Grundlegendes

Lösung Aufgabe 1:

$$
\begin{aligned}
0 &= x^2 + px + q = x^2 + px + (\frac{p}{2})^2 - (\frac{p}{2})^2 + q \\
&= (x + \frac{p}{2})^2 - [(\frac{p}{2})^2 - q] \\
&\Rightarrow\ (x + \frac{p}{2})^2 = \frac{p^2}{4} - q \\
&\Rightarrow\ x + \frac{p}{2} = \pm\sqrt{\frac{p^2}{4} - q} \\
&\Rightarrow\ x = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q} \qquad .
\end{aligned}
$$

Lösung Aufgabe 2:
Wir benutzen die Abkürzung $\sum = \sum_{n=0}^{N} q^n$; Dann gilt

$$
\begin{aligned}
\sum &= 1 + q + q^2 + \cdots + q^N \\
q\sum &= q + q^2 + q^3 + \cdots + q^{N+1} \\
\Rightarrow\ \sum - q\sum &= 1 - q^{N+1} = (1-q)\sum = 1 - q^{N+1} \\
\Rightarrow\ \sum_{n=0}^{N} q^n &= \frac{1 - q^{N+1}}{1-q}
\end{aligned}
$$

Lösung Aufgabe 3:

$$
\frac{\alpha}{2\pi} = \frac{b}{2\pi R} \ \Rightarrow\ b = \alpha R \qquad .
$$

Lösung Aufgabe 4:

(a) Setze $\beta = \alpha$ in Formel für $\sin(\alpha + \beta)$ bzw. $\cos(\alpha + \beta)$

(b) wg. (a) $\cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2} = 2\cos^2\frac{\alpha}{2} - 1$

$\Rightarrow\ \cos^2\frac{\alpha}{2} = \frac{1}{2}(1 + \cos\alpha)\ \Rightarrow\ \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1}{2}(1 + \cos\alpha)}$

und ebenso
$\cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2} = 1 - 2\sin^2\frac{\alpha}{2}$
$\Rightarrow\ \sin^2\frac{\alpha}{2} = \frac{1}{2}(1-\cos\alpha)\ \Rightarrow\ \sin\frac{\alpha}{2} = \pm\sqrt{\frac{1}{2}(1-\cos\alpha)}$

(c) $\frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right] = \frac{1}{2}\left[\cos\alpha\,\cos\beta + \sin\alpha\,\sin\beta - (\cos\alpha\,\cos\beta - \sin\alpha\,\sin\beta)\right] = \sin\alpha\,\sin\beta$
analog für die zwei weiteren Relationen.

(d) $\sin 3\alpha = \sin(2\alpha+\alpha) = \sin 2\alpha\,\cos\alpha + \cos 2\alpha\,\sin\alpha$
$= (2\sin\alpha\,\cos\alpha)\cos\alpha + (\cos^2\alpha - \sin^2\alpha)\sin\alpha$
$= 2\sin\alpha\,\cos^2\alpha + \cos^2\alpha\,\sin\alpha - \sin^3\alpha$
$= 3\sin\alpha\,\cos^2\alpha - \sin^3\alpha$
$= 3\sin\alpha(1-\sin^2\alpha) - \sin^3\alpha$
$= 3\sin\alpha - 4\sin^3\alpha$

(e) $\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\,\cos\beta + \cos\alpha\,\sin\beta}{\cos\alpha\,\cos\beta - \sin\alpha\,\sin\beta}$
$= \frac{\frac{\sin\alpha\,\cos\beta + \cos\alpha\,\sin\beta}{\cos\alpha\,\cos\beta}}{1 - \frac{\sin\alpha\,\sin\beta}{\cos\alpha\,\cos\beta}} = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\,\tan\beta}$.

(f) $\frac{\cos\alpha(1-\tan^2\alpha)}{\tan\alpha(\cot\alpha - 1)} = \frac{\cos\alpha(1-\tan^2\alpha)}{1-\tan\alpha} = \cos\alpha(1+\tan\alpha) = \cos\alpha + \sin\alpha$.

