(1 - \gamma_5) \, \psi \, e^{-i\sigma/a}]] \, . \tag{5}$$

This model possesses a chiral symmetry $\psi \to e^{i\beta\gamma_5}$, $\sigma \to \sigma - 2\beta a$. Apparently, if unbroken, the symmetry would prevent the fermions from having a mass. As we will see, however, the symmetry is not broken, but the fermion has a mass.

The model can be solved exactly by using the boson representation of fermions [5]. We introduce a new boson field c with

$$\bar{\psi} i \partial\!\!\!/ \psi = \tfrac{1}{2}(\partial_\mu c)^2 \, ,$$

$$\bar{\psi}(1 \pm \gamma_5) \, \psi = \exp(\pm i\sqrt{4\pi} c) \, ,$$

$$\bar{\psi}\gamma^{\mu}\psi = -\frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_{\nu}c\ , \tag{6}$$

and the Lagrangian now becomes

$$\mathcal{L} = \int d^2x \left\{ \tfrac{1}{2}(\partial_{\mu}\sigma)^2 + \tfrac{1}{2}(\partial_{\mu}c) \right.$$
$$\left. - \tfrac{1}{2}\lambda\left[\exp i\left[\frac{\sigma}{a} + \sqrt{4\pi}c\right] + \exp -i\left[\frac{\sigma}{a} + \sqrt{4\pi}c\right]\right]\right\} . \tag{7}$$

Introducing new fields

$$\tilde{c} = \frac{\sqrt{4\pi}c + \sigma/a}{\sqrt{4\pi + 1/a^2}}, \qquad \tilde{\sigma} = \frac{-c/a + \sigma\sqrt{4\pi}}{\sqrt{4\pi + 1/a^2}}\ , \tag{8}$$

the Lagrangian takes the form

$$\mathcal{L} = \int d^2x \left[\tfrac{1}{2}(\partial_{\mu}\tilde{c})^2 + \tfrac{1}{2}(\partial_{\mu}\tilde{\sigma})^2 - \lambda\cos(\sqrt{4\pi + 1/a^2}\,\tilde{c})\right] . \tag{9}$$

There is thus a free, massless scalar $\tilde{\sigma}$ and a sine-Gordon field $\tilde{c}$.

The sine-Gordon spectrum is known exactly [6], and in this case, since $\beta = \sqrt{4\pi + 1/a^2}$ is greater than $\sqrt{4\pi}$, it consists only of a massive fermion (the soliton) and a massive antifermion (the antisoliton). These are the original fermion and antifermion ψ and $\bar{\psi}$ of (5); they have acquired masses.

How have the fermion and antifermion managed to acquire masses despite chiral symmetry? To answer this, we should ask what form the original chiral current takes in terms of $\tilde{c}$ and $\tilde{\sigma}$. The chiral current defined from (5) is

$$A_{\mu} = \bar{\psi}\gamma_{\mu}\gamma_5\psi - 2a\partial_{\mu}\sigma\ . \tag{10}$$

In terms of the new variables, one finds simply

$$A_{\mu} = -\sqrt{\frac{1}{\pi} + 4\pi a^2}\,\partial_{\mu}\tilde{\sigma}\ . \tag{11}$$

Thus, the chiral current involves only $\tilde{\sigma}$ and not $\tilde{c}$. This means that the field $\tilde{c}$, and therefore also the physical fermion and antifermion associated with this field, are neutral under chirality. Thus, even though the elementary fermion field ψ has non-zero chirality, the physical fermion particles have zero chirality. This zero chirality of the physical particles enables the theory to evade the apparent connection between chiral symmetry and massless particles.

The point can perhaps be clarified by introducing a new fermion field that has the quantum numbers of the physical particles. We simply introduce a new fermion field $\tilde{\psi}$ with

$$\bar{\tilde{\psi}}\gamma^{\mu}\tilde{\psi} = -\frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_{\nu}\tilde{c}\ . \tag{12}$$

According to the standard rules, (9) can now be written

$$\mathcal{L} = \int d^2x \left[\bar{\tilde{\psi}}i\partial\!\!\!/\tilde{\psi} - \lambda\bar{\tilde{\psi}}\tilde{\psi} + \frac{1}{2\pi}\left(\frac{1}{1 + 4\pi a^2}\right)(\bar{\tilde{\psi}}\gamma^{\mu}\tilde{\psi})^2 + \tfrac{1}{2}(\partial_{\mu}\tilde{\sigma})^2\right] . \tag{13}$$

In this form, it is obvious that our theory consists of a massive fermion with self-interaction, and a free, massless scalar.

