# Dynamical generation of artificial gauge fields in optical lattices

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International School
Anyon Physics of Ultracold Atomic Gases

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# Plan of the lectures

## Introduction

- Ultracold atoms in optical lattice potentials
- Representation of magnetic fields in tight-binding lattices
- Artificial magnetic fields for neutral atoms in optical lattices

## Quantum engineering in time $H(t+T) = H(t) \Longrightarrow H_{\text{eff}}$

- Quantum Floquet theory
- Perturbative computation of $H_{\text{eff}}$

## Dynamical generation of magnetic fields in tight-binding lattices

- General scheme
- Application 1: Staggered-flux triangular lattice (kinetic frustration)
- Application 2: Engineering the Harper Hamiltonian

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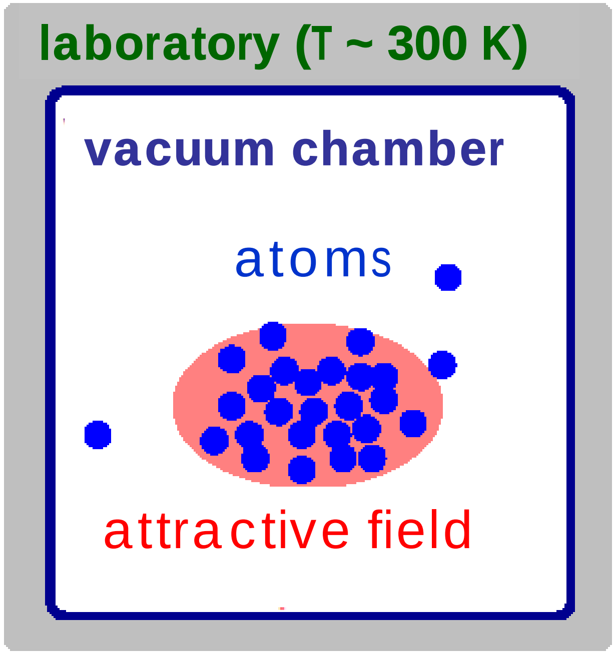
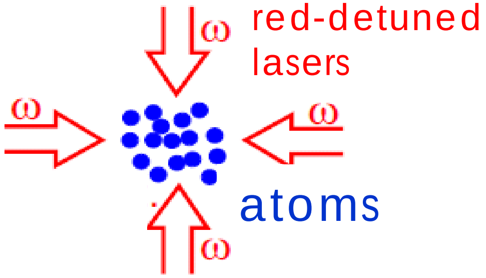
# Ultracold atomic quantum gases

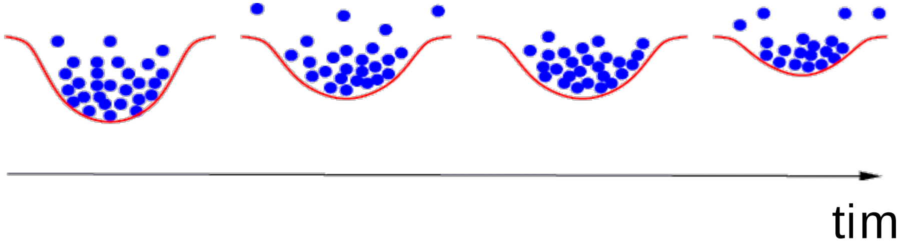
**Trap atoms**



**Laser cooling**



**Evaporative cooling** to quantum degeneracy:



$T \sim$ nano Kelvin

$N \sim 1$ to $10^8$

$\frac{N}{V} \sim 10^{13}$ to $10^{15}$ cm$^{-3}$

(air: $10^{19}$ cm$^{-3}$, solids: $10^{22}$ cm$^{-3}$)

$$T_{\text{degeneracy}} \sim \frac{\hbar^2\pi^2}{k_{\mathrm{B}} m}\left(\frac{N}{V}\right)^{\frac{2}{3}} \sim \left\{ \begin{array}{ll} 10^{-7} \text{ to } 10^{-6}\ \mathrm{K} & \text{trapped atoms} \\ 10^{4}\ \mathrm{K} & \text{electron gas} \\ 1 \text{ to } 10\ \mathrm{K} & \text{liquid helium} \end{array} \right.$$

# Ultracold atomic quantum gases

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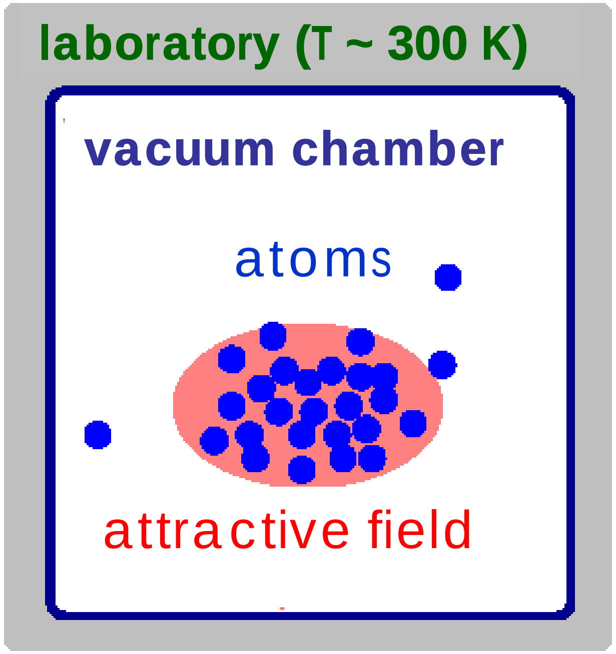

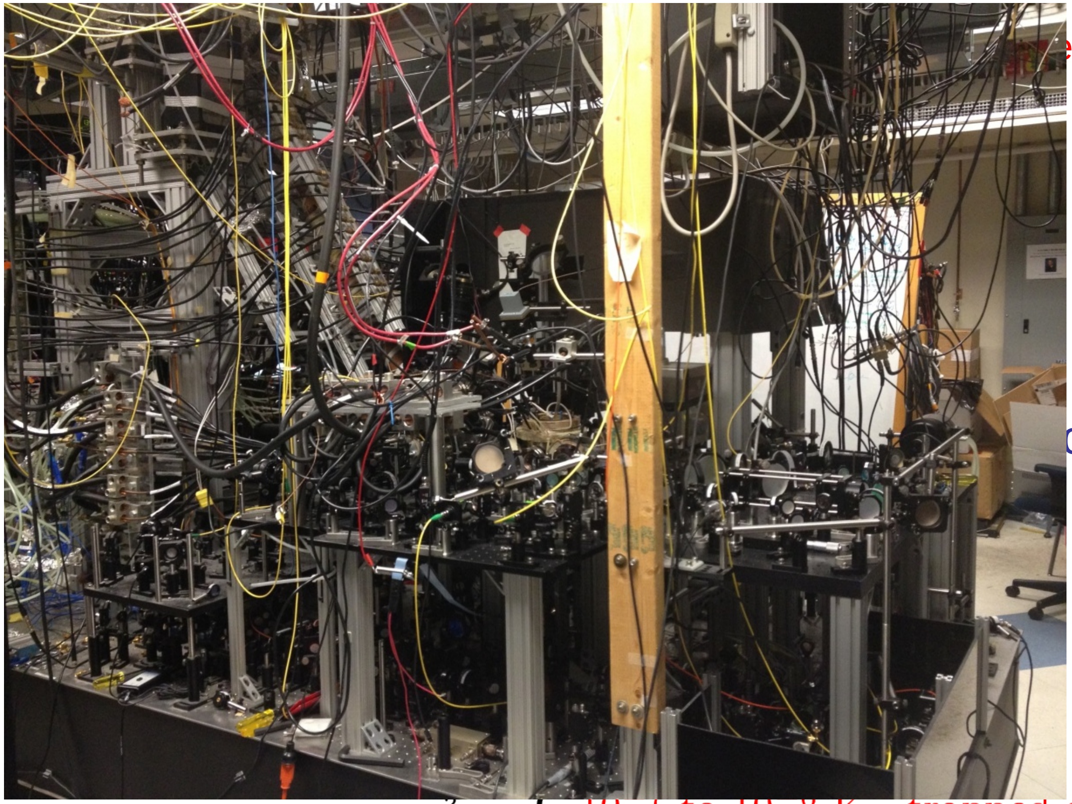

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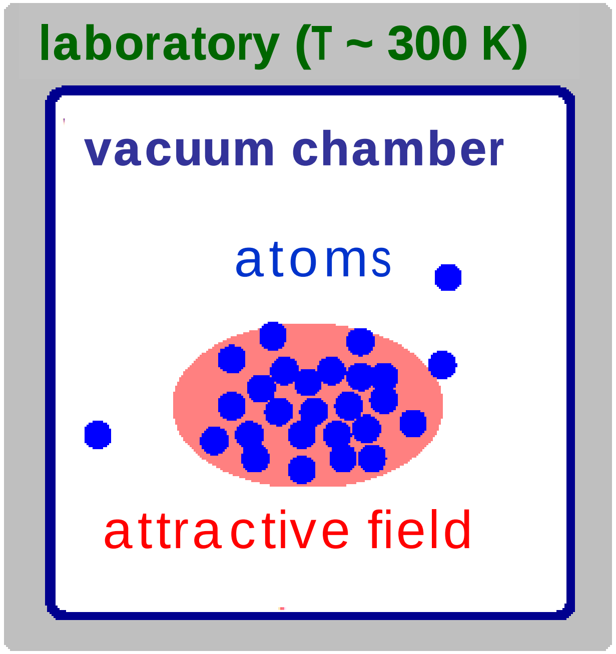

