Lösung Aufgabe 1:

(a)

$$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \qquad \vec{c} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{a} + \vec{b} = \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix} \qquad \vec{a} - \vec{c} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

$\vec{a}\,\vec{b} = 2 + 2 + 9 = 13, \qquad \vec{b}\,\vec{c} = 4 + 0 + 3 = 7.$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 1 & 2 & 3 \\ 2 & 1 & 3 \end{vmatrix} = \vec{e}_x(6-3) - \vec{e}_y(3-6) + \vec{e}_z(1-4) = \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix}$$

$\vec{c}\,(\vec{a} \times \vec{b}) = 6 + 0 - 3 = 3.$

(b)

$$\measuredangle(\vec{a}, \vec{b}) = \arccos \frac{\vec{a}\vec{b}}{ab} = \frac{13}{\sqrt{1+4+9}\sqrt{4+1+9}} = \arccos \frac{13}{14}$$

$$\measuredangle(\vec{b}, \vec{c}) = \arccos \frac{\vec{b}\vec{c}}{bc} == \arccos \frac{7}{\sqrt{70}}$$

$$\measuredangle(\vec{a}, \vec{c}) = \arccos \frac{\vec{a}\vec{c}}{ac} == \arccos \frac{5}{\sqrt{70}}$$

(c)

$$\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

Gesucht: $\vec{c}$ mit $\vec{c} \perp \vec{a}$ und $\vec{c} \perp \vec{b}$, also $\vec{a}\,\vec{c} = \vec{b}\,\vec{c} = 0$.

$$\begin{aligned} c_x + c_y + c_z &= 0 \\ c_x - c_z &= 0 \end{aligned}$$

$c_x = c_z,\ c_y = -2c_x$

$$\vec{c} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

sowie jeder vielfache oder parallelverschobene Vektor,

$$\vec{c} = \vec{\alpha} + \beta \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

mit konstantem Vektor $\vec{\alpha}$ und der Konstanten β.

Lösung Aufgabe 2:

(a) $\vec{A}_1 = \vec{a} + \vec{b}, \qquad \vec{A}_2 = \vec{a} - \vec{b}, \qquad \vec{A}_1 \perp \vec{A}_2 \leftrightarrow \vec{A}_1\vec{A}_2 = 0.$

$$\vec{A}_1 = \begin{pmatrix} a_x + b_x \\ a_y + b_y \\ a_z + b_z \end{pmatrix} \qquad \vec{A}_2 = \begin{pmatrix} a_x - b_x \\ a_y - b_y \\ a_z - b_z \end{pmatrix}$$

$\vec{A}_1\,\vec{A}_2 = a_x^2 - b_x^2 + a_y^2 - b_y^2 + a_z^2 - b_z^2 = a_x^2 + a_y^2 + a_z^2 - (b_x^2 + b_y^2 + b_z^2) = \vec{a}^{\,2} - \vec{b}^{\,2}.$

Antwort: Wenn beide Vektoren die gleiche Länge besitzen.

(b) $|\vec{a} \times \vec{b}| = ab\sin\varphi, \qquad \vec{a}\,\vec{b} = ab\cos\varphi, \qquad ab\sin\varphi = ab\sqrt{1-\cos^2\varphi} = \sqrt{a^2b^2 - a^2b^2\cos^2\varphi} = \sqrt{\vec{a}^{\,2}\vec{b}^{\,2} - (\vec{a}\,\vec{b})^2}$

Lösung Aufgabe 3:

(a)

$$\vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \qquad m = 2 \rightarrow \vec{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \rightarrow \vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

(b)

$$\vec{n}\,\vec{b} = 0 \qquad n_x = \frac{b_y}{b} = \frac{2}{\sqrt{5}}, \qquad \vec{n} = n_x\begin{pmatrix} 1 \\ -b_x/b_y \end{pmatrix} = \frac{2}{\sqrt{5}}\begin{pmatrix} 1 \\ -1/2 \end{pmatrix} = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$a = \sqrt{5}, \qquad |x|_{\text{min}} = a\sin\varphi.\ \varphi = \measuredangle(\vec{a}, \vec{b})$

Entweder formal: $\vec{a}\,\vec{b} = ab\cos\varphi = 5\cos\varphi, \qquad \vec{a}\,\vec{b} = 5 \qquad \rightarrow \cos\varphi = 1, \sin\varphi = 0$

Oder: $\vec{a} \quad || \quad \vec{b} \quad \rightarrow \sin\varphi = 0$

$|x|_{\text{min}} = 0$, d.h. die Gerade läuft durch den Ursprung.

