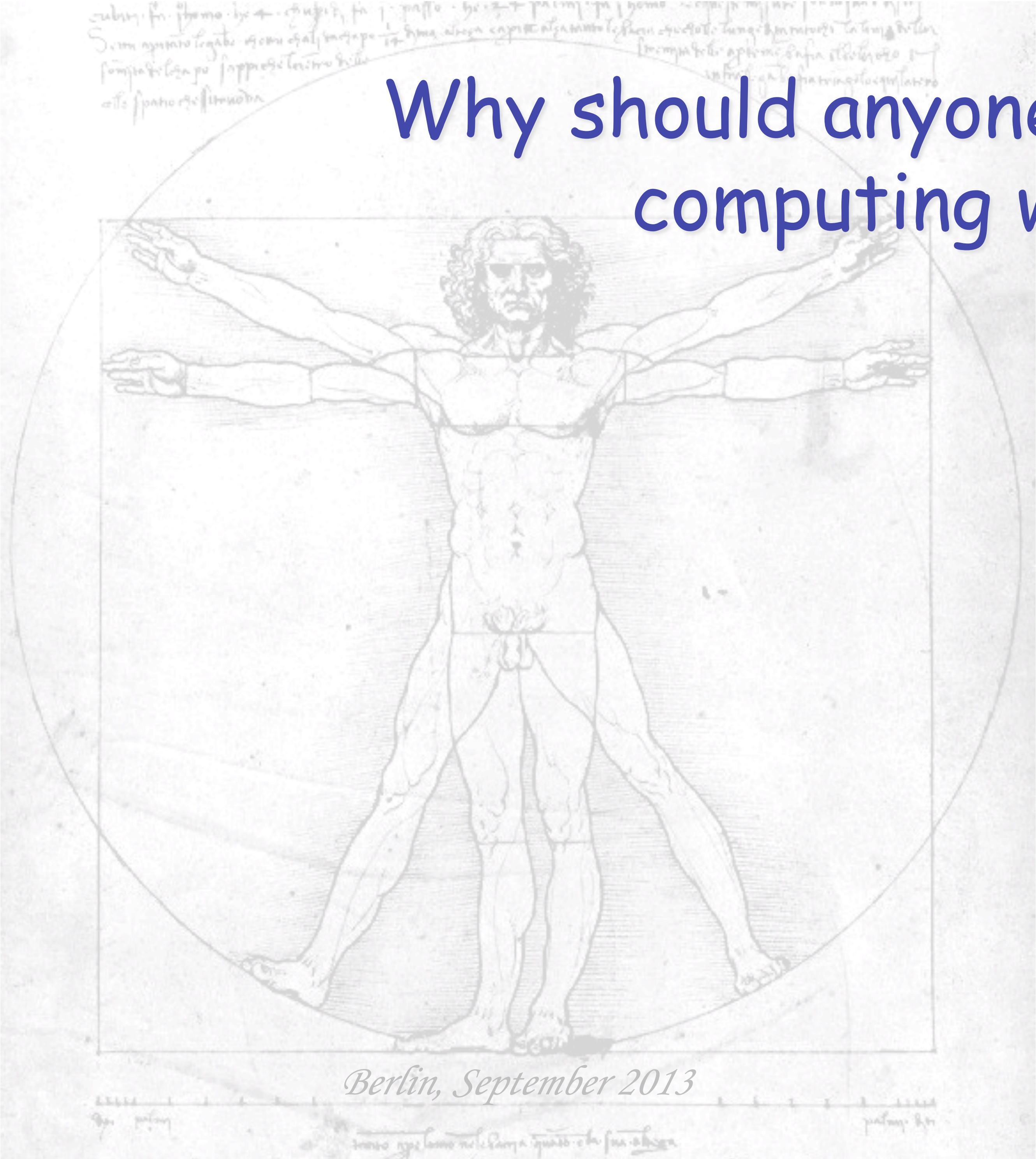


Why should anyone care about computing with anyons?