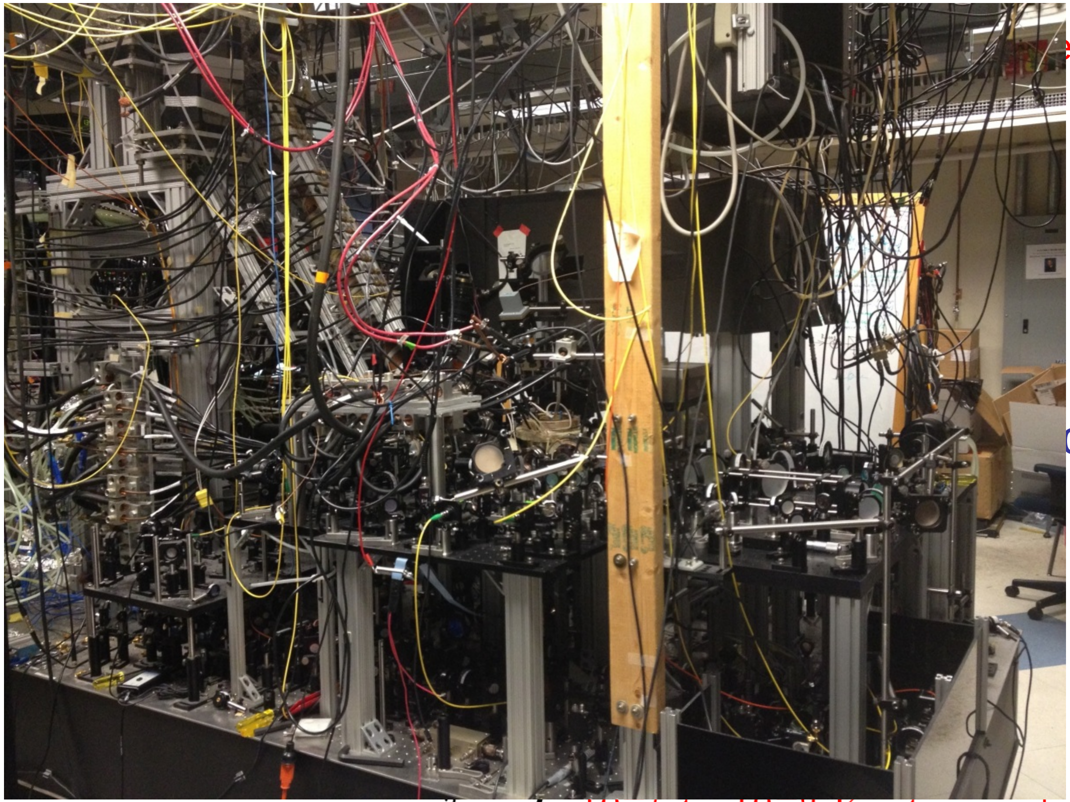

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# Description

Spinless bosons:

$$\hat{H} = \int \mathrm{d}\boldsymbol{r}\, \hat{\psi}^\dagger(\boldsymbol{r}) \left[ \frac{-\hbar^2}{2m} \nabla^2 + V(\boldsymbol{r}) \right] \hat{\psi}(\boldsymbol{r}) + \frac{g}{2} \int \mathrm{d}\boldsymbol{r}\, \hat{\psi}^\dagger(\boldsymbol{r}) \hat{\psi}^\dagger(\boldsymbol{r}) \hat{\psi}(\boldsymbol{r}) \hat{\psi}(\boldsymbol{r})$$

- clean & well isolated from environment
- universal contact interactions $g = \frac{4\pi\hbar^2 a_s}{m}$
- taylorable and controllable, also during experiment

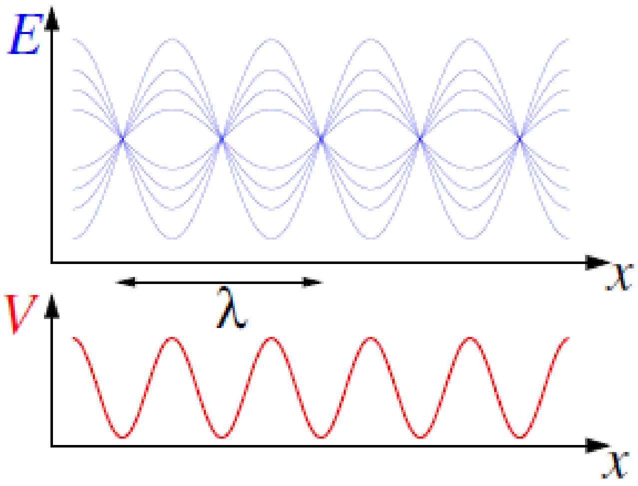
  $V(\boldsymbol{r}) \rightarrow V(\boldsymbol{r}, t)$ $\qquad$ $g \longrightarrow g(t)$

- additional “features” possible

  fermions, spin, dissipation, disorder, ...,

  *artificial magnetic fields, ...*

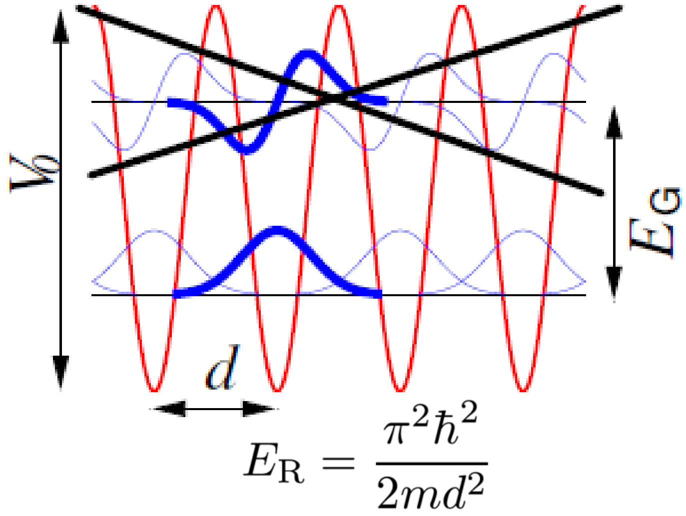
# Optical Lattices



standing light wave

clean periodic potential

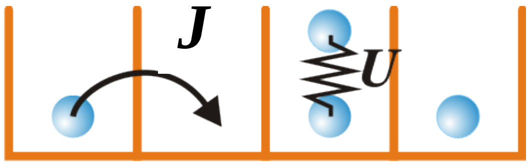
Deep lattices



$$E_{\mathrm{R}} = \frac{\pi^2\hbar^2}{2md^2}$$

# Optical Lattices

**bosons**



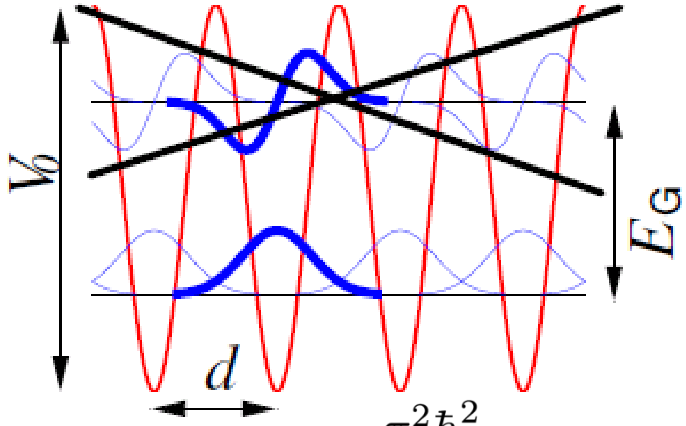
$$\hat{H} = {\color{red}-J} \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + {\color{red}U} \sum_i \hat{n}_i(\hat{n}_i - 1)$$

## Described by Hubbard models

Jaksch et al., PRL (1998)

Ratio ${\color{red}U/J}$ tunable via laser power:

**from weak to strong coupling regime**

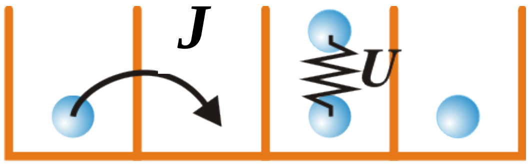


$$E_\mathrm{R} = \frac{\pi^2 \hbar^2}{2md^2}$$

## Deep lattices

# Optical Lattices

**bosons**



$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + U \sum_i \hat{n}_i(\hat{n}_i - 1)$$

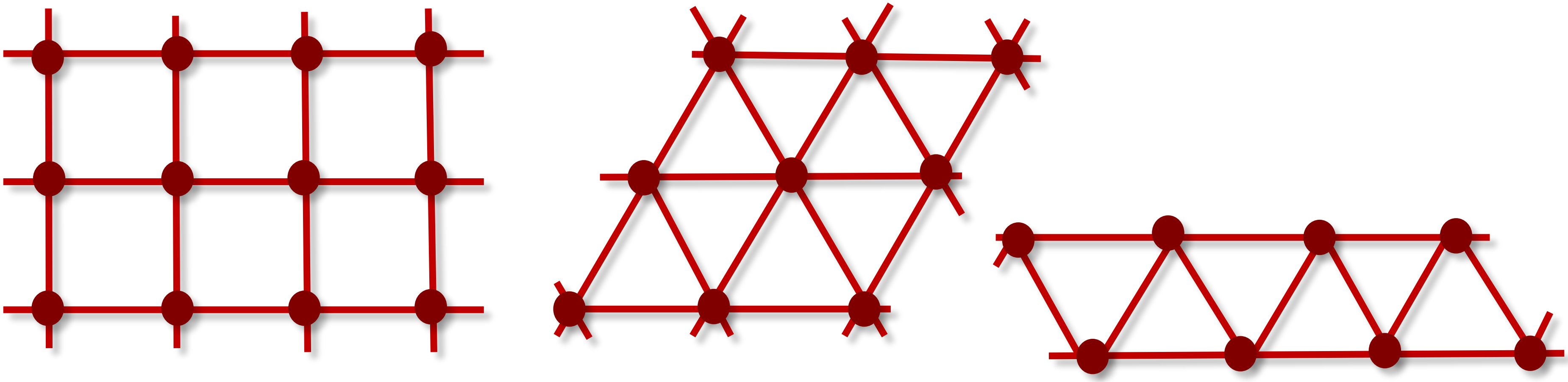
## Described by Hubbard models

Jaksch et al., PRL (1998)

Ratio $U/J$ tunable via laser power:

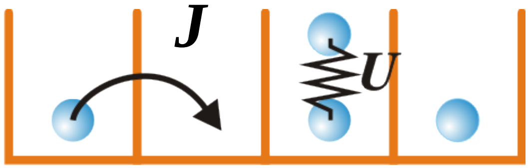
**from weak to strong coupling regime**

## Different lattice geometries / reduction to1D or 2D



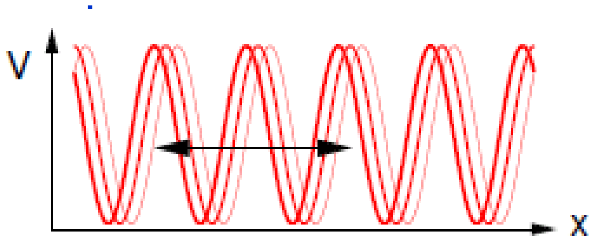
# Cold-atom lattice systems

- **clean & tunable** realizations of minimal many-body models
- **strong interactions** possible
- **well isolated** from environment
- **time-dependent** parameter control
- few-particle correlations directly measurable (single-site resolution)



**=> quantum engineering of many-body systems**

- push boundaries of human control over quantum behavior
- study exotic equilibrium physics
- study coherent many-body quantum dyn
- ...



today:

- **time-periodically driven optical lattices**
- **how to effectively create artificial gauge fields for neutral atoms**

# External fields in tight-binding lattices

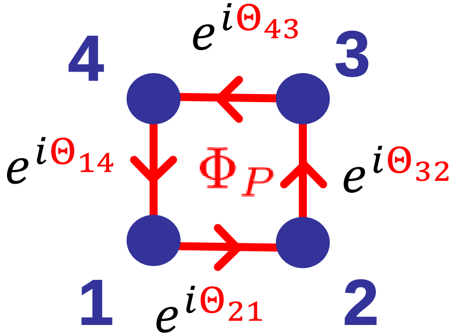
Vector potential $\boldsymbol{A}(\boldsymbol{r})$ represented by Peierls phases $\theta_{ij} = \int_{\text{straight } \boldsymbol{r}_j \to \boldsymbol{r}_i} \mathrm{d}\boldsymbol{r} \cdot \boldsymbol{A}(\boldsymbol{r})$

