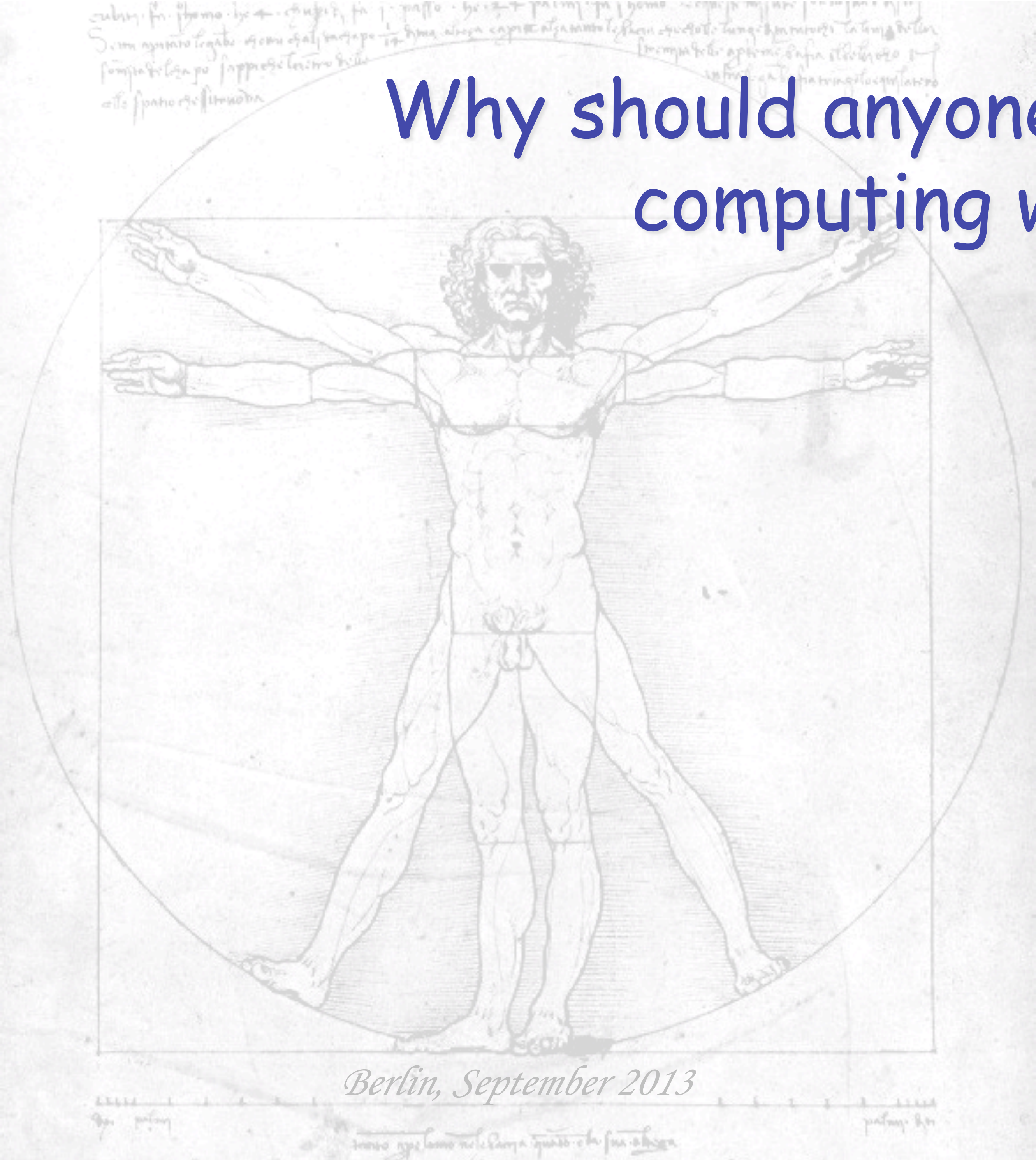
Why should anyone care about computing with anyons?

Jiannis K. Pachos

Introduction



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EPSRC
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Introduction

- **Quantum Computation** is the quest for:
  - » neat quantum **evolutions**
  - » new quantum **algorithms**

Why?

- **2D Topological Quantum Systems:** How?

1) Continuum gases: FQHE **Chern-Simons** $SU(N)_k$

2) Spin lattices: **Quantum Double Models**

Topological Degeneracy
Charge Fractionalization
Effective gauge theories

Anyons

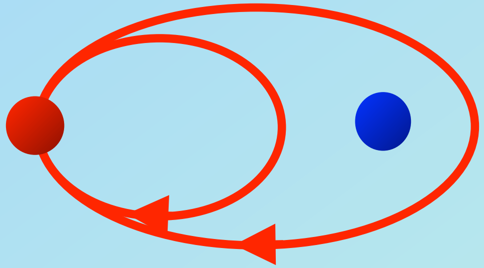
!

[Wilczek, Freedman, Wen, Bais, Wang, Kitaev,...]

Anyons

- Two dimensional systems
- Dynamically trivial (H=0). Only **statistics**.

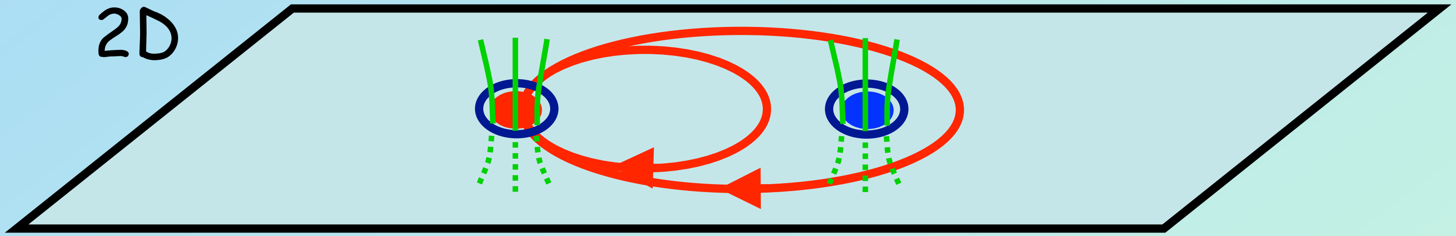
3D



Bosons $|\Psi\rangle \to |\Psi\rangle$

Fermions $|\Psi\rangle \to e^{i2\pi}|\Psi\rangle$

2D



$|\Psi\rangle \to e^{i2\varphi}|\Psi\rangle$

$|\Psi\rangle \to U|\Psi\rangle$

Anyons

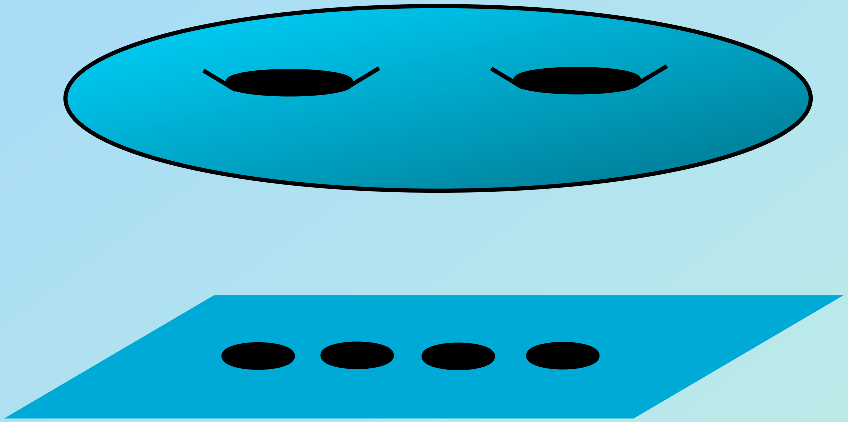
Anyons: vortices with **flux** & **charge (fractional)**.

