Übungen Brückenkurs

Lösungen: Blatt 5

Aufgabe 1)

a) $\int xe^x \sin x dx = \left[\frac{d}{da}\int \sin x e^{ax} dx\right]_{a=1}$, 2 mal partiell integrieren $\Rightarrow$
$\int \sin x e^{ax} dx = -\cos x e^{ax} + a \int \cos x e^{ax} dx = -\cos x e^{ax} + a \sin x e^{ax} - a^2 \int \sin x e^{ax} dx$
$\Rightarrow \int xe^x \sin x dx = \left[\frac{d}{da}\frac{1}{1+a^2}\left(-\cos x + a \sin x\right) e^{ax}\right]_{a=1}$
$= -\frac{2}{4}\left(-\cos x + \sin x\right) e^x + \frac{1}{2}\left(\sin x + \left(-\cos x + \sin x\right) x\right) e^x = \frac{1}{2}\left(\cos x + x\left(\sin x - \cos x\right)\right) e^x$

b) $\int dx \frac{1}{\sqrt{a^2-x^2}} = \int dya \frac{1}{\sqrt{a^2-a^2y^2}} = \int dy \frac{1}{\sqrt{1-y^2}} = \arcsin y = \arcsin \frac{x}{a}, \quad x = ay$

c) $\int dx \left(x^5 - 4x^3\right) = \frac{1}{6}x^6 - x^4$

d) $\int dx\ x^3 \ln x = \int dx\ \left(\frac{1}{4}x^4\right)' \ln x = \frac{1}{4}x^4 \ln x - \frac{1}{4}\int dx\ x^4 \frac{1}{x} = \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$

Aufgabe 2)

a) $\sin x = x - \frac{1}{3!}x^3 + \ldots$

b) $\cos x = 1 - \frac{1}{2}x^2 + \ldots$

c) $\sinh x = x + \frac{1}{3!}x^3 + \ldots$

d) $\cosh x = 1 + \frac{1}{2}x^2 + \ldots$

Aufgabe 3)

a) $y = x^2 \Rightarrow y' = 2x$
$\Rightarrow L = \int_0^1 \sqrt{1+4x^2} dx = 1.478\,9 = \int_0^A dy\ \frac{1}{2}\cosh y \sqrt{1+\sinh^2 y} = \int_0^A dy\ \frac{1}{2}\cosh^2 y$
(mit $x = \frac{1}{2}\sinh y,\ A = \operatorname{arsinh} 2 = \sinh^{-1} 2$)
$L = \frac{1}{8}\int_0^A dy \left(e^{2y} + e^{-2y} + 2\right) = \frac{1}{8}\left[\frac{1}{2}e^{2y} - \frac{1}{2}e^{-2y} + 2y\right]_0^A = \frac{1}{8}\left(\sinh\left(2 \operatorname{arsinh} 2\right) + 2 \operatorname{arsinh} 2\right)$
$= \frac{1}{2}\sqrt{5} + \frac{1}{4}\ln\left(\sqrt{5}+2\right) \approx 1.478\,9$ (wegen $\operatorname{arsinh} y = \sinh^{-1} y = \ln\left(y + \sqrt{y^2+1}\right)$)

b) $y = \cosh x \Rightarrow y' = \sinh x \Rightarrow$
$L = \int_0^1 \sqrt{1+\sinh^2 x} dx = \int_0^1 \cosh x dx = \sinh 1 \approx 1.175\,2$

c)

$y = \sqrt{x}$ ist Umkehrfunktion von $y = x^2$
und damit Spiegelbild von $y = x^2 \Rightarrow$ gleiche Länge.