Scalar potential $V(\boldsymbol{r})$ represented by on-site energies $v_i = V(\boldsymbol{r}_i)$

$$\hat{H} = -J \sum_{\langle ij \rangle} \mathrm{e}^{i\theta_{ij}} \hat{a}_i^\dagger \hat{a}_j + \sum_i v_i \hat{n}_i$$

Constant vector potential: $\hat{H} = -J \sum_{\langle i,j \rangle} \mathrm{e}^{\mathrm{i}(\boldsymbol{r}_i - \boldsymbol{r}_j)\boldsymbol{A}} \hat{a}_i^\dagger \hat{a}_j = \sum_{\boldsymbol{k}} \varepsilon(\boldsymbol{k} - \boldsymbol{A}) \hat{n}_{\boldsymbol{k}}$

Magnetic flux through a lattice plaquette $P$



$$\Phi_P = \theta_{14} + \theta_{43} + \theta_{32} + \theta_{21} = \oint_{\partial_P} \mathrm{d}\boldsymbol{r} \cdot \boldsymbol{A}(\boldsymbol{r}) = \int_{S_P} \mathrm{d}^2 r \, B(\boldsymbol{r})$$

Flux quantum $\Phi_0 = 2\pi$

Invariant under gauge transformations $\hat{U} = \exp\left(\frac{i}{\hbar} \sum_i \chi_i \hat{n}_i\right)$

$$\theta'_{ij} = \theta_{ij} + \hbar^{-1}(\chi_i - \chi_j) \qquad v'_i = v_i - \dot{\chi}_i$$

# Why artificial gauge fields in optical lattices?

- Complete the toolbox for mimicking charged particles

- Reach Quantum Hall regime

  # magnetic flux quanta ~ # particles

- Intriguing interplay between lattice and gauge field
  - strong-field regime (fractal Hofstadter butterfly spectrum relevant)

    # magnetic flux quanta ~ # lattice cells

  - Chern/topological insulators

    gauge-field changes on length scale of the lattice

    → Bloch bands with quantized (spin) Hall conductivity (like Landau level)

- Intriguing interplay with interactions
  - Fractional Quantum Hall effect / Fractional Chern insulators
  - Mimic quantum antiferromagnetism with hard-core bosons

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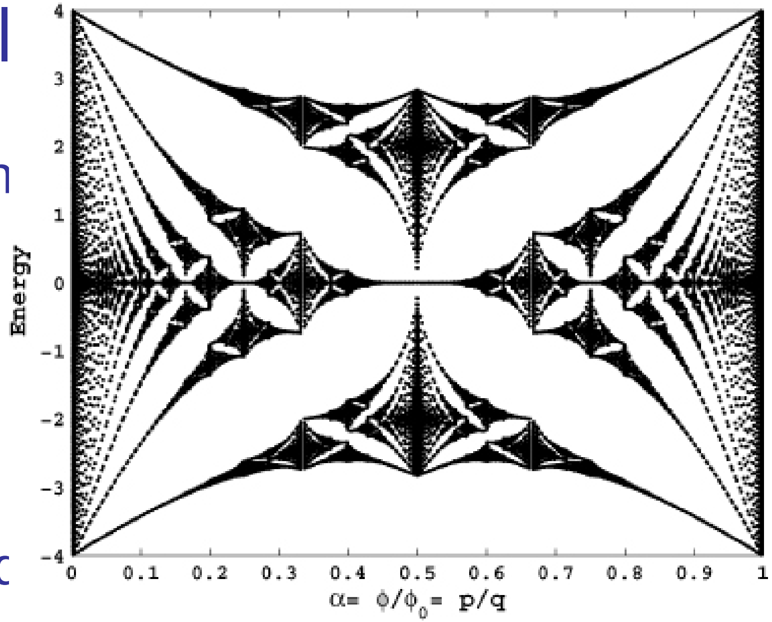

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# How to create artificial gauge fields in optical lattices?

## Using internal atomic structure

**State-dependent lattices**

**+ Laser-assisted tunneling**

- Jaksch & Zoller, NJP 2003
- Mueller, PRA 2004
- Gerbier & Dalibard, NJP 2010

**Optical Flux lattice**

**Lattice and gauge field created on same footing**

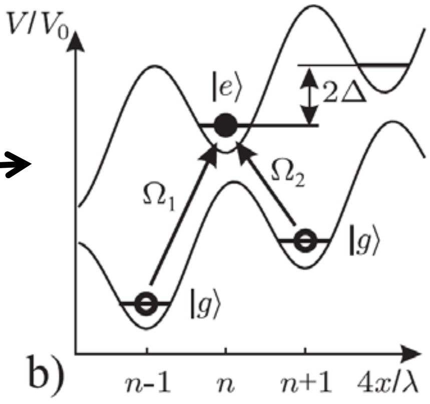
- Cooper PRL 2011
- Dalibard & Cooper, EPL 2011
- Cooper & Moessner, PRL 2012
- Juzeliūnas & Spielman, NJP 2012
- Dalibard & Cooper, PRL 2013

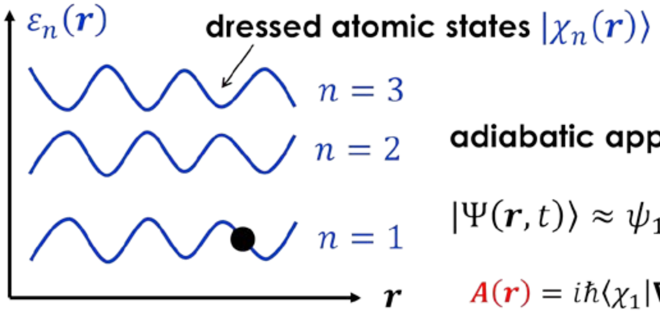
**Proposals for non-abelian gauge fields**

- Osterloh et al. PRL 2005
- … more …

**Experiment: tunable1D gauge potential**

Jiménez-García et al PRL 2012 (Spielman)





**adiabatic approx.:**

$$|\Psi(\boldsymbol{r},t)\rangle \approx \psi_1(\boldsymbol{r},t)|\chi_1(\boldsymbol{r})\rangle$$

$$\boldsymbol{A}(\boldsymbol{r}) = i\hbar\langle\chi_1|\boldsymbol{\nabla}_{\boldsymbol{r}}\chi_1\rangle$$

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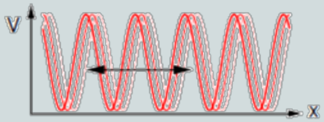
## Dynamically w/o internal structure

### Lattice shaking (kHz-regime)



- EPL **89**, 10010 (2010)
  *π-flux triangular lattice*
- PRL **108**, 225304 (2012 )
  *tunable magnetic fields*
- PRL **109,** 145301 (2012)
  *Chern/topological insulators, non-abelian gauge fields*

### Moving superlattice

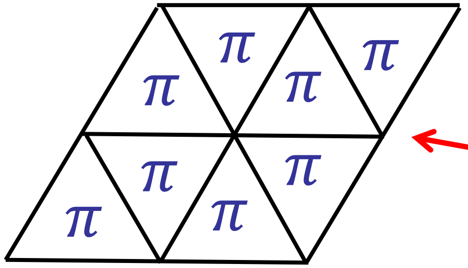
- Kolovsky EPL (2011)
  (tuanble) magnetic *fields*

### Stirring potentials

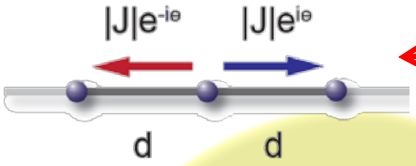
- Lim, Morais Smith & Hemmerich PRL (2008)
  *tunable stagered square lattice*
- Kitagawa et al. PRB (2010)
  *topological insulatior on hexagonal lattice*

# First experiments:

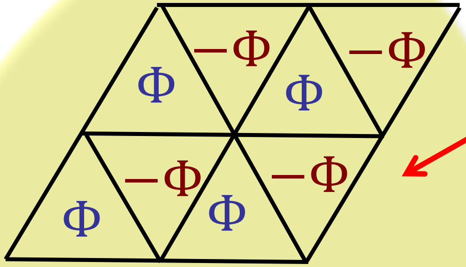
# How to create artificial gauge fields in optical lattices?



Science **333**, 996 (2011)



Nature Phys. (2013)
doi:10.1038/nphys2750



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| $\pi/2$ | $\pi/2$ | $\pi/2$ |
| $\pi/2$ | $\pi/2$ | $\pi/2$ |
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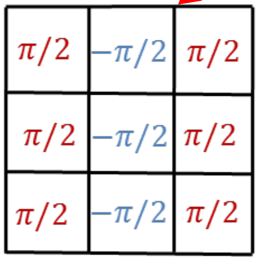
Aidelsburger et al. arXiv:1308.0321
Miyake eta l. arXiv:1308.1431



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- Quantum Floquet theory
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**Dynamical generation of magnetic fields in tight-binding lattices**

- General scheme
- Application 1: Staggered-flux triangular lattice (kinetic frustration)
- Application 2: Engineering the Harper Hamiltonian

# In a nutshell

Time-periodic Hamiltonian (Floquet system)

$$\hat{H}(t+T) = \hat{H}(t)$$

**Effective time-independent Hamiltonian** for time-evolution over one period**:**

$$\hat{U}(T,0) = \mathcal{T} \exp\Big(-\frac{\mathrm{i}}{\hbar}\int_0^T \mathrm{d}t\, \hat{H}(t)\Big) \equiv \exp\big(-\frac{\mathrm{i}}{\hbar}\hat{H}_{\text{eff}}T\big)$$

**Useful? Yes!** If $\hat{H}_{\text{eff}}$ has simple form (at least approximatly)

**Quantum engineering in time:**

*Engineer a time-periodic many-body system that realizes an effective time-independent Hamiltonian of interest!*

# Floquet states

Schrödinger equation with time-periodic Hamiltonian

$$\hat{H}(t+T) = \hat{H}(t)$$

possesses solutions $|\psi_\alpha(t+T)\rangle = \mathrm{e}^{-i\varepsilon_\alpha T/\hbar}|\psi_\alpha(t)\rangle$

equivalently $|\psi_\alpha(t)\rangle = \mathrm{e}^{-i\varepsilon_\alpha t/\hbar}|u_\alpha(t)\rangle \quad \text{with} \quad |u_\alpha(t+T)\rangle = |u_\alpha(t)\rangle$

Floquet state

Quasienergy

Floquet mode

Floquet states form complete orthonormal basis at every time $t$

# Floquet states

**Proof:**

Time evolution operator: $\hat{U}(t_2, t_1) \equiv \mathcal{T} \exp\left(-\frac{i}{\hbar}\int_{t_1}^{t_2} \mathrm{d}t\, \hat{H}(t)\right)$

Monodromy operator: $\hat{U}_T(t) \equiv \hat{U}(t+T, t)$

$$\hat{U}_T(t)|\psi_\alpha(t)\rangle = \underbrace{\mathrm{e}^{-i\varepsilon_\alpha T/\hbar}}_{\lambda_\alpha}|\psi_\alpha(t)\rangle$$