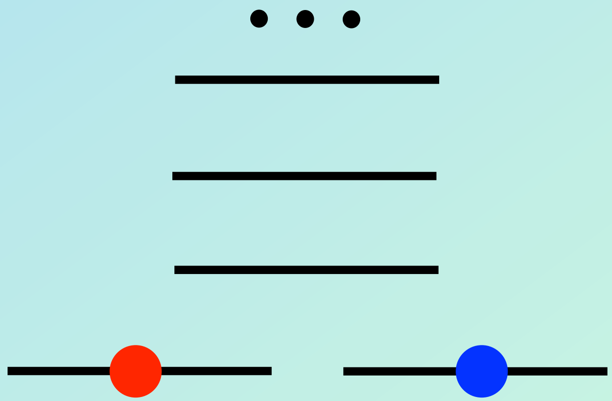
Aharonov-Bohm effect ⇔ **Berry Phase**.

Topological Degeneracy

System with **degenerate ground states** where:

- The degeneracy is **protected by topology** (genus).
- Degenerate states are **not locally distinguishable**.



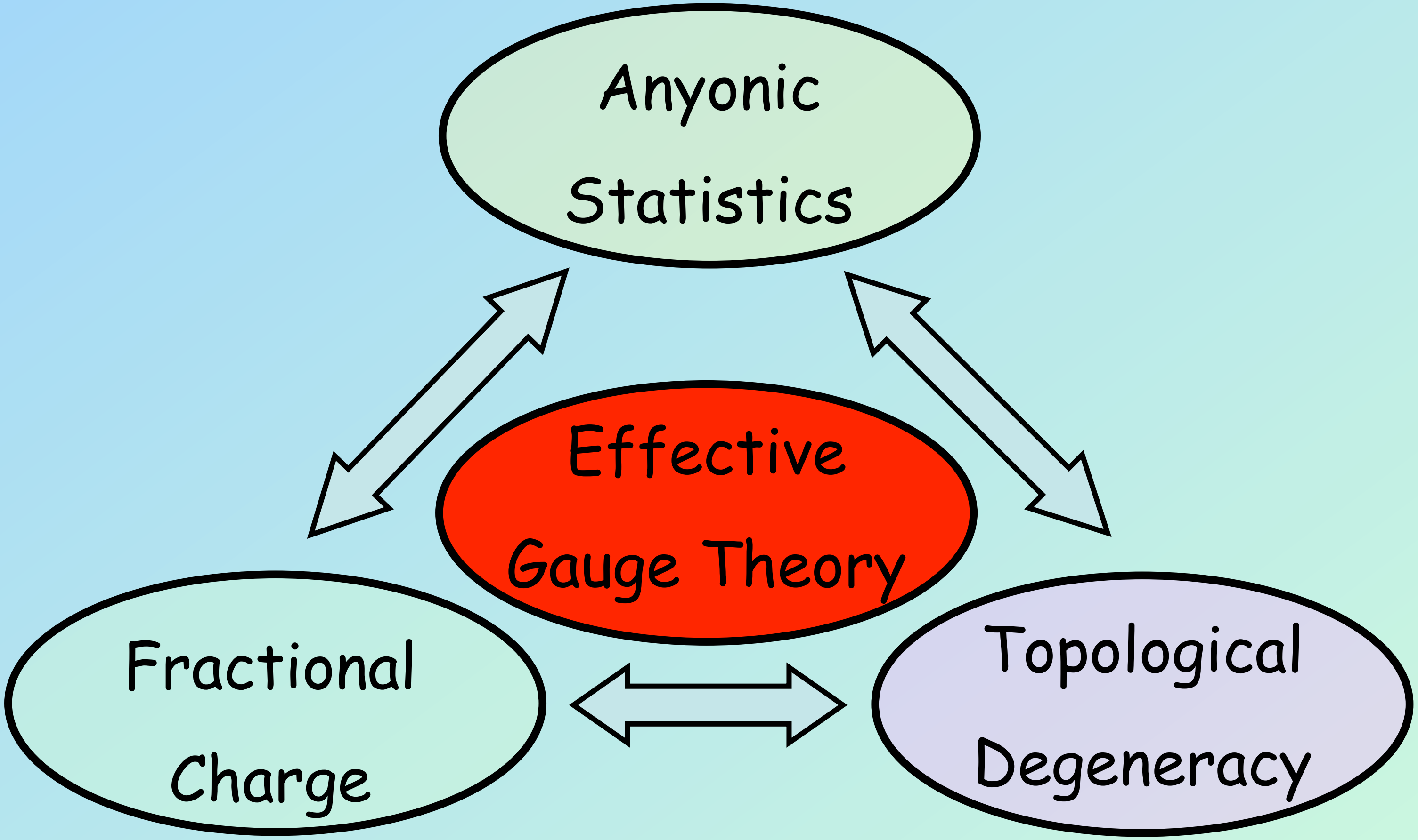


$\Rightarrow$ **encode information in degenerate subspace**

Index Theorem: n fermionic zero energy modes

=> 2^n degenerate ground states [Atiyah, Singer (1963)]