Jiannis K. Pachos

Outlook

Berlin, September 2013

EPSRC
Engineering and Physical Sciences Research Council

UNIVERSITY OF LEEDS

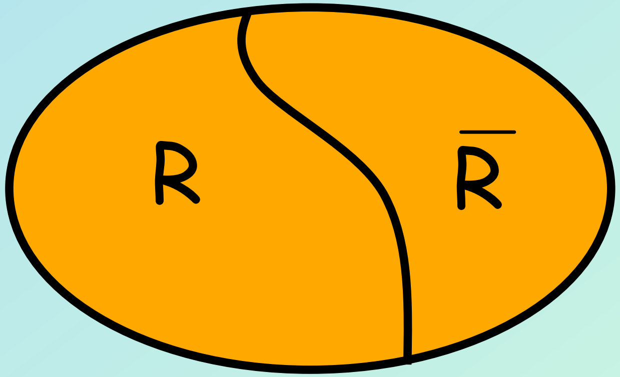
Various topics

- Topological order **witnesses**
- Topological entropy
- **Errors** and topological order
- **Topological memories**
- **Protection against errors**
- **Physical proposals:** Kitaev's honeycomb lattice

Topological Entropy

- Pure system $|\xi\rangle$
- Partition in R and $\overline{\mathrm{R}}$ with boundary ∂R
- Reduced density matrix of R:

$$\rho_R = \mathrm{tr}_{\overline{R}} |\xi\rangle\langle\xi|$$



- Von Neumann entropy:

$$S_R = tr(\rho_R \ln \rho_R)$$

- We expect:

$$S_R = \alpha\,|\,\partial R\,| + \gamma + \cancelto{0}{\varepsilon(|\,\partial R\,|)}$$

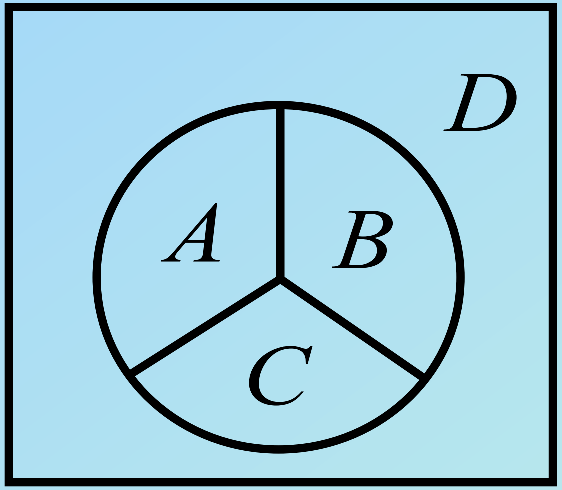
- Topological entropy: $\gamma = \ln D, \quad D = \sqrt{\sum_q d_q^2}$

!

Topological Entropy

- Consider partition of single system Σ:

Σ



System is **gapped** ->

finite correlation length

Size of areas -> **infinity**

$$\varepsilon(|\partial R|) \to 0$$

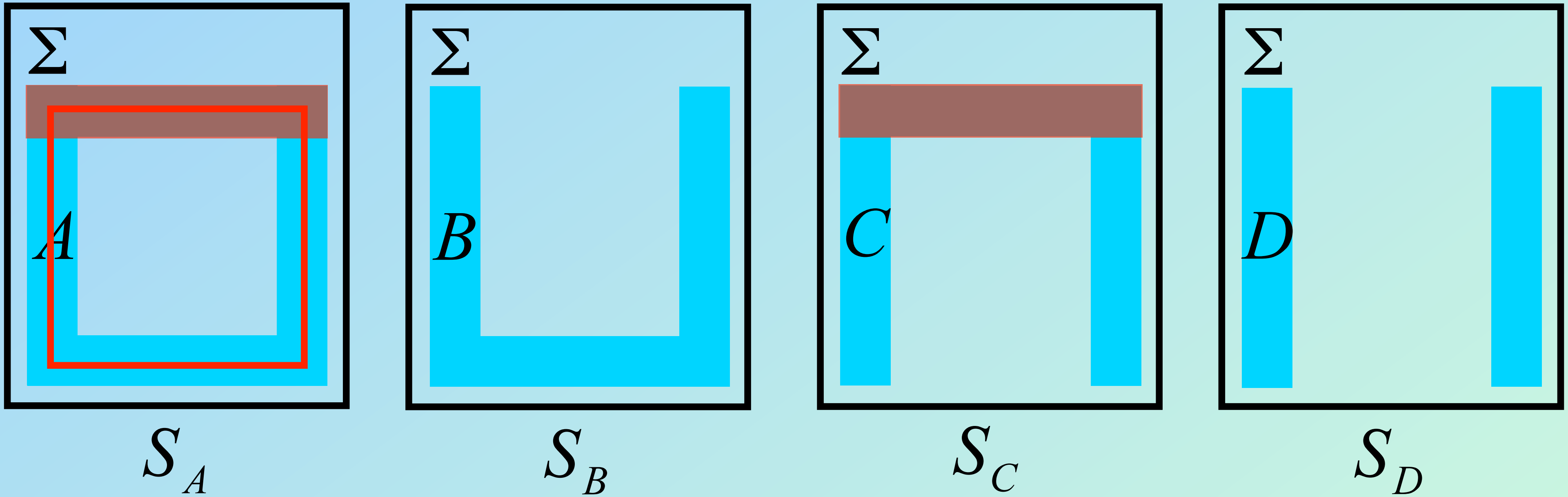
- Topological entropy:

$$\gamma = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC}$$

The area terms disappear! [Kitaev & Preskill]

Topological Entropy

- Consider four different partitions:



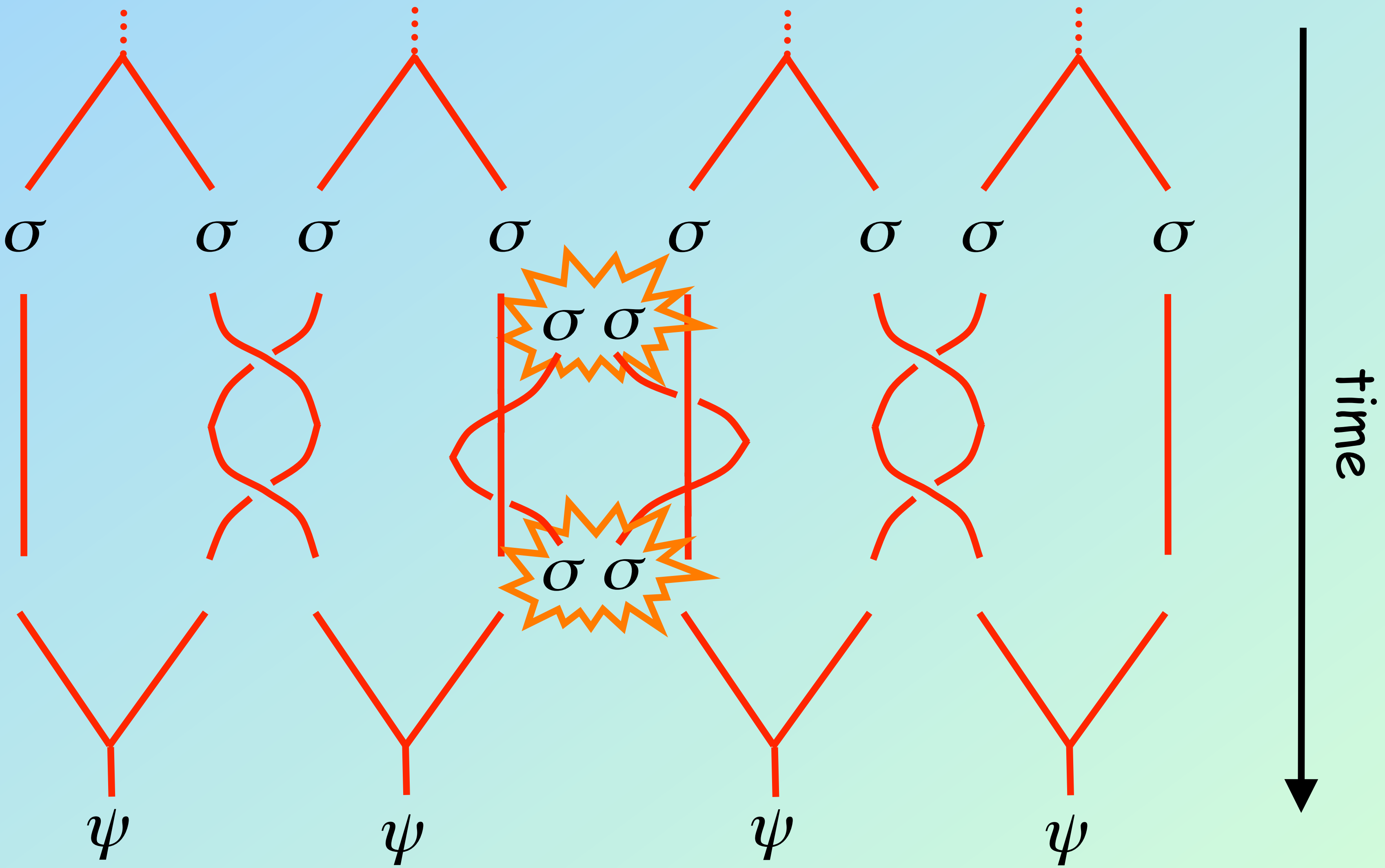
- Topological entropy:

$$\gamma = -\frac{1}{2}\left[(S_A - S_B) - (S_C - S_D)\right]$$

- Only loop contributions survive! [Levin & Wen]

Topological Errors

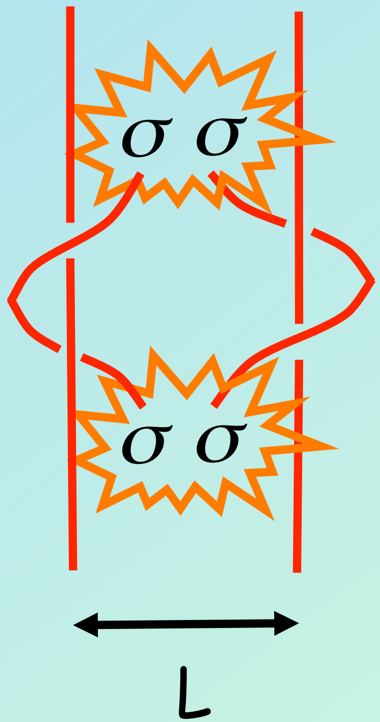
Errors can appear in the form of **virtual** anyons:



Topological Errors

Errors can appear in the form of virtual anyons

They can be avoided by keeping data anyons far apart: $\boxed{P_{error} \sim e^{-\Delta L/v}}$



Δ :Energy Gap for σ pair cration

L :distance between σ anyons

v :characteristic velocity of anyons

Topological Memory

Can you create a system that can **resist errors**, virtual or actual (temperature) for *long times*?

1) Toric code coupled with bosonic field:

   then errors (anyons) **attract** each other and annihilate! [Hamma, Castelnovo & Chamon]

2) Induce a **repulsion** between toric code anyons:

   it can be shown that it generates a stable anyonic phase. [Chesi, Roethlisberger & Loss]

3) Alternative one can perform **topologically inspired error correction**.