- Eigenstates form complete orthonormal basis (from unitarity)

$$\langle\psi_\alpha(t)|\psi_\beta(t)\rangle = \delta_{\alpha\beta} \qquad \sum_\alpha |\psi_\alpha(t)\rangle\langle\psi_\alpha(t)| = 1$$

- Eigenvalues are phase factors (from unitarity)

$$|\lambda_\alpha(t)| = 1 \quad \text{or} \quad \lambda_\alpha = \mathrm{e}^{i\mu(t)} \quad \text{with} \quad \mu^* = \mu$$

- Eigevalues are independent of *t* (from $\hat{H}(t+T) = \hat{H}(t)$ )

$$\lambda_\alpha(t) = \lambda_\alpha(t') = \lambda_\alpha \equiv \mathrm{e}^{-i\varepsilon_\alpha T/\hbar} \quad \text{with real quasienergy } \varepsilon_\alpha$$

**Eigenstates are Floquet states** $|\psi_\alpha(t+T)\rangle = \mathrm{e}^{-i\varepsilon_\alpha T/\hbar}|\psi_\alpha(t)\rangle$

# Time evolution generated by time-periodic Hamiltonian $\hat{H}(t+T) = \hat{H}(t)$

$$|\psi(t)\rangle = \hat{U}(t,0)|\psi(0)\rangle = \sum_\alpha \underbrace{\langle u_\alpha(0)|\psi(0)\rangle}_{\text{constant } c_\alpha} \mathrm{e}^{-i\varepsilon_\alpha t/\hbar}|u_\alpha(t)\rangle \qquad \text{with} \qquad |u_\alpha(t+T)\rangle = |u_\alpha(t)\rangle$$

- If prepared in Floquet state: purely periodic
- If prepared in superposition of Floquet states:
  stroboscopic time-evolution determined by quasienergies $\varepsilon_\alpha$

$$|\psi(t)\rangle = \hat{U}(\delta t, 0)\exp(-i\hat{H}_{\text{eff}}\, nT/\hbar)|\psi(0)\rangle \quad \text{with} \quad t = nT + \delta t \quad \text{and} \quad n = \lfloor t/T \rfloor$$

Time evolution
on short times
within one period
(micromotion)

Long-time
behavior

$$\hat{U}(nT,0) = [\hat{U}(T,0)]^n$$

$$\hat{U}(T,0) \equiv \exp(-i\hat{H}_{\text{eff}}\, T/\hbar)$$

$$\hat{U}(\delta t,0) = \sum_\alpha \mathrm{e}^{-i\varepsilon_\alpha\, \delta t/\hbar}\, |u_\alpha(\delta t)\rangle\langle u_\alpha(0)|$$

$$\hat{H}_{\text{eff}} \equiv \sum_\alpha \mathrm{e}^{-i\varepsilon_\alpha T/\hbar}\, |u_\alpha(0)\rangle\langle u_\alpha(0)|$$

How to compute Floquet states and quasienergy practically?

numerically ?

analytic approximations?

# Eigenvalue problem of monodromy operator

$$\hat{U}(T)|\psi_\alpha(0)\rangle = \mathrm{e}^{-\mathrm{i}\varepsilon_\alpha T/\hbar}\,|\psi_\alpha(0)\rangle$$

use together with $$|\psi_\alpha(t)\rangle = \hat{U}(t,0)|\psi_\alpha(0)\rangle$$

$$|\psi_\alpha(t)\rangle = \mathrm{e}^{-i\varepsilon_\alpha t/\hbar}|u_\alpha(t)\rangle \quad \text{with} \quad |u_\alpha(t+T)\rangle = |u_\alpha(t)\rangle$$

**Usefull for numerical computation of small systems:**

- Compute $\hat{U}(T,0)$ by integrating the time-evolution $\hat{U}(T,0)|\xi_\alpha\rangle$ for a complete set of basis states $\langle\xi_\beta|\hat{U}(T,0)|\xi_\alpha\rangle$

- Diagonalize $\langle\xi_\beta|\hat{U}(T,0)|\xi_\alpha\rangle$ **fully**

# Quasienergy eigenvalue problem

[Sambe PRA (1973)]

Ambiguity in definition of Floquet modes $|u_\alpha(t)\rangle$ (here $\omega = \frac{2\pi}{T}$)

$$|\psi_\alpha(t)\rangle = |u_\alpha(t)\rangle \mathrm{e}^{-\mathrm{i}\varepsilon_\alpha t/\hbar} = \underbrace{|u_\alpha(t)\rangle \mathrm{e}^{\mathrm{i}m\omega t}}_{|u_{\alpha m}(t)\rangle}\, \mathrm{e}^{-\mathrm{i}\overbrace{(\varepsilon_\alpha + m\hbar\omega)}^{\varepsilon_{\alpha m}} t/\hbar}$$

Hermitian quasienergy eigenvalue problem **(time plays role of a coordinate)**

$$\underbrace{[\hat{H}(t) - i\hbar\partial_t]}_{\hat{\hat{Q}}}\, |u_{\alpha m}\rangle\rangle = \varepsilon_{\alpha m}|u_{\alpha m}\rangle\rangle$$

Quasienergy operator

extended space = state space $\otimes$ $T$-periodic functions

$$\langle\langle\cdot|\cdot\rangle\rangle = \frac{1}{T}\int_0^T \mathrm{d}t\, \langle\cdot|\cdot\rangle$$

$$\{\mathrm{e}^{\mathrm{i}m\omega t}\}$$

− drastically enlarged Hilbert space

\+ *Stationary* perturbation theory applicable

\+ Adiabatic principle works

\+ Intuitive Framework for resonance effects

# The Floquet Picture

[Breuer & Holthaus, Phys. Lett. A **140**, 507 (1989)]

Arbitrary time-dependent Hamiltonian $\hat{H}(t) = \hat{H}^{P(t)}(t)$ with $\hat{H}^{P}(t) = \hat{H}^{P}(t+T)$

e.g.: $\hat{H}^{P}(t) = \hat{H}_0 + P\cos(\omega t)\hat{V}$

## Two-times formalism

Consider generalized Schrödinger equation in extended Hilbert space:

$$i\hbar\partial_\tau|\Psi(\tau;t)\rangle\rangle = \hat{\hat{Q}}(\tau;t)|\Psi(\tau;t)\rangle\rangle \qquad \hat{\hat{Q}}(\tau;t) = [\hat{H}^{P(\tau)}(t) - i\hbar\partial_t]$$

Project back to original state space:

$$|\psi(t)\rangle = |\Psi(\tau;t)\rangle\rangle\Big|_{\tau=t}$$

$$\begin{aligned} \mathrm{i}\hbar\partial_t|\psi(t)\rangle &= (\mathrm{i}\hbar\partial_\tau + \mathrm{i}\hbar\partial_t)|\Psi(\tau;t)\rangle\rangle\Big|_{\tau=t} \\ &= (\hat{Q}(\tau;t) + \mathrm{i}\hbar\partial_t)|\Psi(\tau;t)\rangle\rangle\Big|_{\tau=t} \\ &= \hat{H}^{P(t)}(t)|\psi(t)\rangle \end{aligned}$$

**Use tools and intuition of non-driven systems**

**Stationary perturbation theory for eigenvalue problem of** $\hat{Q}$

**Adiabatic principle for parameter variation**

# Perturbation theory for effective Hamiltonian

## Quasienegy eigenvalue problem

$$\underbrace{[\hat{H}(t) - i\hbar\partial_t]}_{\hat{\hat{Q}}}|u_{\alpha m}\rangle\rangle = \varepsilon_{\alpha m}|u_{\alpha m}\rangle\rangle$$

$$\langle\langle\cdot|\cdot\rangle\rangle = \frac{1}{T}\int_0^T \mathrm{d}t\,\langle\cdot|\cdot\rangle$$

$$\text{extended space} = \text{state space} \otimes T\text{-periodic functions}$$

Strategy for choosing $\hat{U}_T^{(0)}(t)$

Integrate out large terms $\sim\hbar\omega$

Appropriately chosen basis $|n,m\rangle\rangle \equiv \mathrm{e}^{im\omega t}\hat{U}_T^{(0)}(t)|n\rangle$

$$\hat{V}_{\Delta m} = \frac{1}{T}\int_0^T \mathrm{d}t\,\mathrm{e}^{-i\Delta m\omega t}\hat{U}_T^{(0)}(t)^\dagger\,[\hat{H}(t) - i\hbar\partial_t]\,\hat{U}_T^{(0)}(t)$$

$$\hat{\hat{Q}} = \begin{array}{cccc|c} -1 & 0 & 1 & 2 & m \\ \ddots & & & & \\ \hat{V}_0 - \hbar\omega & \hat{V}_1 & \hat{V}_2 & \iddots & -1 \\ \hat{V}_{-1} & \hat{V}_0 & \hat{V}_1 & \hat{V}_2 & 0 \\ \hat{V}_{-2} & \hat{V}_{-1} & \hat{V}_0 + \hbar\omega & \hat{V}_1 & 1 \\ \iddots & \hat{V}_{-2} & \hat{V}_{-1} & \hat{V}_0 + 2\hbar\omega & 2 \\ & & & \ddots & \end{array}$$

$m$ plays role of a ``photon'' number

If “ $\hat{V}_0, \hat{V}_{\Delta m\neq 0} \ll \hbar\omega$ ”

neglect off-diagonal blocks $\hat{V}_{\Delta m\neq 0}$

⇒ **effective Hamiltonian**

$$\hat{H}_{\text{eff}} \simeq \hat{V}_0$$

(1st order degenerate perturbation theory, systematic corrections from higher orders)

# Plan of the lectures

**Introduction**

- Ultracold atoms in optical lattice potentials
- Representation of magnetic fields in tight-binding lattices
- Artificial magnetic fields for neutral atoms in optical lattices

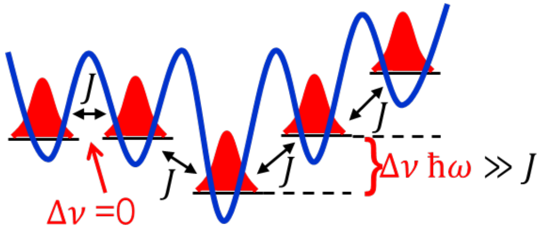
**Quantum engineering in time** $H(t+T) = H(t) \Longrightarrow \; H_{\text{eff}}$

- Quantum Floquet theory
- Perturbative computation of $H_{\text{eff}}$

**Dynamical generation of magnetic fields in tight-binding lattices**

- General scheme
- Application 1: Staggered-flux triangular lattice (kinetic frustration)
- Application 2: Engineering the Harper Hamiltonian

# Basic scheme for generation of gauge fields



Hubbard Hamiltonian with periodic driving

$$\hat{H}(t) = -\sum_{\langle ij\rangle} J_{ij}\hat{a}_i^\dagger \hat{a}_j + \sum_i [v_i^\omega(t) + \nu_i \hbar\omega]\hat{n}_i + \hat{H}_{\text{on-site}}$$

tunneling

periodic driving

possible static tilt

weak trap, interactions,

Unitary transformation (interaction picture / change of gauge)