Moreover, it is instructive to compare perturbation theory for (5) to perturbation theory for (13). Perturbation theory in these models means an expansion in powers of $1/a$ for large a. If we naively expand (5) in perturbation theory, we find that the fermion has a mass λ, the scalar is massless, and the theory becomes a free field theory at $a = \infty$. However, in this approximation chiral symmetry is spontaneously broken, something we know to be impossible, and this may lead us to distrust all of the perturbation theory results. But looking at (13), for which perturbation theory does not involve any dubious spontaneously broken symmetry, we see that in fact the fermion has a mass λ, the scalar is massless, and the theory becomes a free-field theory for large a. Thus, perturbation theory is a good guide for all properties of the theory, except the question of whether the symmetry is spontaneously broken.

Now, let us discuss and attempt to resolve the various paradoxes that could be posed in connection with Lagrangian (5).

In perturbation theory of (5) the ψ particle has a mass, and the chirally asymmetric part of its propagator is non-vanishing. We have seen that the mass is actually present. This in itself does not mean that chirality conservation has been broken, as long as the physical fermion is neutral under chirality, which we have seen to be the case. But the chirally asymmetric part of the propagator must vanish, by Coleman's theorem. If it were non-zero, this would mean spontaneously broken chiral symmetry. Let us see how it vanishes.

Let ψ_+ and ψ_- be the positive and negative chirality components of the original ψ field, and ψ_+^* and ψ_-^* their Hermitian conjugates. A chiral transformation $\psi \to e^{i\beta\gamma_5}\psi$ has

$$\psi_+ \to e^{i\beta}\psi_+ , \qquad \psi_+^* \to e^{-i\beta}\psi_+^* ,$$
$$\psi_- \to e^{-i\beta}\psi_- , \qquad \psi_-^* \to e^{i\beta}\psi_- . \tag{14}$$

The chirality violating (but fermion-number conserving) fermion propagator is $G(x,y) = \langle \psi_+(x)\, \psi_-^*(y)\rangle$. To calculate this, we should express the original ψ field in terms of the free field $\tilde{\sigma}$ and the fermion field $\tilde{\psi}$ which creates the physical fermions. The connection can be made using Mandelstam's operator form of the boson representation [7]. Roughly speaking, the correct formulas, obtained by combining Mandelstam's formulas with (6) and (12), are

$$\psi_+ = e^{ib\tilde{\sigma}}\tilde{\psi}_+ , \qquad \psi_+^* = e^{-ib\tilde{\sigma}}\tilde{\psi}_+^* ,$$
$$\psi_- = e^{-ib\tilde{\sigma}}\tilde{\psi}_- , \qquad \psi_-^* = e^{ib\tilde{\sigma}}\tilde{\psi}_-^* , \tag{15}$$

where $b = \sqrt{\pi}/\sqrt{1 + 4\pi^2 a^2}$. (We say "roughly speaking" because Mandelstam's formulas actually require a splitting of $\tilde{\sigma}$ into left-moving and right-moving parts. This is extraneous for our purposes and we will ignore it.)

We now see that the chirality violating propagator is

$$G(x,y) = \langle \psi_+(x)\ \psi_-^*(y)\rangle$$
$$= \langle e^{i\beta\widetilde{\sigma}(x)}\widetilde{\psi}_+(x)\ e^{i\beta\widetilde{\sigma}(y)}\widetilde{\psi}_+^*(y)\rangle\ . \tag{16}$$

Because $\widetilde{\sigma}$ and $\widetilde{\psi}$ are decoupled, this factorizes:

$$G(x,y) = \langle e^{ib\widetilde{\sigma}(x)}\ e^{ib\widetilde{\sigma}(y)}\rangle\langle\widetilde{\psi}_+(x)\ \widetilde{\psi}_+^*(y)\rangle\ . \tag{17}$$

But for the free massless field $\widetilde{\sigma}$, a classic calculation [3,5] shows that $\langle e^{ib\widetilde{\sigma}(x)}\ e^{ib\widetilde{\sigma}(y)}\rangle = 0$. Thus, the chirally asymmetric part of the fermion propagator vanishes.