Topological Quantum Systems

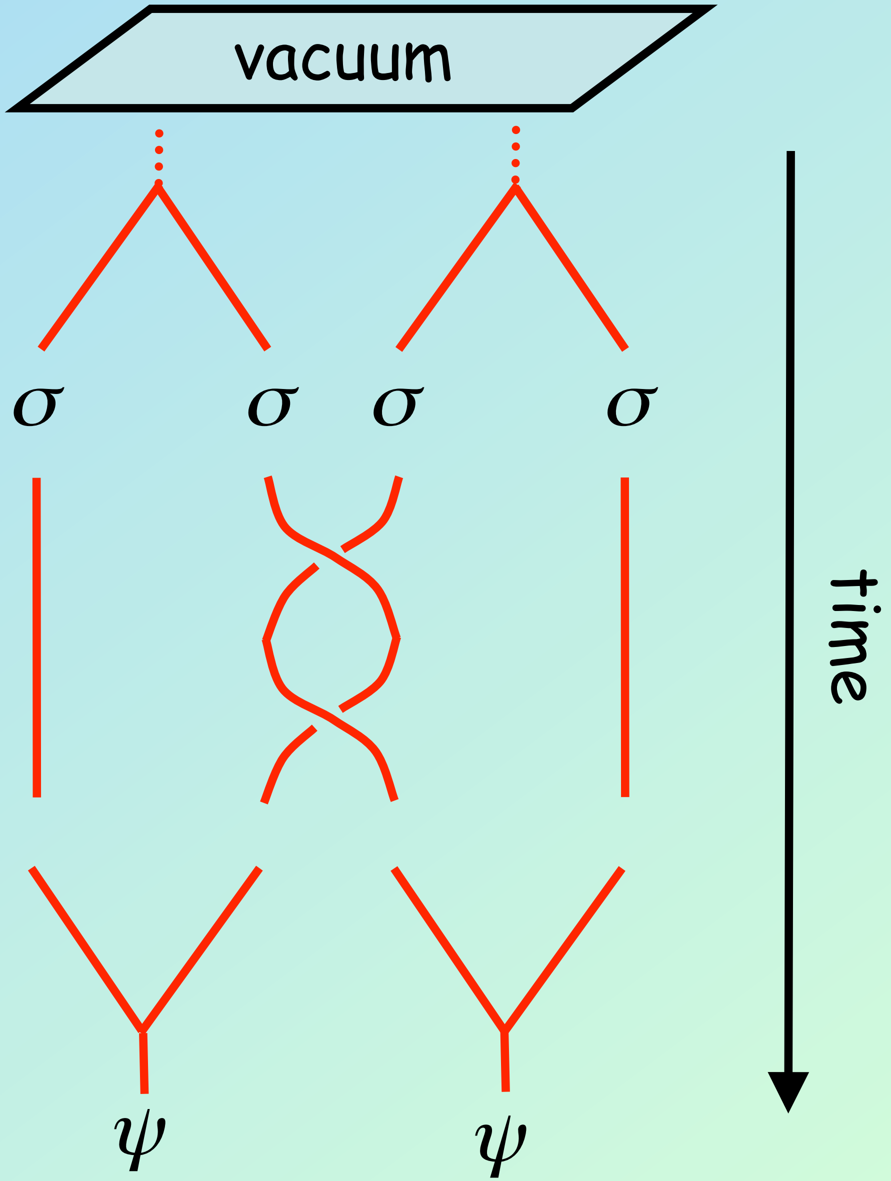


Anyon Properties

- Assume we can:
  - **Create** identifiable anyons
    vacuum pair creation
  - **Braid** anyons
    Statistical evolution:
    braid representation B
  - **Fuse** anyons
    e.g. $\sigma \times \sigma = 1 + \psi$

Fusion Hilbert space:

$$|\sigma, \sigma \to 1\rangle, |\sigma, \sigma \to \psi\rangle$$



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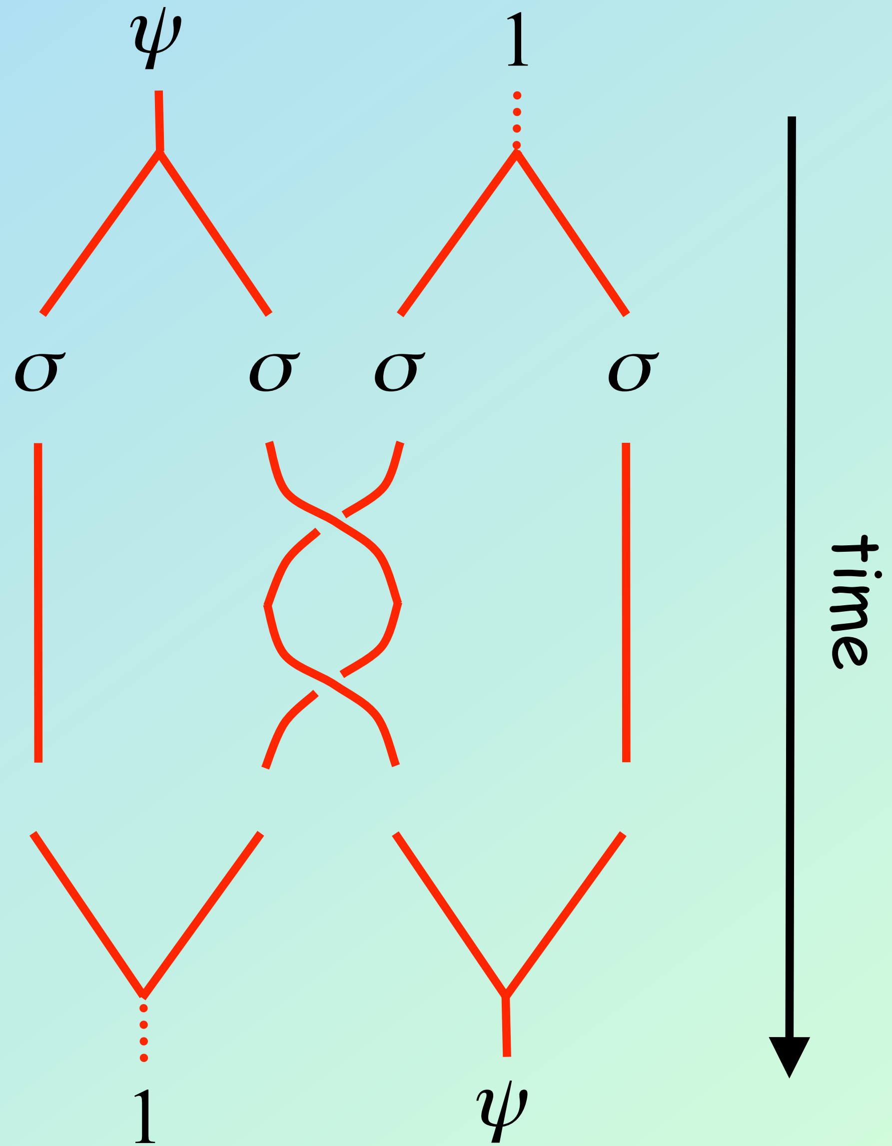


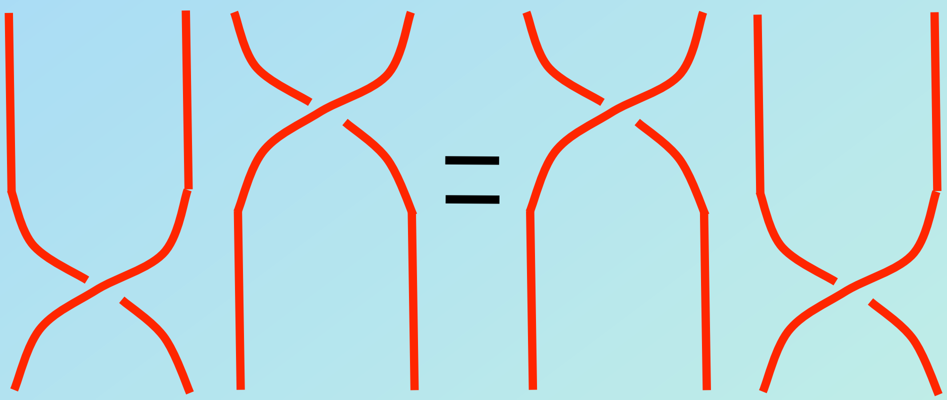
The braid group B_n

The braid group B_n has elements $b_1, b_2, \ldots, b_{n-1}$ that satisfy:

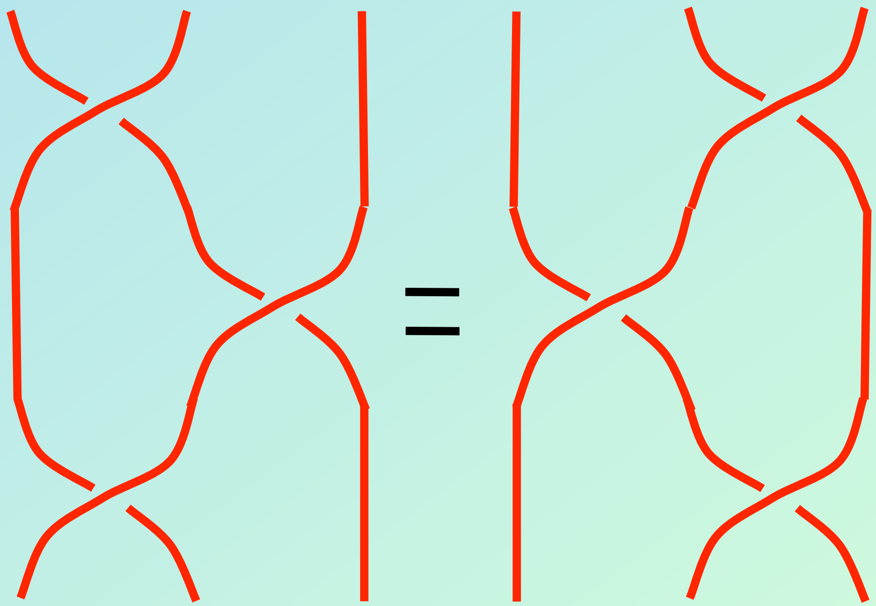
$$b_i b_j = b_j b_i, \quad \text{for } |i-j| \geq 2$$

$$b_i b_{i+1} b_i = b_{i+1} b_i b_{i+1} \quad \text{for } 1 \leq i < n$$

Pictorially:



$$b_i b_j = b_j b_i$$



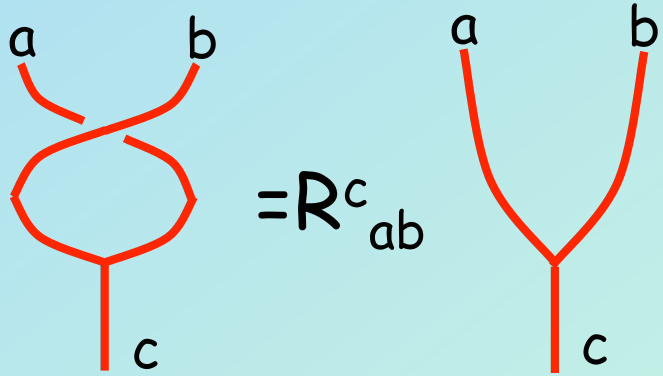
$$b_i b_{i+1} b_i = b_{i+1} b_i b_{i+1}$$

Braiding and Fusion properties

- The action of braiding of two anyons depends on their fusion outcome:

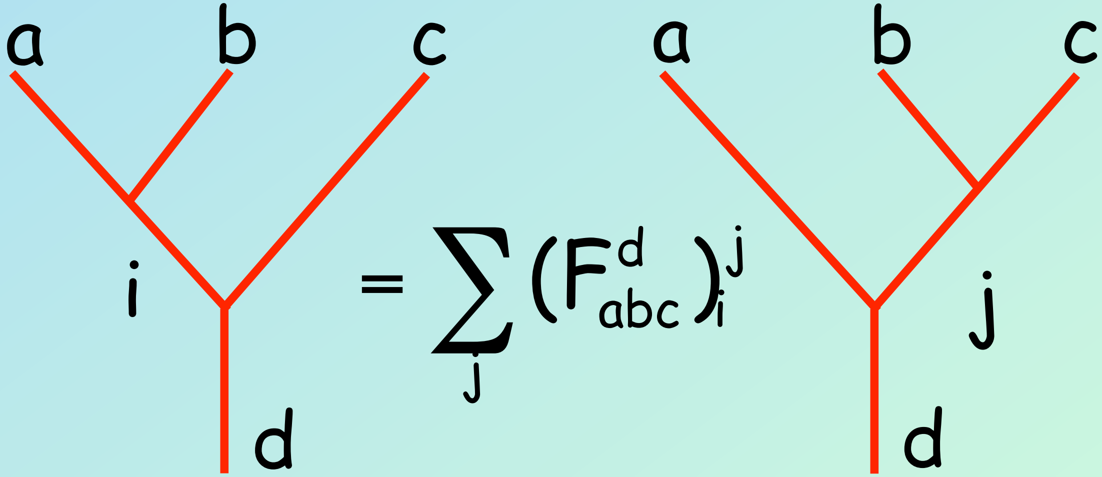
E.g. for c: fermion

then $R^{c}_{ab}=i$ is possible



$$= R^{c}_{ab}$$

- Changing the order of fusion is non-trivial:



$$= \sum_{j} \left(F^{d}_{abc}\right)^{j}_{i}$$

Construction of Anyonic Models

1. Take a certain number of **different anyons**

   1, a, b, ...

   the vacuum (1) and one or more non-trivial particles

2. Define **fusion rules** between them

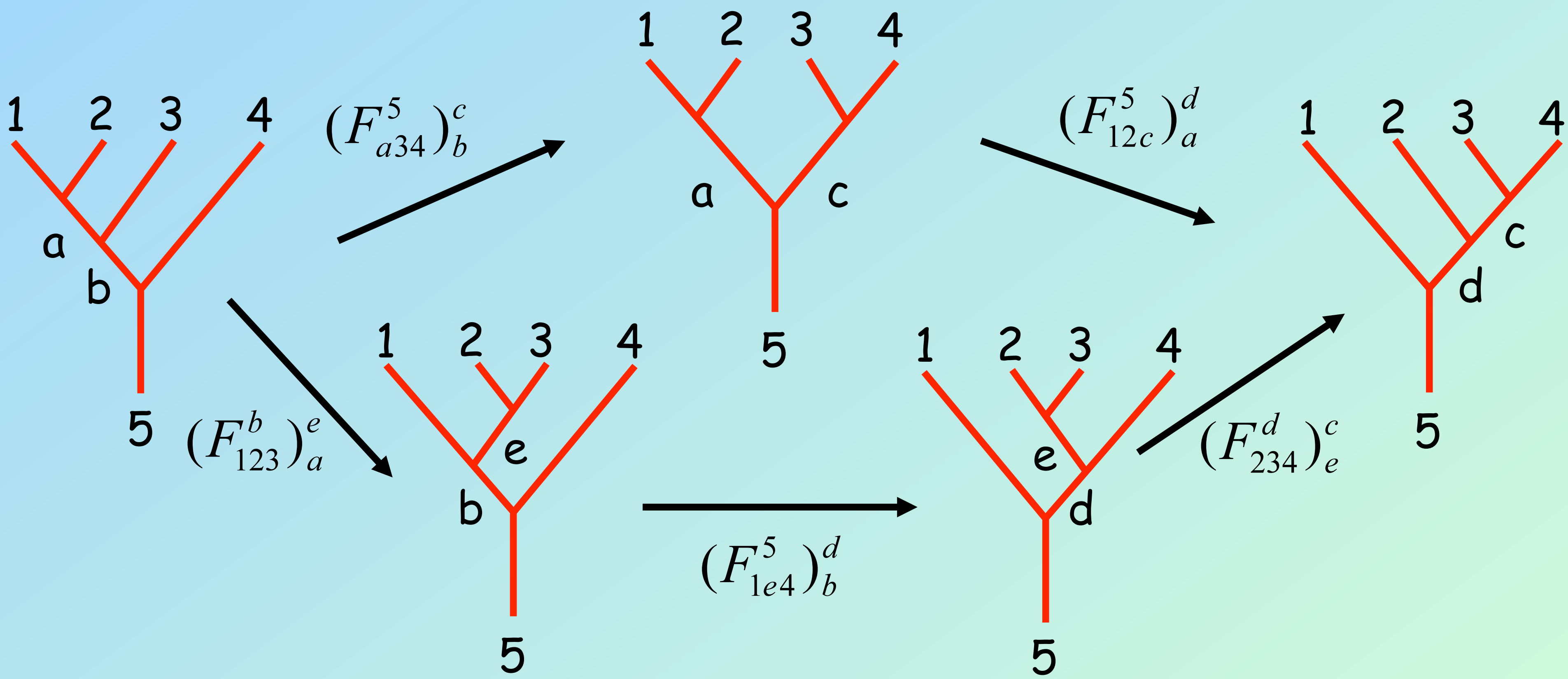
   1xa=a, axb=c+d+..., axa=1+...