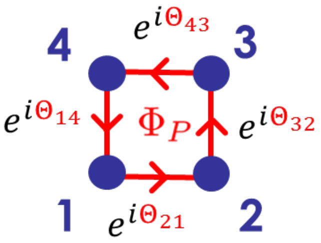
$$\hat{H}'(t) = \hat{U}^\dagger \hat{H}\hat{U} - \mathrm{i}\hbar\hat{U}^\dagger(\mathrm{d}_t\hat{U}) \qquad \hat{U} = \exp\left(\mathrm{i}\sum_i \chi_i(t)\hat{n}_i\right) \qquad \chi_i(t) = -\int_0^t \mathrm{d}t'\, v_i(t') - \nu_i\omega t$$

$$\hat{H}'(t) = -\sum_{\langle ij\rangle} \mathrm{e}^{-\mathrm{i}[\chi_i(t)-\chi_j(t)]} J_{ij}\hat{a}_i^\dagger \hat{a}_j + \hat{H}_{\text{on-site}}$$

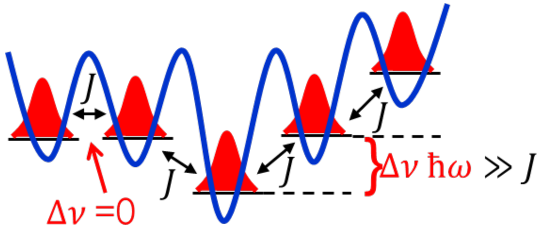
Effective tunneling matrix elements (can be complex)

$$\boxed{J_{ij}^{\text{eff}} = J\langle \mathrm{e}^{\mathrm{i}[\chi_i(t)-\chi_j(t)]}\rangle_T \equiv |J_{ij}^{\text{eff}}|\mathrm{e}^{\mathrm{i}\theta_{ij}}}$$

# Basic scheme for generation of gauge fields



$$\Phi_P = \theta_{14} + \theta_{43} + \theta_{32} + \theta_{21}$$

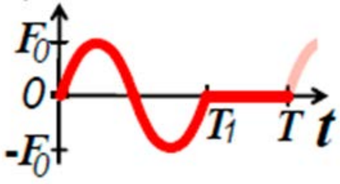


Plaquette fluxes $\Phi_P = 0, \pi$ (time-revresal symmetry not broken)

if *global* reflection symmetry $v_i^\omega(t-\tau) = v_i^\omega(-t-\tau)$

$$v_{ij}^\omega = v_i^\omega - v_j^\omega$$

if the $\nu_i = 0$ and *local* reflection symmetry $v_{ij}^\omega(t-\tau_{ij}) = v_{ij}^\omega(-t-\tau_{ij})$



or shift antisymmtry $v_i^\omega(t) = -v_i^\omega(t-T/2)$

$$J_{ij}^{\mathrm{eff}} = J\langle \mathrm{e}^{\mathrm{i}[\chi_i(t)-\chi_j(t)]}\rangle_T \equiv |J_{ij}^{\mathrm{eff}}|\mathrm{e}^{\mathrm{i}\theta_{ij}}$$

These symmtries also prevent ratchet-type rectification

Flach et al. PRL **84**, 2358 (2000),
Denisov et al. PRA **75**, 063424 (2007).

$$\chi_i(t) = -\int_0^t \mathrm{d}t'\, v_i(t') - \nu_i\omega t$$

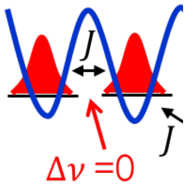
# Basic scheme for generation of gauge fields

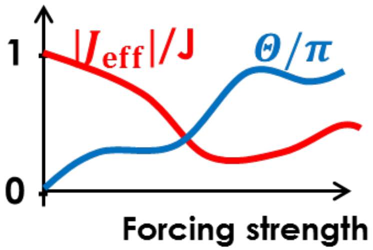
$$\hat{H}(t) = -\sum_{\langle ij \rangle} J_{ij} \hat{a}_i^\dagger \hat{a}_j + \sum_i [v_i^\omega(t) + \nu_i \hbar\omega] \hat{n}_i + \hat{H}_{\text{on-site}}$$

$$\hat{H}_{\text{eff}} = -\sum_{\langle ij \rangle} J_{ij}^{\text{eff}} \hat{a}_i^\dagger \hat{a}_j + \hat{H}_{\text{on-site}}$$

$$J_{ij}^{\text{eff}} = |J_{ij}^{\text{eff}}| \mathrm{e}^{i\theta_{ij}}$$

## Case 1: AC-modified tunneling (no off-sets $\nu_i - \nu_j = 0$)

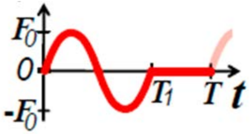




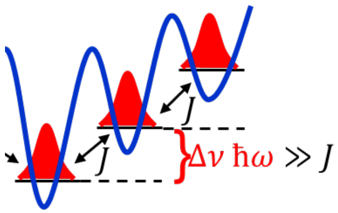
Plaquette fluxes $\Phi_P \neq 0, \pi$ requires to break

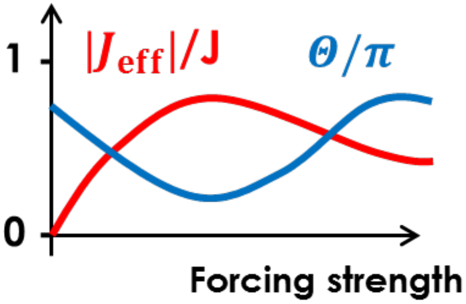
$$v_{ij}^\omega(t - \tau_{ij}) = v_{ij}^\omega(-t - \tau_{ij})$$

$$v_i^\omega(t) = -v_i^\omega(t - T/2)$$



## Case 2: AC-induced tunneling (strong off-sets $\nu_i - \nu_j \neq 0$)





Plaquette fluxes $\Phi_P \neq 0, \pi$ requires to break

$$v_i^\omega(t - \tau) = v_i^\omega(-t - \tau)$$

Easier to break, e.g. a moving Superlattice is enough

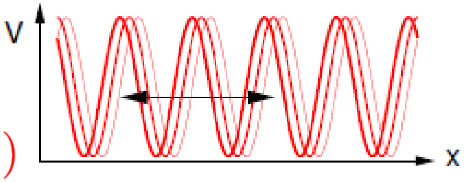
# AC-modified tunneling via lattice shaking

$$\hat{H}(t) = -J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger\hat{a}_j + \sum_i v_i(t)\hat{n}_i + \hat{H}_{\text{on-site}}$$

inertial force

$$\boldsymbol{F}(t) = -m\ddot{\boldsymbol{x}}(t)$$

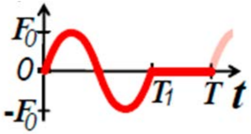
$$v_i^\omega(t) = -\boldsymbol{r}_i\cdot\boldsymbol{F}(t)$$



$$\hat{H}_{\text{eff}} = -\sum_{\langle ij\rangle} J_{ij}^{\text{eff}}\hat{a}_i^\dagger\hat{a}_j + \hat{H}_{\text{on-site}}$$

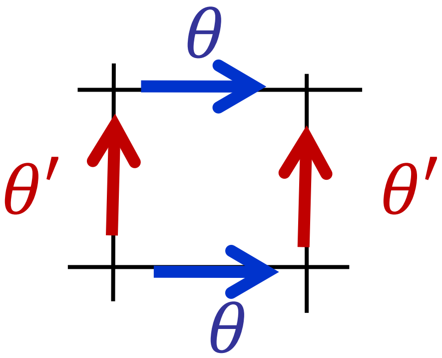
$$J_{ij}^{\text{eff}} = |J_{ij}^{\text{eff}}|\mathrm{e}^{i\theta_{ij}}$$

Break symmetry



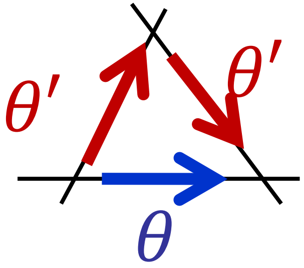
**Square plaquettes remain trivial**

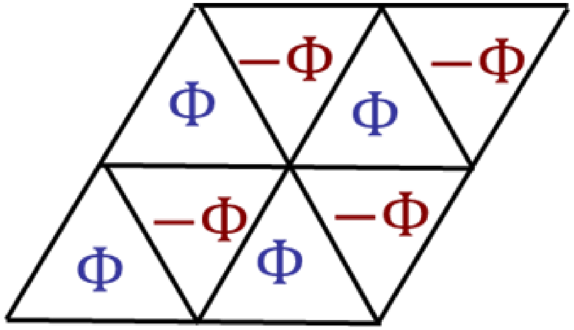
$$\Phi_P = \theta + \theta' - \theta - \theta' = 0$$



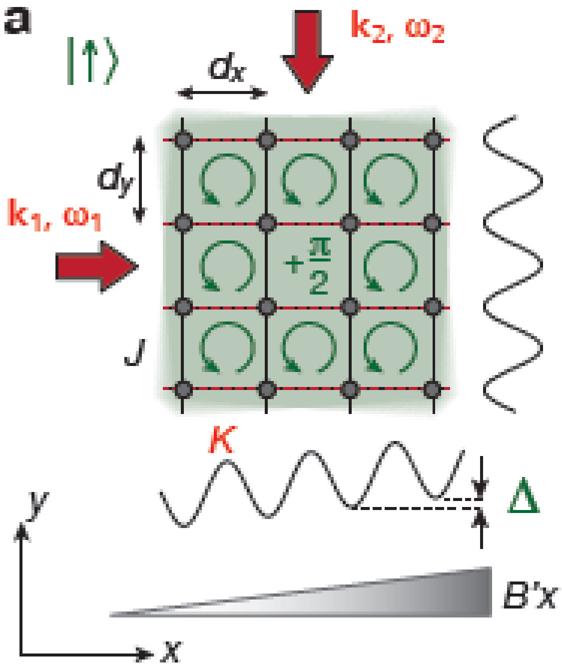
**Triangular plaquette flux tunable**

$$\Phi_P = \theta - 2\theta' \neq 0$$

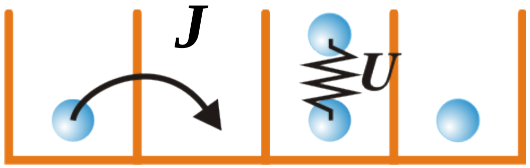




# AC-induced tunneling in tilted lattice via moving superlattice (Kolovsky proposal & Bloch/Ketterle experiments)



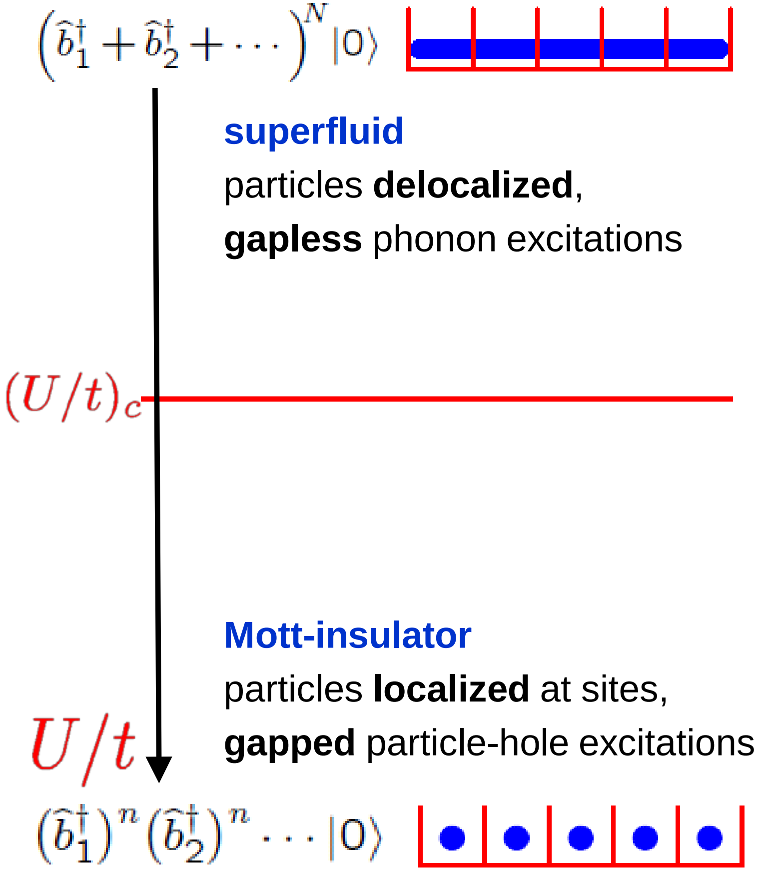
# Dynamically induced quantum phase transition