(c)

$$\text{Aus} \qquad \vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ folgt:} \qquad x = 1 + 2, \quad y = 2 + 2s \quad \rightarrow y = 2x.$$

Lösung Aufgabe 4:

(a) f: Zahl der Schülerinnen, $\qquad m$: Zahl der Schüler

$$\begin{aligned} f - 1 &= 2m \\ f &= 2.6\,(m-1) \end{aligned}$$

$2m + 1 = 2.6(m-1), \qquad 1 + 2.6 = 0.6m, \qquad m = 6, \qquad f = 2m + 1 = 13.$

(b) $\vec{v}$: Geschwindigkeit des Flugzeugs ohne Wind
$\vec{w}$: Geschwindigkeit des Windes

Eindimensionale Bewegung:

Mit Wind: $v + w = s/t$
Gegen Wind: $v - w = 2/1.2t = 5s/(6t)$

$$v = \frac{11}{12}\frac{s}{t}, \qquad w = \frac{1}{12}\frac{s}{t}$$

Die Aufgaben 5-7 siehe jpg-Dateien

Lösungen: Blatt 4

Aufgabe 4)

b) $\left.\begin{array}{l} v = \text{Geschwindigkeit des Flugzeugs relativ zur Luft} \\ w = \text{Geschwindigkeit des Windes} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} s = (v+w)t = (v-w)1.2\,t \\ \quad = (v-w)\frac{6}{5}\,t \end{array}\right.$

$$\Rightarrow v + w = \frac{s}{t}, \quad v - w = \frac{5}{6}\frac{s}{t} \Rightarrow \left\{\begin{array}{l} v = \dfrac{1}{2}\left(1+\dfrac{5}{6}\right)\dfrac{s}{t} = \dfrac{11}{12}\dfrac{s}{t} \\ v = \dfrac{1}{2}\left(1-\dfrac{5}{6}\right)\dfrac{s}{t} = \dfrac{1}{12}\dfrac{s}{t} \end{array}\right.$$

Aufgabe 5)

a) $\left\{\begin{array}{l} 8x - 3y = 11 \\ 5x + 2y = 34 \end{array}\right. \Rightarrow \left\{\begin{array}{l} 16x - 6y = 22 \\ 15x + 6y = 102 \end{array}\right. \Rightarrow \left\{\begin{array}{l} 31x = 124, \text{ dh. } \boxed{x = 4} \\ y = \dfrac{1}{2}(34 - 5x) = \dfrac{1}{2}(34 - 20), \text{ dh. } \boxed{y = 7} \end{array}\right.$

b) $\left\{\begin{array}{l} 12x + 16y = 28 \\ 15x + 20y = 35 \end{array}\right. \Rightarrow \left\{\begin{array}{l} 3x + 4y = 7 \\ 3x + 4y = 7 \end{array}\right. \Rightarrow y = -\dfrac{3}{2}x + \dfrac{7}{4}$, d.h. keine eindeutige Lösung

c) $\left\{\begin{array}{l} 2x - 2y = -3 \\ -3x + 3y = 9 \end{array}\right. \Rightarrow \left\{\begin{array}{l} x - y = -3/2 \\ x - y = -9/3 = -3 \end{array}\right. \Rightarrow$ Widerspruch, d.h. keine Lösung

d) $\left\{\begin{array}{l} 8x - 6y = 2 \\ 2x + 3y = 2 \end{array}\right. \Rightarrow \left\{\begin{array}{l} 4x - 3y = 1 \\ 2x + 3y = 2 \end{array}\right. \Rightarrow \left\{\begin{array}{l} 6x = 3, \text{ dh. } \boxed{x = 1/2} \\ y = \dfrac{1}{3}(4x - 1) = \dfrac{1}{3}(2 - 1), \text{ dh. } \boxed{y = 1/3} \end{array}\right.$

Aufgabe 6)

a) $\left\{\begin{array}{l} y(x) = ax^2 + bx + c \\ y'(x) = 2ax + b \end{array}\right.$ mit $y(1) = 0,\ y(2) = 6,\ y'(2) = 8$

$$\Rightarrow \left\{\begin{array}{l} a + b + c = 0 \\ 4a + 2b + c = 6 \\ 4a + b = 8 \end{array}\right. \Rightarrow \left\{\begin{array}{l} 3a + b = 6 \\ 4a + b = 8 \\ a + b + c = 0 \end{array}\right. \Rightarrow \left\{\begin{array}{l} a = 2 \\ b = 6 - 3a = 0 \\ c = -a - b = -2 \end{array}\right. \Rightarrow \boxed{y(x) = 2x^2 - 2}$$

b) $\left\{\begin{array}{l} \alpha + \beta + \gamma = \pi \\ \alpha = 2\beta \\ \alpha + \beta = \gamma \end{array}\right. \Rightarrow \left\{\begin{array}{l} 2(\alpha + \beta) = \pi \\ \alpha - 2\beta = 0 \\ \alpha + \beta = \gamma \end{array}\right. \Rightarrow \left\{\begin{array}{l} 3\alpha = \pi \text{ dh. } \alpha = \pi/3 \\ \beta = \pi/6 \\ \gamma = \alpha + \beta = \pi/2 \end{array}\right.$

c) $\left\{\begin{array}{l} x + z = 14 \\ x + y = -1 \\ y + z = 1 \end{array}\right. \Rightarrow \left\{\begin{array}{l} x + z = 14 \\ x - z = -2 \\ y + z = 1 \end{array}\right. \Rightarrow \left\{\begin{array}{l} x = 6 \\ z = 8 \\ y = -7 \end{array}\right.$

Aufgabe 7)

$$\begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2+2 \\ -4+3 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a-b \\ a+b \end{pmatrix}$$