Lösung Aufgabe 5:

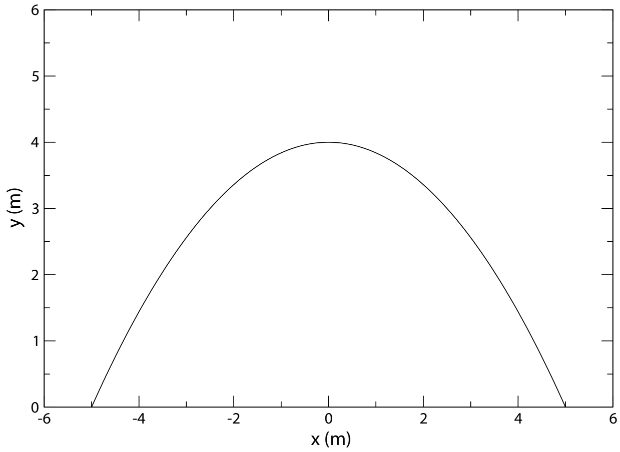
(a) $e^x = \cosh x + \sinh x$
$e^{-x} = \cosh x - \sinh x$
$\rightarrow\ \sinh(x+y) = \frac{1}{2}\left[e^{x+y} - e^{-x-y}\right] = \frac{1}{2}\left[e^x e^y - e^{-x}e^{-y}\right]$
$= \frac{1}{2}\left[(\cosh x + \sinh x)(\cosh y + \sinh y) - (\cosh x - \sinh x)(\cosh y - \sinh y)\right]$
$= \frac{1}{2}\left[2\cosh x\,\sinh y + 2\sinh x\,\cosh y\right]$

$\Rightarrow\ \sinh(x+y) = \cosh x\,\sinh y + \sinh x\,\cosh y$.

(b) $\cosh(x+y) = \frac{1}{2}\left[(\cosh x + \sinh x)(\cosh y + \sinh y) + (\cosh x - \sinh x)(\cosh y - \sinh y)\right]$
$= \frac{1}{2}\left[2\cosh x\,\cosh y + 2\sinh x\,\sinh y\right]$

$\Rightarrow\ \cosh(x+y) = \cosh x\,\cosh y + \sinh x\,\sinh y$.

Lösung Aufgabe 6:



$h = 4\,m$
$x_0 = 5\,m$

allg. Parabel: $y = ax^2 + bx + c$
mit

$$(i)\, y = h \qquad \text{für}\, x = 0$$
$$(ii)\, h = 0 \qquad \text{für}\, x = \pm x_0$$

$$(i) \Rightarrow c = h \qquad (ii) \Rightarrow 0 = ax_0^2 \pm bx_0 + h$$

$$\Rightarrow \begin{cases} 2bx_0 = 0 \\ 0 = ax_0^2 + h \end{cases}$$

$$\rightarrow b = 0 \qquad \text{und} \qquad a = -\frac{h}{x_0^2}$$

$$\Rightarrow y = -h\left[(\frac{x}{x_0})^2 - 1\right] \qquad .$$

oder eleganter:

allg. Parabel: $y = ax^2 + bx + c = a(x - x_{N1})(x - x_{N2})$

x_{N1}, x_{N2} sind die zwei Nullstellen des Polynoms aus Aufgabenstellung: $x_{N1} = -x_{N2} = x_0$

$\rightarrow\ y = a(x - x_0)(x + x_0) = a(x^2 - x_0^2)$

$y = h$ für $x = 0$ erfordert $h = -ax_0^2$ oder $a = -\frac{h}{x_0^2}$

Lösung Aufgabe 7:

(a) $(x^2 - 2x + 3) : (x^2 + 1) = 1 - \frac{2}{x} + \frac{2}{x^2} + \mathcal{O}(\frac{1}{x^3})$

$$\begin{array}{l} \underline{x^2 + 1} \\ -2x + 2 \\ \underline{-2x - \frac{2}{x}} \\ 2 + \frac{2}{x} \\ \underline{2 + \frac{2}{x^2}} \\ \frac{2}{x} - \frac{2}{x^2} \end{array}$$

$\rightarrow \ f(x) \simeq 1 - \frac{2}{x} + \frac{2}{x^2} + \mathcal{O}(\frac{1}{x^3})$

(b) $(x^3 - 3x^2 + 4x + 1) : (x^2 + 2) = x - 3 + \frac{2}{x} + \frac{7}{x^2} + \mathcal{O}(\frac{1}{x^3})$

$$\begin{array}{l} \underline{x^3 + 2x} \\ -3x^2 + 2x + 1 \\ \underline{-3x^2 - 6} \\ 2x + 7 \\ \underline{2x + \frac{4}{x}} \\ 7 - \frac{4}{x} \\ \underline{7 + \frac{14}{x^2}} \\ -\frac{4}{x} - \frac{14}{x^2} \\ \ldots \end{array}$$

$\rightarrow \ f(x) = x - 3 + \frac{2}{x} + \frac{7}{x^2} + \mathcal{O}(\frac{1}{x^3})$