$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

**bosonic ground state:**

MPA Fisher et al., PRB (1989),
for cold atoms: Jaksch et al., PRL (1998)



$$\left(\hat{b}_1^\dagger + \hat{b}_2^\dagger + \cdots\right)^N |0\rangle$$

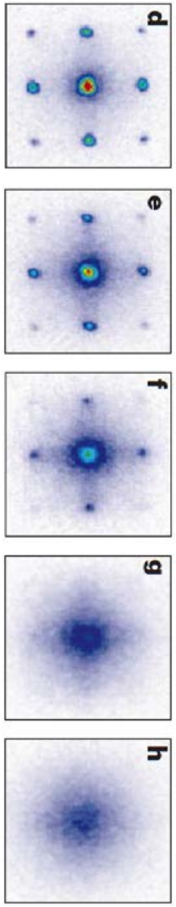
**superfluid**
particles **delocalized**,
**gapless** phonon excitations

$(U/t)_c$

**Mott-insulator**
particles **localized** at sites,
**gapped** particle-hole excitations

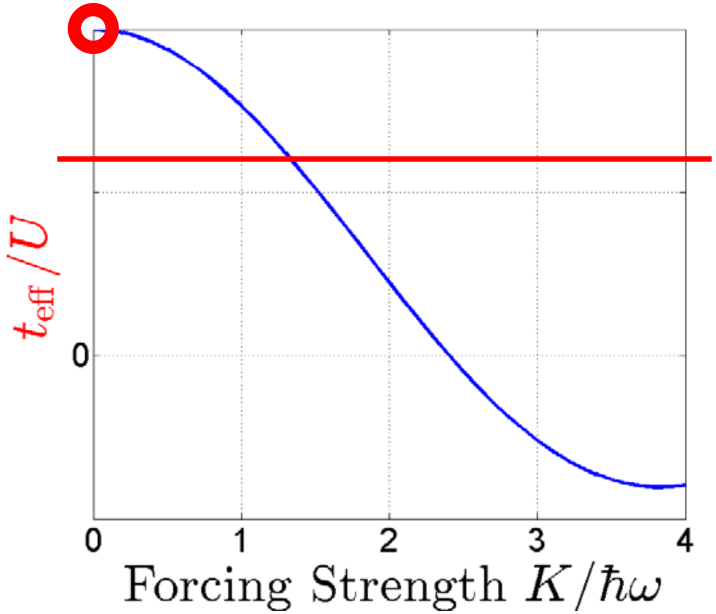
$U/t$

$$(\hat{b}_1^\dagger)^n (\hat{b}_2^\dagger)^n \cdots |0\rangle$$



Greiner et al., Nature (2002)

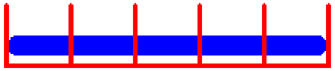
# Dynamically induced quantum phase transition



**bosonic ground state:**

MPA Fisher et al., PRB (1989),
for cold atoms: Jaksch et al., PRL (1998)

$\left(\hat{b}_1^\dagger + \hat{b}_2^\dagger + \cdots\right)^N |0\rangle$

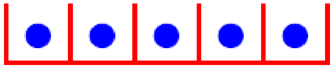


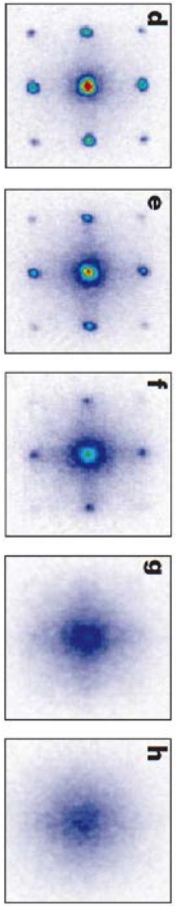
**superfluid**
particles **delocalized**,
**gapless** phonon excitations

$(U/t)_c$

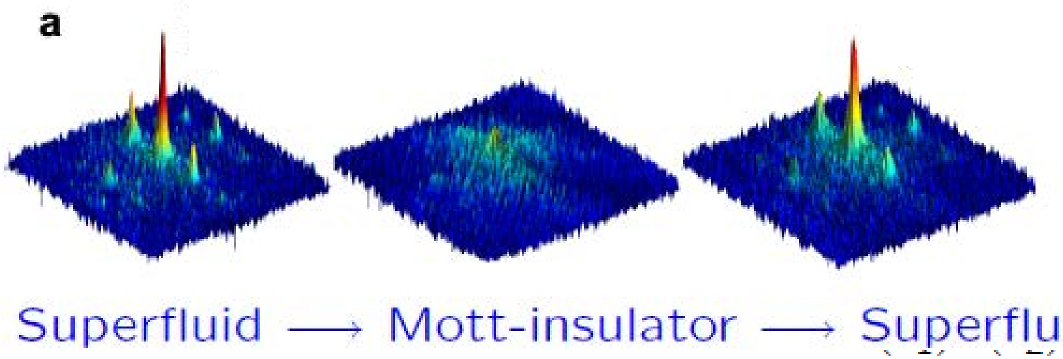
**tt-insulator**
ticles **localized** at sites,
**ped** particle-hole excitations

$\cdot\cdot|0\rangle$





Greiner et al., Nature (2002)



experiment: Zenesini et al., PRL (2009)
proposal: Eckardt et al., PRL (2005)

# Perturbation theory for effective Hamiltonian

## Quasienegy eigenvalue problem

$$\underbrace{[\hat{H}(t) - i\hbar\partial_t]}_{\hat{\hat{Q}}} |u_{\alpha m}\rangle\rangle = \varepsilon_{\alpha m} |u_{\alpha m}\rangle\rangle$$

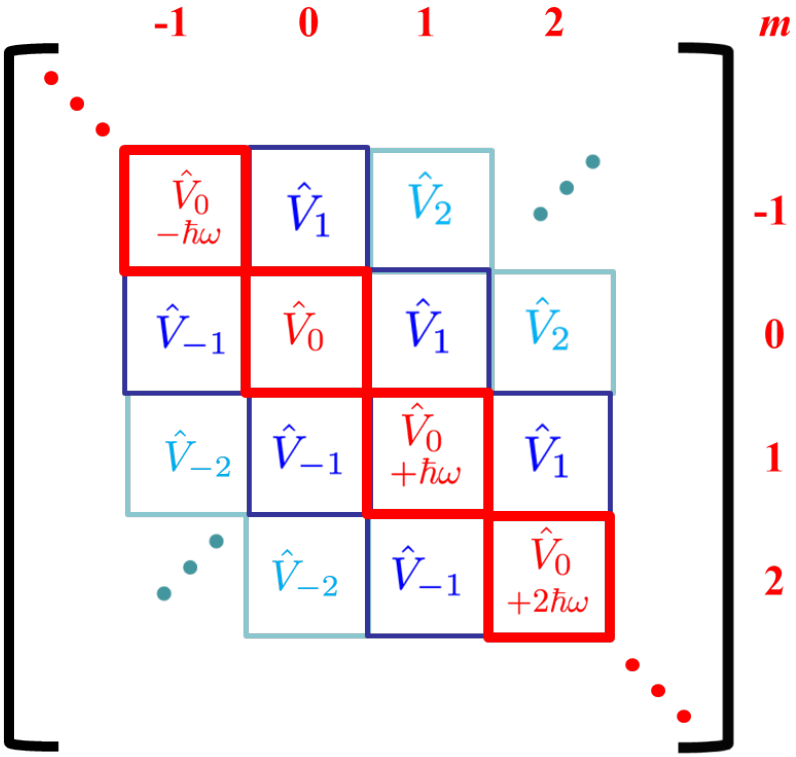
$$\langle\langle\cdot|\cdot\rangle\rangle = \frac{1}{T}\int_0^T \mathrm{d}t\, \langle\cdot|\cdot\rangle$$

extended space = state space $\otimes$ $T$-periodic functions

**Strategy for choosing $\hat{U}_T^{(0)}(t)$**
Integrate out large terms $\sim\hbar\omega$

Appropriately chosen basis $|n, m\rangle\rangle \equiv \mathrm{e}^{im\omega t}\hat{U}_T^{(0)}(t)|n\rangle$

$$\hat{V}_{\Delta m} = \frac{1}{T}\int_0^T \mathrm{d}t\, \mathrm{e}^{-i\Delta m\omega t}\hat{U}_T^{(0)}(t)^\dagger\, [\hat{H}(t) - i\hbar\partial_t]\, \hat{U}_T^{(0)}(t)$$



$$\hat{\hat{Q}} = \left[\begin{array}{ccccc|c} & -1 & 0 & 1 & 2 & m \\ \ddots & & & & & \\ & \hat{V}_0 - \hbar\omega & \hat{V}_1 & \hat{V}_2 & \iddots & -1 \\ & \hat{V}_{-1} & \hat{V}_0 & \hat{V}_1 & \hat{V}_2 & 0 \\ & \hat{V}_{-2} & \hat{V}_{-1} & \hat{V}_0 + \hbar\omega & \hat{V}_1 & 1 \\ & \iddots & \hat{V}_{-2} & \hat{V}_{-1} & \hat{V}_0 + 2\hbar\omega & 2 \\ & & & & \ddots & \end{array}\right]$$