   0.75% tolerance [Raussendorf & Harrington]

Resilience to Errors

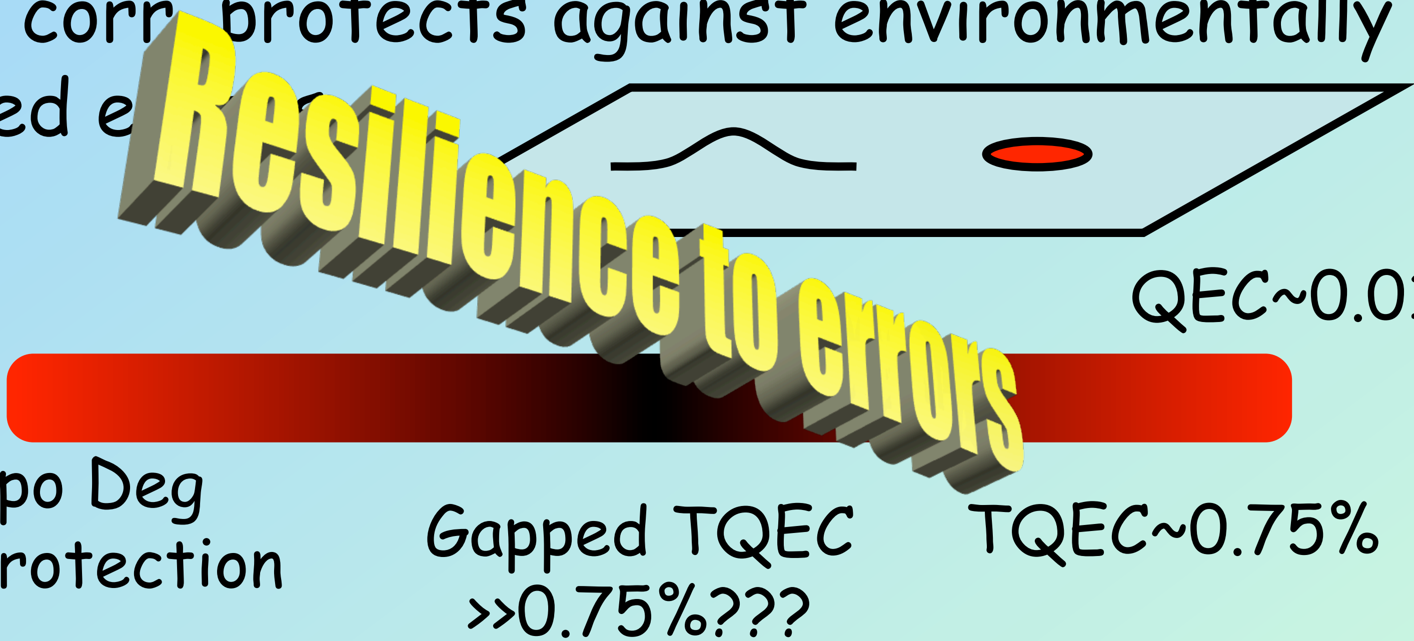
- **Abandon** the idea of separate **subsystems** for qubits. Encode info in **macroscopic degree of freedom** (non-locally). Direct observation of anyons does not reveal their total state.

  => local decoherence (environment "measures") does not destroy information.
- The **unitary transformations** resulting from braiding are virtually **errorless** as they depend only on the topological characteristics of the anyonic trajectories.

  We should $\sim 10^{-4}$ we get $\sim 10^{-30}$

Resilience to Errors

- Hamiltonian (energy gap) protects against local perturbations.
- Error corr. protects against environmentally induced errors.



- Could a hybrid of quantum error correction and TQC perform *much* better?
- **Fault tolerance**: physically inspired mechanism.