   The vacuum acts trivially. Each particle has an anti-particle (might be itself or not).

   - Abelian anyons axb=c
   - Non-Abelian anyons axb=c+d+...

Construction of Anyonic Models

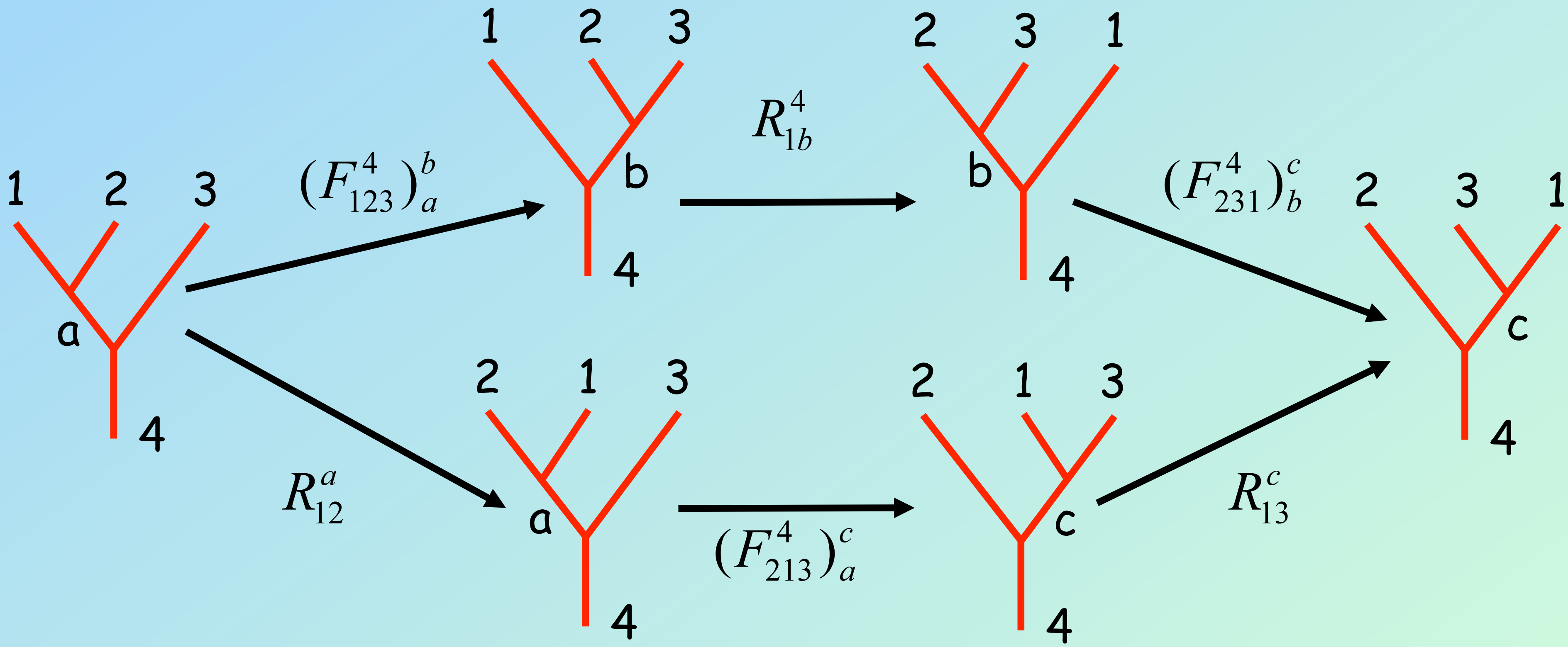
3. The F and B matrices are determined from the **Pentagon** and Hexagon identities



$$(F^5_{12c})^d_a (F^5_{a34})^c_b = \sum_e (F^d_{234})^c_e (F^5_{1e4})^d_b (F^b_{123})^e_a$$

Construction of Anyonic Models

3. The F and B matrices are determined from the Pentagon and **Hexagon** identities

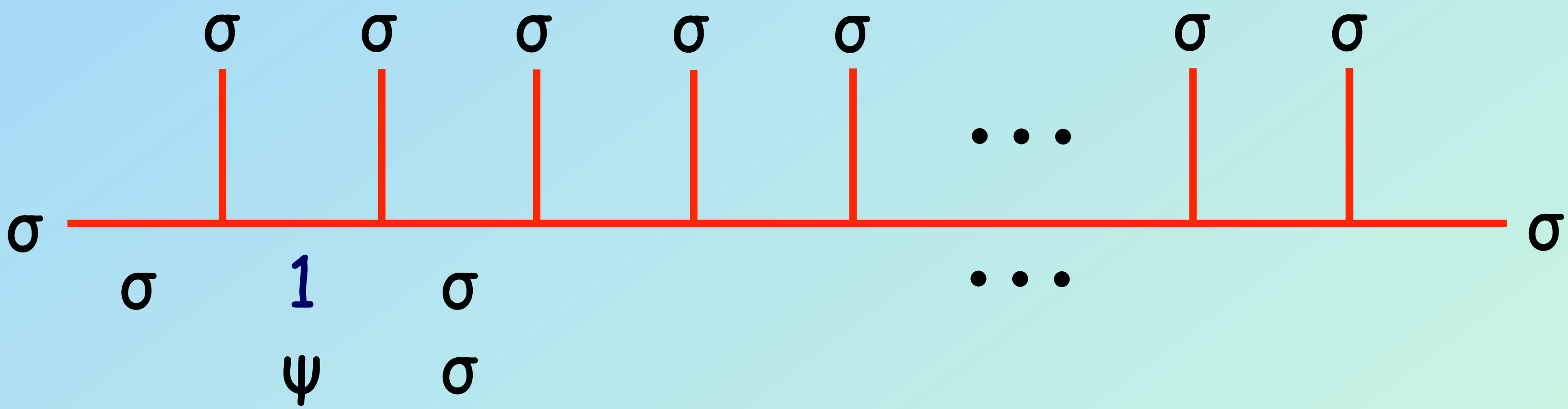


$$\sum_b (F^4_{231})^c_b R^4_{1b} (F^4_{123})^b_a = R^c_{13} (F^4_{213})^c_a R^a_{12}$$