$m$ plays role of a ``photon'' number

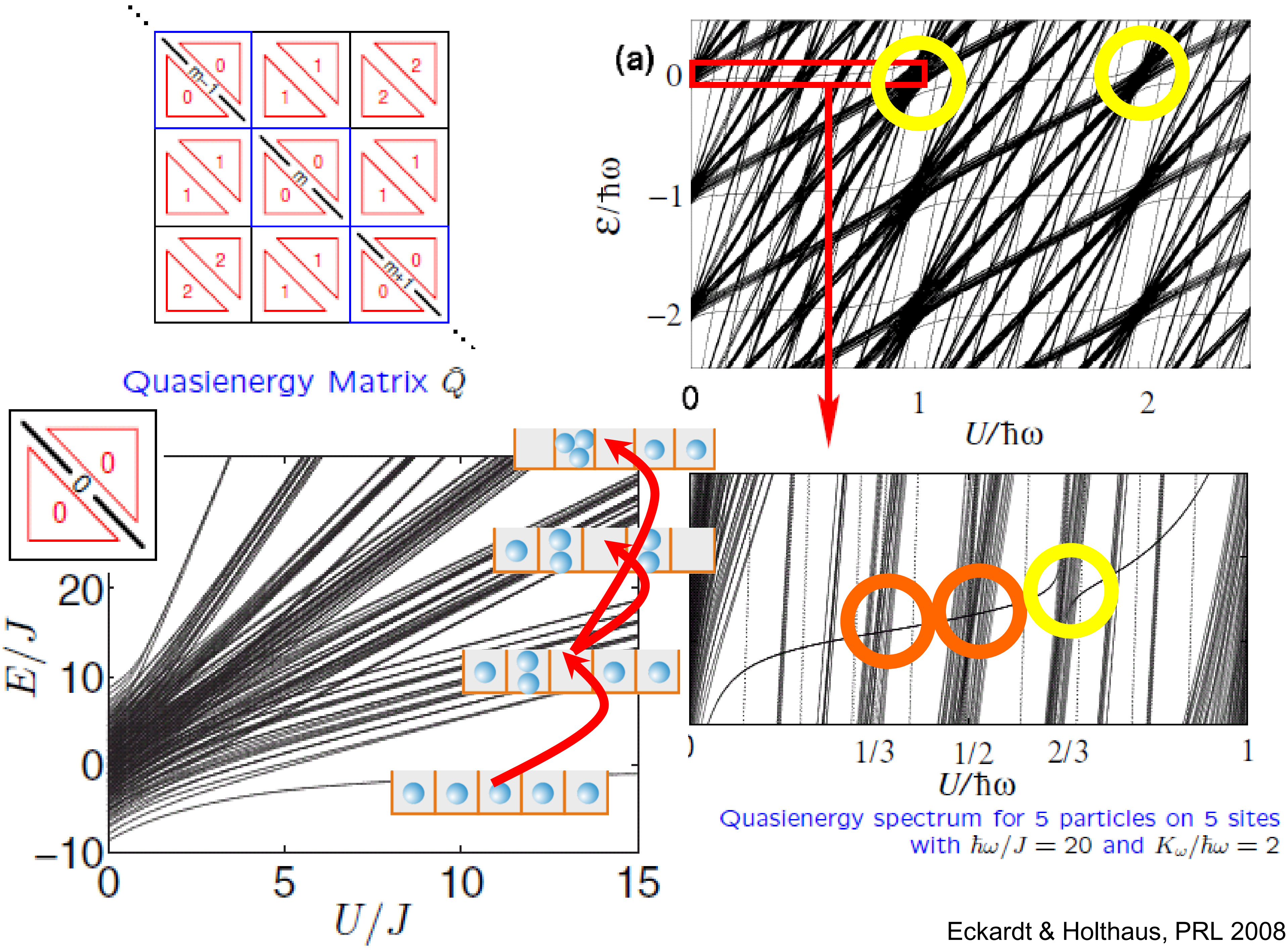
If “ $\hat{V}_0, \hat{V}_{\Delta m\neq 0} \ll \hbar\omega$ ”

neglect off-diagonal blocks $\hat{V}_{\Delta m\neq 0}$

$\Rightarrow$ **effective Hamiltonian**

$$\hat{H}_{\mathrm{eff}} \simeq \hat{V}_0$$

(1st order degenerate perturbation theory,
systematic corrections from higher orders)



Quasienergy Matrix $\hat{Q}$

Quasienergy spectrum for 5 particles on 5 sites with $\hbar\omega/J = 20$ and $K_\omega/\hbar\omega = 2$

Eckardt & Holthaus, PRL 2008

# Dynamically induced frustration in a triangular lattice

Joint work with experimentalists from Sengstock group in Hamburg

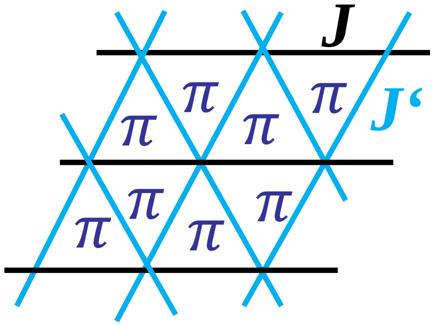
Eckardt et al. EPL 2010

Struck et al., Science 2011

# Shaken triangular lattice

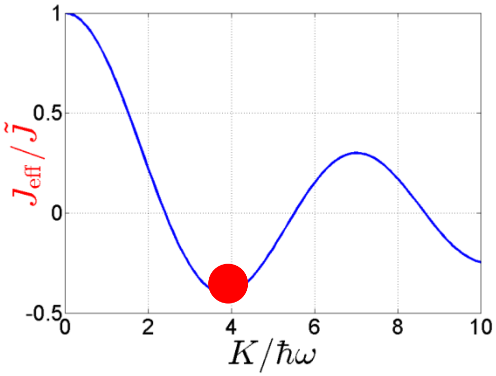
## Elliptically shaken triangular lattice

$$\hat{H}_{\text{eff}} = -\sum_{\langle ij\rangle}\left(J^{\text{eff}}_{ij}\hat{b}^\dagger_i\hat{b}_j + \text{ h.c. }\right) + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1)$$



Frustrated kinetics for $-J^{\text{eff}}_{ij} > 0$

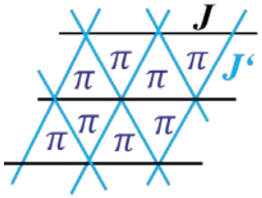
$$J^{\text{eff}}_{ij} \equiv \tilde{J}\,\mathrm{J}_0\left(\frac{K_{ij}}{\hbar\omega}\right)$$



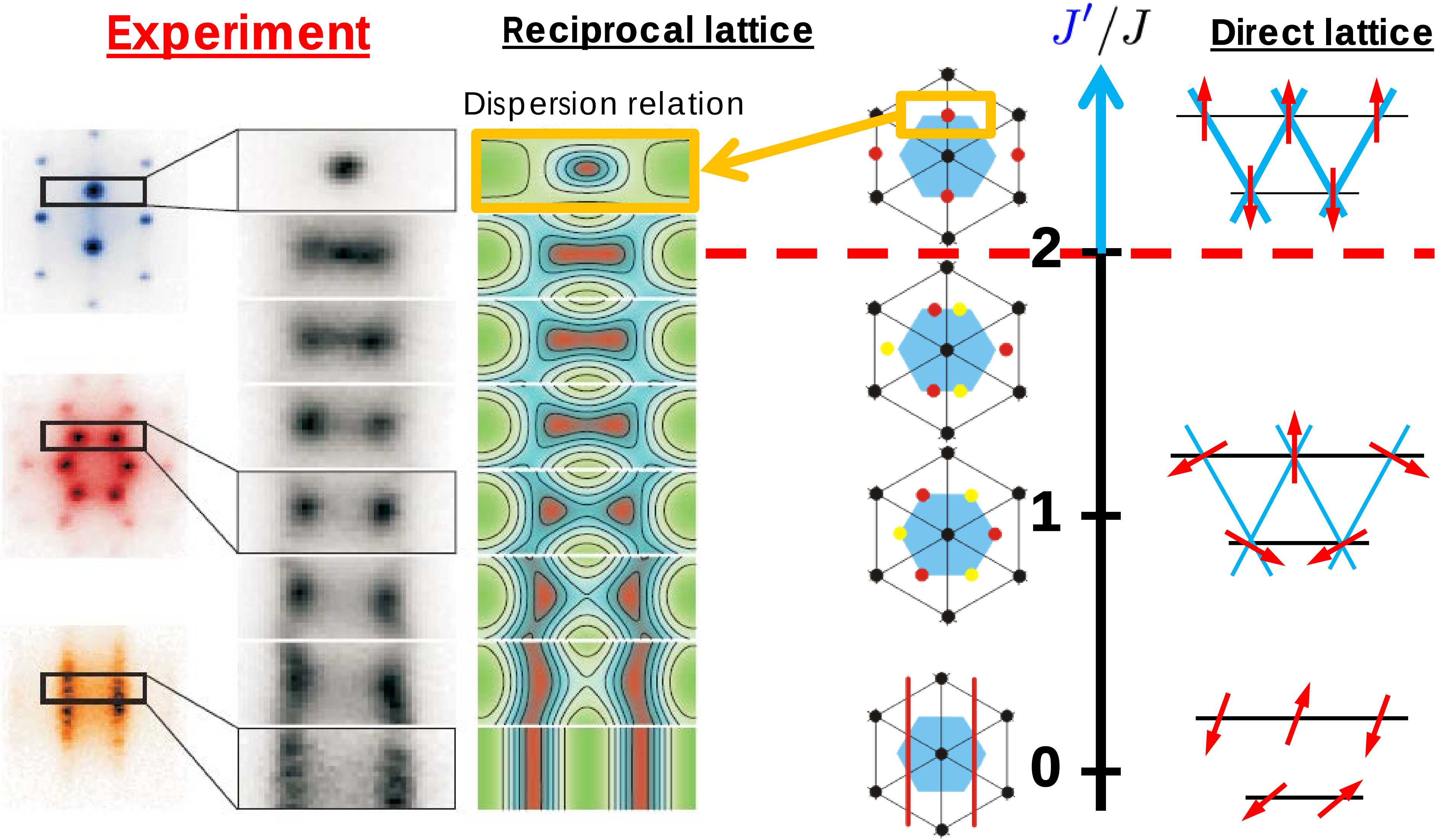
# Limit of weak interactions

Condensate with wave function: $\psi_\ell = \sqrt{n}\exp(\mathrm{i}\varphi_\ell)$