Honeycomb lattice model

- **Kitaev's Honeycomb Lattice Model**:

  an exactly solvable 2D spin-1/2 lattice model

- Two different topological phases:
  - **Abelian anyons** (Toric code)
  - **Non-Abelian anyons** (Ising model, Majorana fermions)

- Physical realizations:
  - **Rydberg atoms** and stroboscopic techniques

    [Wootton & J.K.P.]
  - Ultra-cold atoms or **polar molecules**

    [Micheli *et al. Nature Physics*, 2005]

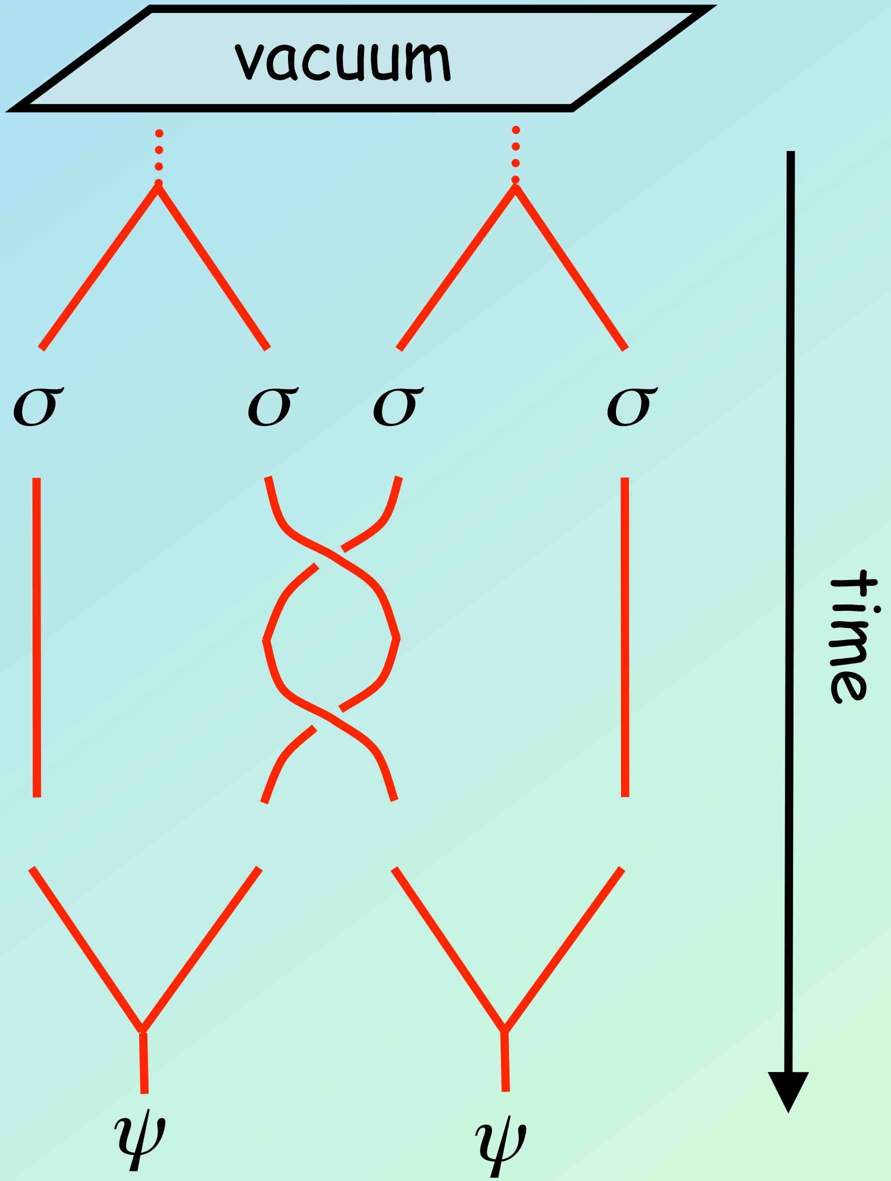
[A. Kitaev, *Ann. of Phys.*, 2006; J.K.P., *Ann. of Phys.*, 2006]