It may be useful to see wherein lies the subtlety. As we have mentioned, perturbation theory for (5) is an expansion in powers of $1/a$, or equivalently, in powers of b. In an expansion in powers of b, one would expect

$$\langle e^{ib\widetilde{\sigma}(x)}\ e^{ib\widetilde{\sigma}(y)}\rangle = 1 + O(b^2)\ ,$$

whereas in fact this object, because of infrared divergences, vanishes.

It is also interesting to compute the chirally symmetric part of the fermion propagator:

$$\langle \psi_+(x)\ \psi_+^*(0)\rangle = \langle e^{ib\widetilde{\sigma}(x)}\ e^{-ib\widetilde{\sigma}(0)}\rangle\langle\widetilde{\psi}_+(x)\ \widetilde{\psi}_+^*(0)\rangle\ . \tag{18}$$

But [3,5]

$$\langle e^{ib\widetilde{\sigma}(x)}\ e^{-ib\widetilde{\sigma}(0)}\rangle = |x|^{-b^2/4\pi}\ ,$$

while $\langle\widetilde{\psi}_+(x)\ \widetilde{\psi}_+^*(0)\rangle$ is the propagator of a massive fermion, and therefore behaves for large $|x|$ as $e^{-m|x|}$, m being the mass. So the long-distance behavior of (18) is

$$\langle \psi_+(x)\ \psi_+^*(0)\rangle = |x|^{-b^2/4\pi}\ e^{-m|x|}\ . \tag{19}$$

The appearance of a power law correction to the exponential decay means that the ψ spectral function does not have a one-particle pole but rather begins with a cut. This is because the ψ field has non-zero chirality while the physical fermion has chirality zero, so the physical fermion cannot appear as a pole.

Finally, we may consider the behavior of fermion bilinears, such as $\langle\overline{\psi}(1+\gamma_5)\ \psi(x)\ \overline{\psi}(1-\gamma_5)\ \psi(0)\rangle$. In view of Coleman's theorem, this must vanish for large $|x|$. Actually,

$$\overline{\psi}(1\pm\gamma_5)\ \psi(x) = \overline{\widetilde{\psi}}(1\pm\gamma_5)\ \widetilde{\psi}(x)\ e^{\pm 2ib\widetilde{\sigma}(x)}\ ,$$

so we have

$$\langle\overline{\widetilde{\psi}}(1+\gamma_5)\ \widetilde{\psi}(x)\ \overline{\widetilde{\psi}}(1-\gamma_5)\ \widetilde{\psi}(0)\rangle\langle e^{2ib\widetilde{\sigma}(x)}\ e^{-2ib\widetilde{\sigma}(0)}\rangle\ .$$

The first factor approaches a constant for large $|x|$, but the second decays like $|x|^{-b^2/\pi}$. The two-point function for a chirality violating fermion bilinear thus decays according to a power law. The power-law decay shows that we are, in fact, dealing with a low-temperature phase analogous to (4).

The main lessons to be learned from this exactly soluble model are as follows.

The theorem concerning the absence of spontaneous breaking of a continuous symmetry means that chirality violating Green functions must vanish. Indeed they do; we have seen, for instance, that the chirality violating part of the fermion propagator vanishes.

This theorem also implies that there are no Goldstone bosons in two dimensions. A Goldstone boson is a massless boson whose singular contributions to Ward identities enable the identities to be satisfied even though some symmetry breaking Green functions are non-zero. This is not possible in two dimensions. But massless bosons are possible. The massless boson in this theory is not a Goldstone boson; it satisfies no pertinent low-energy theorems; in this theory the chirality violating Green functions are zero and there is no room for Goldstone-boson contributions in the Ward identities.