$\varphi_\ell$ resemble **classical rotors** with **antiferromagnetic** coupling



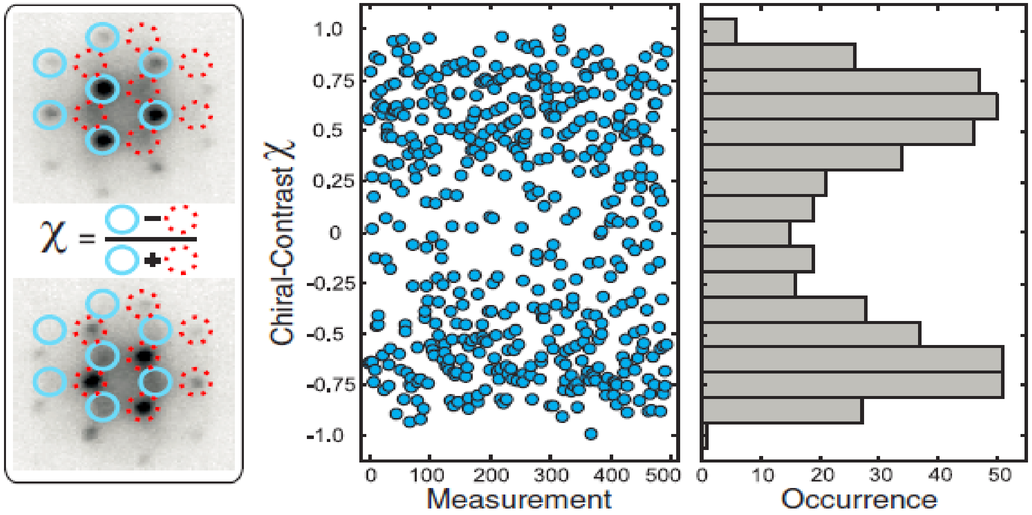
**Experiment** **Reciprocal lattice** **Direct lattice**



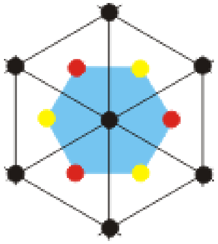
# Spontaneous breaking of time-reversal symmetry

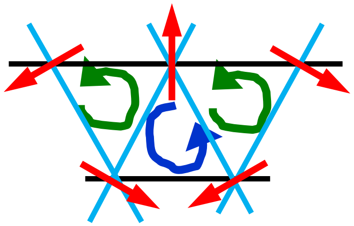
Condensate with wave function: $\psi_\ell = \sqrt{n}\,\exp(\mathrm{i}\varphi_\ell)$

$\varphi_\ell$ resemble **classical rotors** with **antiferromagnetic** coupling



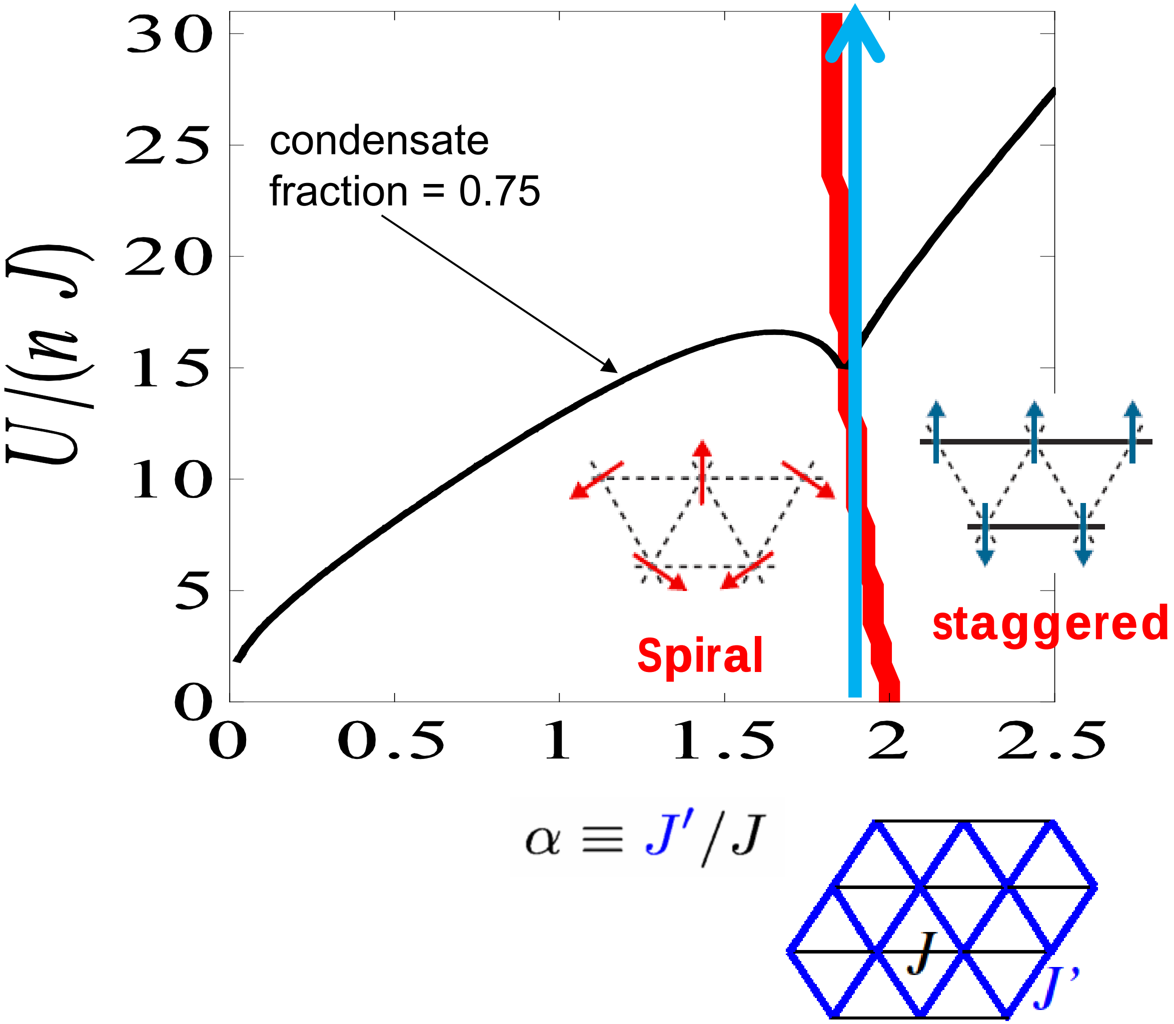
**Circular plaquette currents**

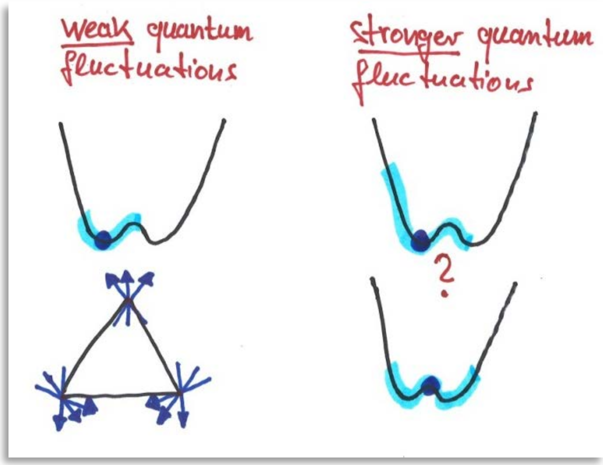


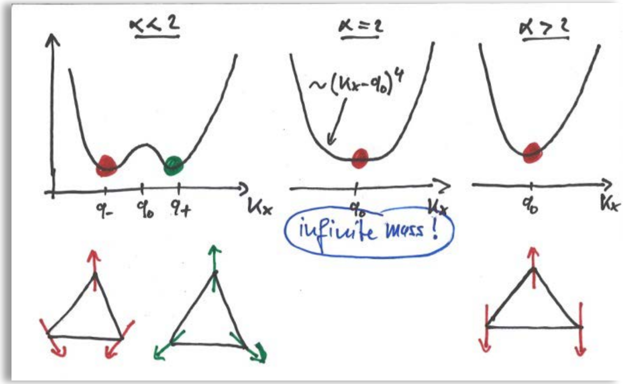


# Corrections for intermediate interaction

“Order-by-disorder-type effect”







# Strong interaction

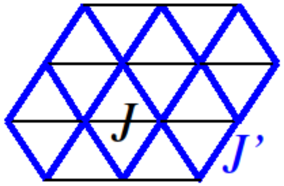
## Hard-core boson limit $U \gg J$

**System resembles frustrated quantum antiferromagnet**

$$\hat{H}_{\mathrm{XY}} = \sum_{\langle ij \rangle} 2J^{\mathrm{eff}}_{ij} (\hat{S}^x_i \hat{S}^x_j + \hat{S}^y_i \hat{S}^y_j)$$

$$|\uparrow_i\rangle \equiv |n_i = 0\rangle$$

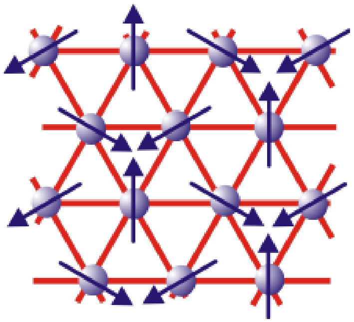
$$|\downarrow_i\rangle \equiv |n_i = 1\rangle$$



**Ground state difficult to predict**

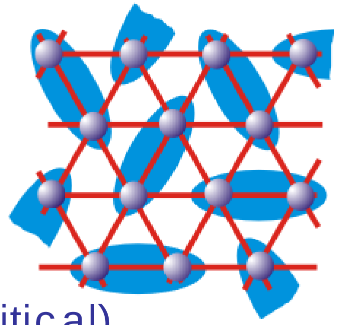
Two simple Ansaetze give the same energy per spin $-(3/8)\ J$

**Classical Neel order**



**Cover of singlets (exponentially degenerate!)**

=> Valence bond solid or
Spin liquid (gapped or critical)



# Strong interaction

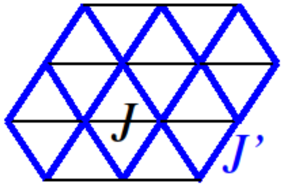
## Hard-core boson limit $U \gg J$

**System resembles frustrated quantum antiferromagnet**

$$\hat{H}_{\mathrm{XY}} = \sum_{\langle ij \rangle} 2J_{ij}^{\mathrm{eff}} (\hat{S}_i^x \hat{S}_j^x + \hat{S}_i^y \hat{S}_j^y)$$

$$|\uparrow_i\rangle \equiv |n_i = 0\rangle$$

$$|\downarrow_i\rangle \equiv |n_i = 1\rangle$$



**Novel type of quantum spin simulator**

- *built on **easy-to-cool** bosonic motional („charge“) degrees of freedom*
- **large coupling** of the order of boson tunneling (no superexchange)
- **different adiabatic preparation schemes** (tunable frustration & „quantumness“)
- can host **quantum disordered spin-liquid-like phases**
- generalizable to **further lattice geometries, e.g. Kagome (Berkeley group)**
- **easy to implement experimentally**

# Strong interaction

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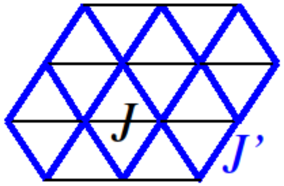

**Conjectured phase diagram at half filling:**

**Schmied et al. NJP 2008:** PEPS and exact diagonalization

algebraic

gapped spin liquid

Spiral Neel order

gapped spin liquid

Staggered Neel order

# Conclusions

**Lattice shaking is a low-demanding method for the creation of artificial gauge fields (both abelian and non-abelian) for neutral atoms.**

**Opens novel routes for engineering many-body physics in optical lattices**

# Thanks to collaborators of the presented work

Barcelona (Theory):

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Philipp Hauke

(now in Innsbruck)

Olivier Tieleman

(now in Dresden)

Alessio Celi

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