Ising Anyons

Consider the particles: 1, σ and ψ

Fusion rules: σxσ=1+ψ, ψxψ=1, σxψ=σ



$$|\Psi\rangle = |1,1,\ldots\rangle$$

$$|\Psi\rangle = |1,\psi,\ldots\rangle$$

All these states span the **fusion Hilbert space**

Ising Anyons

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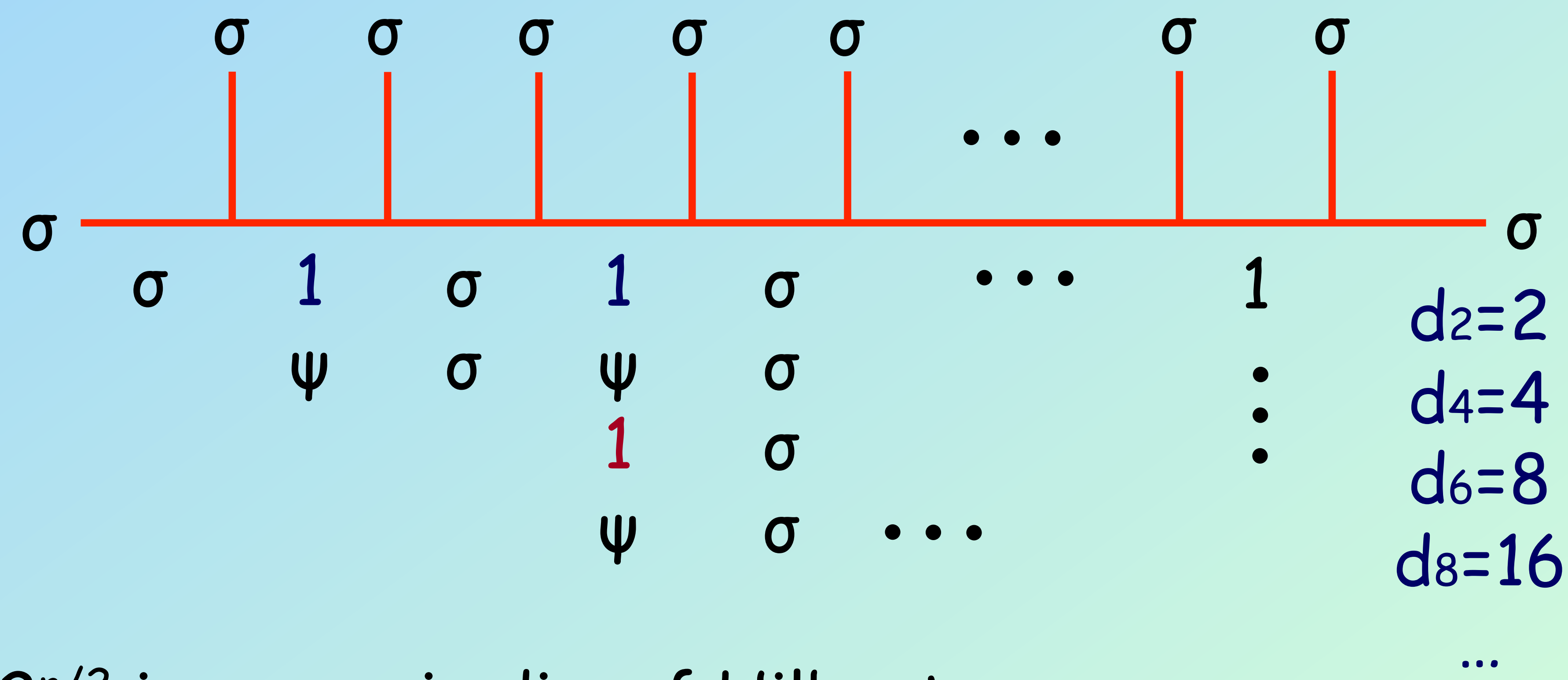
Fusion rules: σxσ=1+ψ, ψxψ=1, σxψ=σ



$2^{n/2}$ increase in dim of Hilbert space

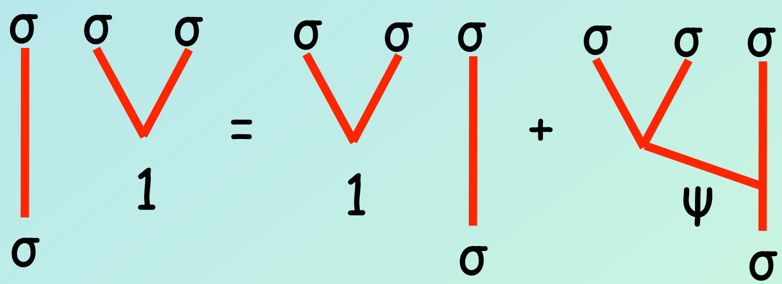
Ising Anyons

Consider the particles: 1, σ and ψ

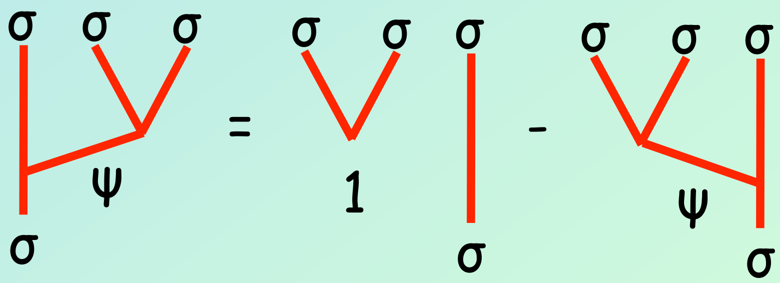
Fusion rules: σxσ=1+ψ, ψxψ=1, σxψ=σ

From 5-gon and 6-gon identities we have:

$$F^{\sigma}_{\sigma\sigma\sigma} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

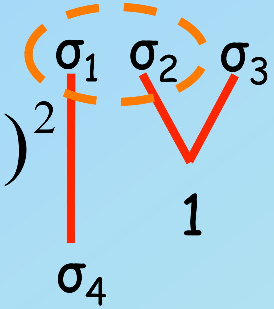


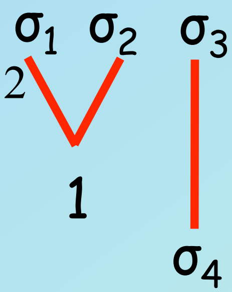
Rotation of basis states

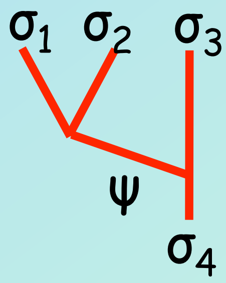


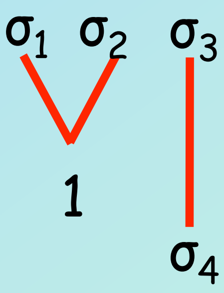
Ising Anyons

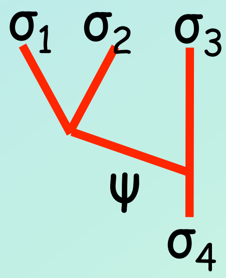
Braiding $R^{1}_{\sigma\sigma} = e^{-i\pi/8}$ and $R^{\psi}_{\sigma\sigma} = ie^{-i\pi/8} \Rightarrow R_{\sigma\sigma} = e^{-i\pi/8}\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

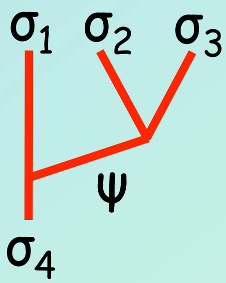
$(R_{\sigma_1\sigma_2})^2$ 

$= (R^{1}_{\sigma_1\sigma_2})^2$ 

$+ (R^{\psi}_{\sigma_1\sigma_2})^2$ 

$= \quad e^{-i\pi/4}$ 

$- \quad e^{-i\pi/4}$ 

$= \quad e^{-i\pi/4}$ 

$H\sigma^z H = \sigma^x$

Clifford group:
non-universal!