In this theory there is a Fermi field ψ with a chiral symmetry $\psi \to e^{i\beta\gamma_5}\psi$. In such a situation one usually feels that if the chiral symmetry is unbroken, the physical fermions will be massless. In perturbation theory this is true. But the correct statement is that if the symmetry is unbroken, *and* the physical fermion appears as a pole in the two-point function of ψ, then the fermion must be massless. If the physical fermion does not have the same quantum numbers as ψ, and so does not appear in the ψ two-point function as a one-particle state, then chiral symmetry does not tell us whether the fermion will have zero mass.

The physical properties of this model may be summarized as follows. Although $\bar\psi\psi$ has no long-range order, it is so close to having long-range order (we have seen that its two-point function decays only like a power law) that the usual physical consequence of long-range order, a fermion mass, is present even though long-range order is not quite present.

For this "almost long-range order" with power-law decay of correlations a massless particle is clearly necessary. That the massless particle decouples is not necessary. By adding extra fields and interactions, one could get a model with similar physical properties, but no longer exactly soluble, and with the massless particle no longer decoupling. The tempting idea that "a Goldstone boson is allowed in two dimensions if it decouples" is a misunderstanding. The massless particle of this model is not a decoupled Goldstone boson; it is a non-Goldstone, massless boson.

3. The SU(N) Thirring model

Now we turn to the model of main interest in this paper, the SU(N) Thirring model. This is a system of N Dirac fields ψ_k, $k = 1, \dots N$, with Lagrangian

$$\mathcal{L} = \int d^2x \left[i\bar\psi_k i\not\partial \psi_k + \frac{g}{N}((\bar\psi_k \psi_k)^2 - (\bar\psi_k \gamma_5 \psi_k)^2) \right] , \tag{20}$$

where the factor of N is included so as to have a smooth limit as $N \to \infty$. The Lagrangian possesses a U(1) chiral symmetry $\psi_k \to e^{i\beta\gamma_5}\psi_k$.

Following ref. [1], we may equivalently introduce auxiliary fields $\sigma = \overline{\psi}\psi$, $\pi = i\overline{\psi}\gamma_5\psi$, and write

$$\mathcal{L} = \int d^2x \left[i\overline{\psi}_k \partial\!\!\!/ \psi_k - \tfrac{1}{2}\sigma^2 - \tfrac{1}{2}\pi^2 + \sqrt{\frac{g}{N}}\,\overline{\psi}_k(\sigma + i\pi\gamma_5)\,\psi_k \right] . \tag{21}$$

The symmetry is now

$$\psi \to e^{i\beta\gamma_5}\psi , \qquad \sigma + i\pi \to e^{2i\beta}(\sigma + i\pi) .$$

To carry out the $1/N$ expansion, one now integrates over the Fermi fields, yielding the effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}} = iN \,\mathrm{Tr}\,\ln\left(i\partial\!\!\!/ + \sqrt{\frac{g}{N}}(\sigma + i\pi\gamma_5)\right) - \tfrac{1}{2}\int d^2x (\sigma^2(x) + \pi^2(x)) . \tag{22}$$

As explained by Gross and Neveu, this Lagrangian is apparently of the symmetry breaking type, in the sense that the minimum of the action occurs for non-zero values of σ and π. But if one expands around a non-zero value of σ and π, one is introducing symmetry breaking and Goldstone bosons, which are not possible in two dimensions.

The resolution of this problem is very simple: it is just the Berezinski-Kosterlitz-Thouless phenomenon. One writes $\sigma + i\pi = \rho e^{i\theta}$. We can expand around a non-zero vacuum expectation value of ρ without breaking the symmetry. The dangerous step, which must be avoided, is to assume for θ a definite vacuum expectation value, such as zero. This would violate the symmetry. In these variables, chiral symmetry is $\theta \to \theta + c$.