Anyon Properties

- Assume we can:
  - **Create** identifiable anyons
    vacuum pair creation
  - **Braid** anyons
    Statistical evolution:
    braid representation B
  - **Fuse** anyons
    e.g. $\sigma \times \sigma = 1 + \psi$

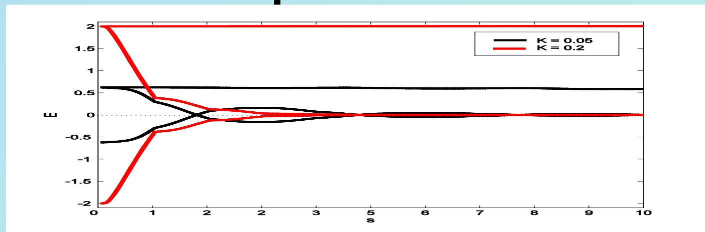
Fusion Hilbert space:

$$|\sigma,\sigma \rightarrow 1\rangle, |\sigma,\sigma \rightarrow \psi\rangle$$



What can we do

- **Exact** *quantities* for large systems > 20x20 spins:



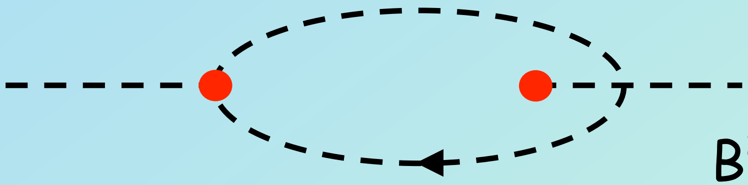
  - **Interactions** between vortices

V(r)

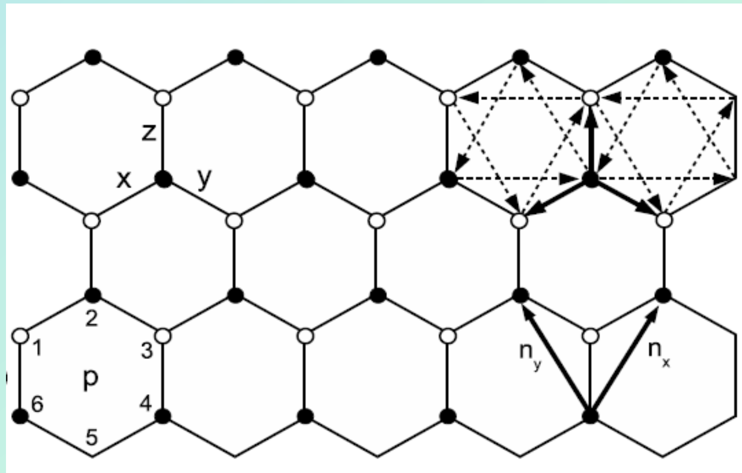
r

  - Braiding **statistics** of vortices

360 spins



$$B^2 = e^{-\frac{\pi}{4}i}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



[Lahtinen *et al*, *Ann. of Phys*. 2008; Lahtinen & J.K.P., *NJP*, 2009, *PRL*...]

Outlook

- **Quantum information** has a lot to offer to the study of topological systems.

- **Topological quantum computation** is a very promising way of storing and manipulating quantum information.

- Research on topological quantum computation has **applications** to many relevant fields of condensed matter, statistical physics, biology,...

for your great kindness in the matter of the names respecting which I applied to you; but I hoped to have met you last Saturday at Kensington and therefore delayed expressing my obligations

I have taken your advice and the names used are <u>anode</u> <u>cathode</u> <u>anions</u> <u>cations</u> and <u>ions</u> the last I shall have but little occasion for. I had some hot objections made to them here and found myself very much in the condition of the man with his son and Ass who tried to please every body; but when

Letter from Faraday to Whewell (1834)