Ising Anyons

Qubit initialization:

State |0> State |1>

Measurement: Outcome of pairwise fusion, 1 or ψ

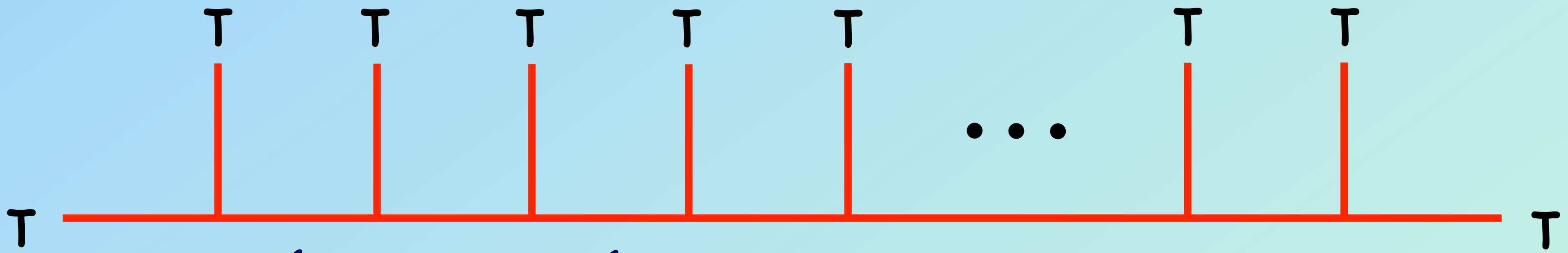
$$H\sigma^z H = \sigma^x$$

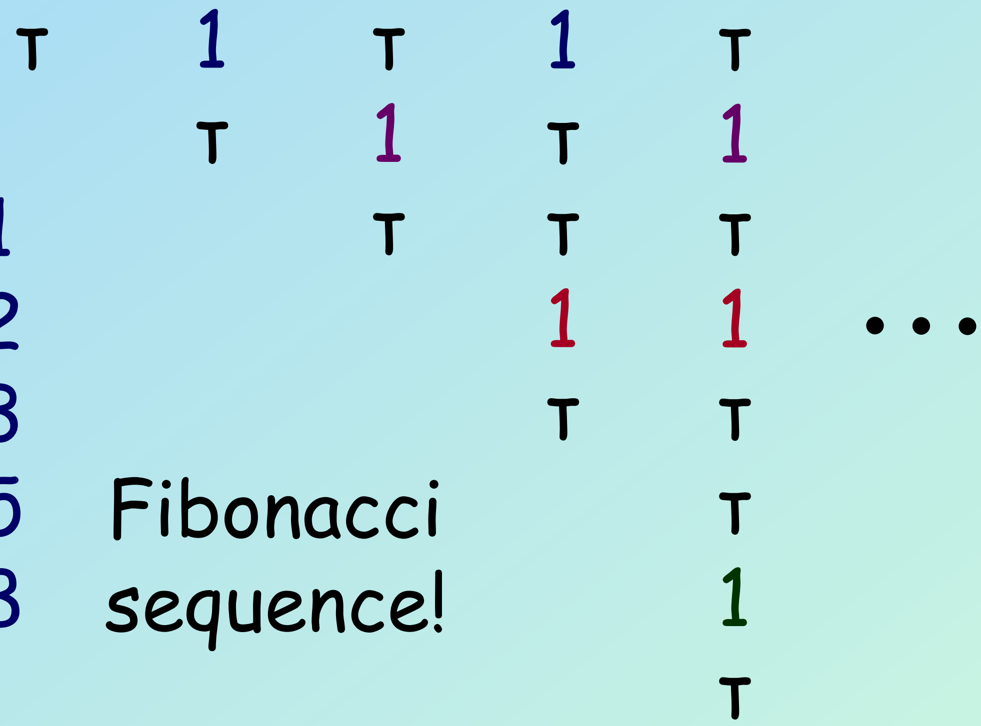
Gates: Clifford group. Non-universal!

Can be employed as a quantum memory.
One needs a **phase gate**: employ interactions between anyons.

Fibonacci Anyons

Consider anyons with labels 1 or τ with the fusion properties: **1x1=1, 1xτ = τ, τxτ =1+τ**





$d_1=1$
$d_2=2$
$d_3=3$
$d_4=5$
$d_5=8$
...

Fibonacci sequence!

Dimension of Hilbert space

$$d_n \propto \phi^n$$

$$\phi = (1+\sqrt{5})/2$$

Golden mean

Fibonacci Anyons and QC

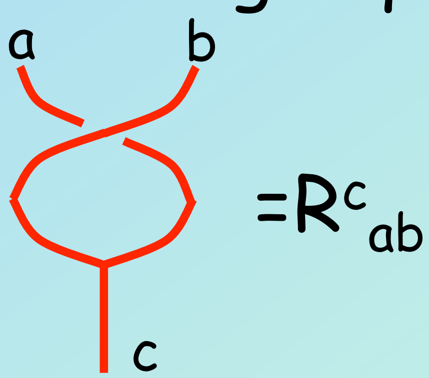
Qubit encoding:

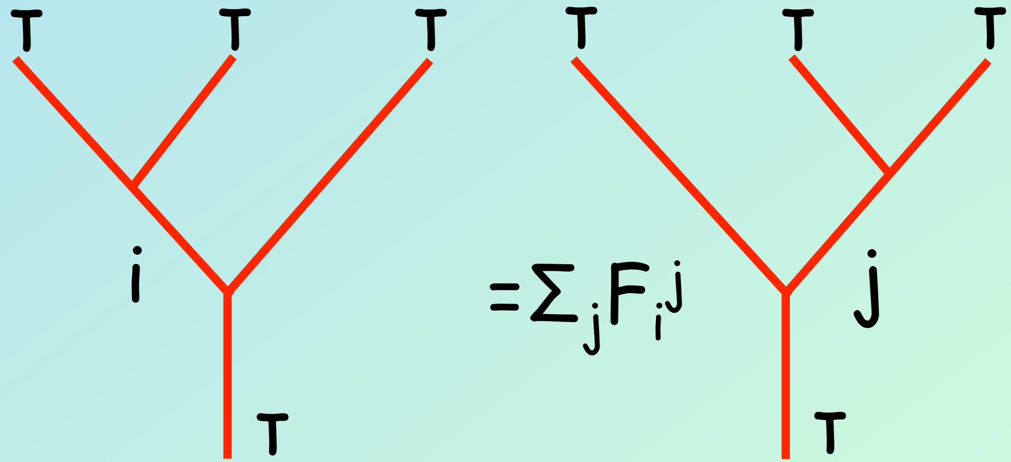
State |0>   State |1>

$$|\tau,\tau \rightarrow 1\rangle = |0\rangle$$

$$|\tau,\tau \rightarrow \tau\rangle = |1\rangle$$

Evolving a qubit:

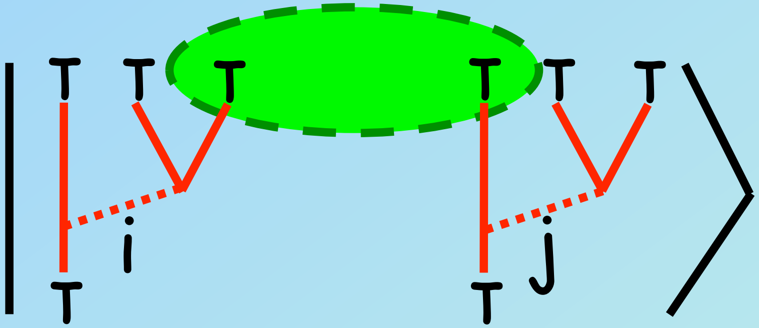




Unitaries B and F are dense in SU(2).
[Freedman, Larsen, Wang, CMP 228, 177 (2002)]

Fibonacci Anyons and QC

Qubit encoding:



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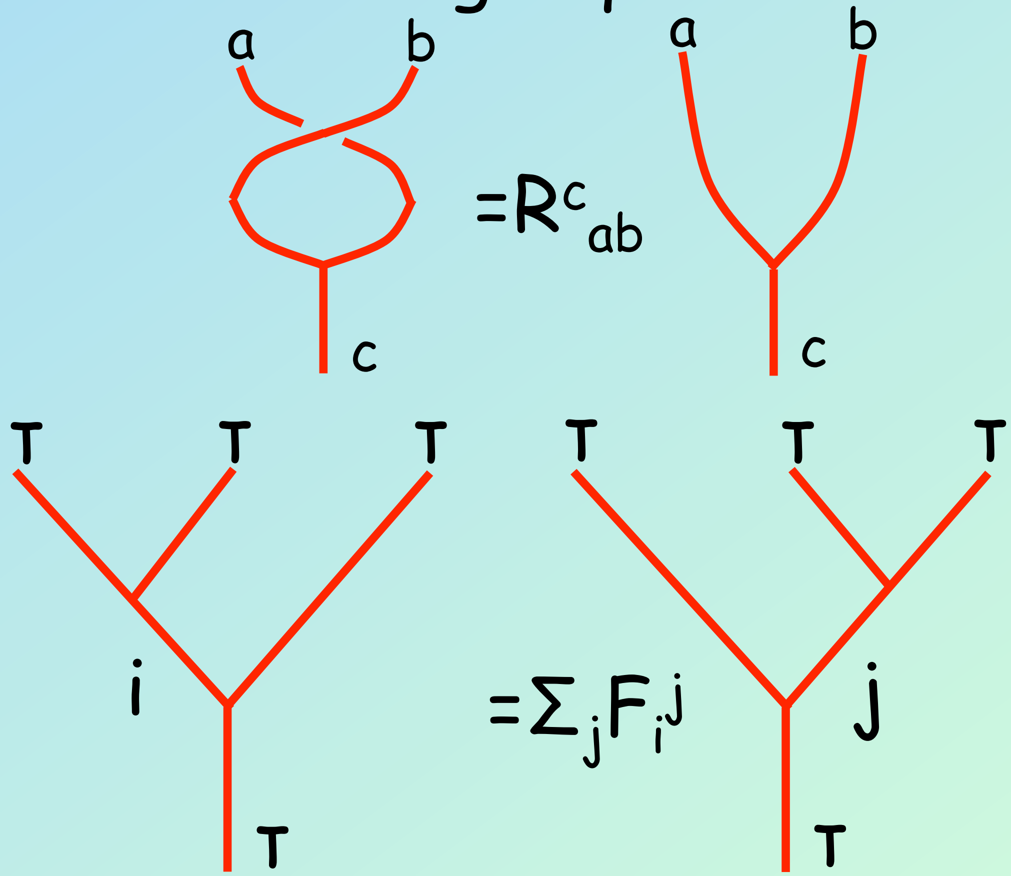

Unitaries B and F are dense in SU(2).

Extends to $SU(d_n)$ when n anyons are employed.

Fibonacci Anyons and QC

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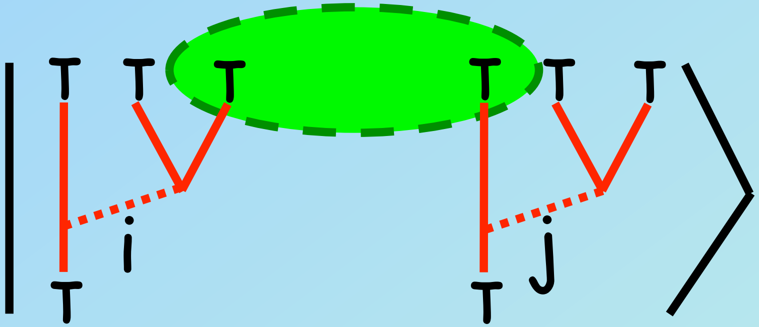

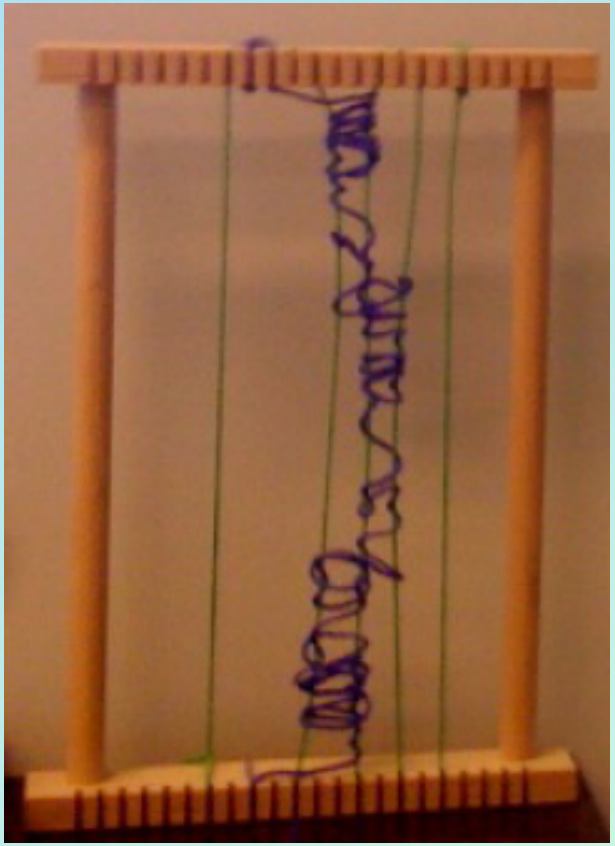

CNOT

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Extends to SU(d_n) when n anyons are employed.

Conclusions

- Topological Quantum Computation promises to **overcome** the problem of **decoherence** and errors in the most direct way.
- There is lots of work to be done to make anyons work for us.
- Is it worth it?

Aesthetics says YES!