In terms of ρ and θ, our effective Lagrangian is

$$\mathcal{L}_{\mathrm{eff}} = iN \,\mathrm{Tr}\,\ln\left(i\partial\!\!\!/ + \sqrt{\frac{g}{N}}\tfrac{1}{2}\rho\, e^{i\theta\gamma_5}\right) - \tfrac{1}{2}\int d^2x\, \rho^2(x) . \tag{23}$$

In analyzing the infrared behavior of this theory, one may just as well set ρ equal to a c-number, its vacuum expectation value, since the infrared behavior will be determined by the massless field θ. As far as the θ dependent terms are concerned, only the term proportional to $(\nabla\theta)^2$ is important, since other terms have higher dimensions and will not be important in the infrared. The $(\nabla\theta)^2$ term is readily calculated, and we find

$$\mathcal{L}_{\mathrm{eff}} = \frac{N}{4\pi}\int d^2x (\nabla\theta)^2 , \tag{24}$$

plus terms that will not be important in the infrared.

We would like to calculate the behavior in the SU(N) Thirring model of matrix elements that could show symmetry breaking and long-range order. For instance, let us calculate the large x form of $\langle\bar{\psi}(1+\gamma_5)\,\psi(x)\,\bar{\psi}(1-\gamma_5)\,\psi(0)\rangle$. With

$$\bar{\psi}(1\pm\gamma_5)\,\psi=\sigma\mp i\pi=\rho\,e^{\mp i\theta}\,,$$

we must calculate $\langle\rho(x)\,e^{-i\theta(x)}\rho(0)\,e^{i\theta(0)}\rangle$. Replacing ρ by its c-number, vacuum expectation value, the x dependence will be that of $\langle e^{-i\theta(x)}\,e^{i\theta(0)}\rangle$ in the free field theory with Lagrangian (24). This, in turn, is [3,5] $|x|^{-1/N}$. So we conclude that, in the SU(N) Thirring model,

$$\langle\bar{\psi}(1+\gamma_5)\,\psi(x)\,\bar{\psi}(1-\gamma_5)\,\psi(0)\rangle\sim c|x|^{-1/N}\,, \tag{25}$$

for large $|x|$.

The procedure followed in arriving at (25) may seem rather cavalier. However, it is shown in the literature on the X-Y model [3] that this sort of behavior is stable against perturbations. The reason for this is simply that, as we have said, the infrared behavior is determined by the massless field θ, and θ-dependent terms other than the free field term $(\nabla\theta)^2$ have higher dimension and are unimportant in the infrared (remember that terms like θ^4 or $\theta^2\,(\nabla\theta)^2$ are forbidden by the symmetry $\theta\to\theta+c$).

(25) is certainly the nicest result in this paper. If in (25) one sets $N=\infty$ one finds the two-point function approaching a constant for large x, erroneously indicating long-range order and symmetry breaking. For any finite N, on the other hand, this two-point function vanishes as $|x|\to\infty$, but it vanishes very slowly if N is large.

This "almost long-range order" is sufficient to justify many of the physical results that come from the large-N expansion. There is no reason to doubt the prediction of the $1/N$ expansion that the physical fermions have masses. On the contrary we should expect this, since we have seen in the last section that "almost long-range order" like (25) is sufficient to generate Fermi masses. The $1/N$ expansion correctly predicts that there is a massless particle, the θ particle, although it is wrong in indicating that it is a Goldstone boson. The $1/N$ expansion can be validly used to calculate the elementary fermion S matrix as an expansion in powers of $1/N$. As long as one interprets the results carefully and as long as one does not break the symmetry by attributing a mean value to θ, the $1/N$ expansion is a quite reliable guide to the properties of this model.

For completeness, although it is outside the main purpose of this paper, we should note that there is a simple (and fairly well-known) field theoretic argument for the existence of a massless particle θ in this model. The argument also shows that θ decouples from the S matrix, and it applies equally well to the soluble model of sect. 2. The argument is that, as one can see from the equations of motion, the chiral current A_μ of this model satisfies a free, massless wave equation, $\nabla^2 A_\mu=0$, and therefore, when acting on the vacuum, it creates a free, massless particle, which we have called θ. The decoupling of θ, incidentally, can be seen in leading order from the fact that (24) is a free field Lagrangian.

We should also note that, independently of the arguments given above, the fact that the $1/N$ expansion gives the correct spectrum for this model can be seen from the bosonized version of this model, which was described by Banks, Horn and Neuberger and by Halpern [8].

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