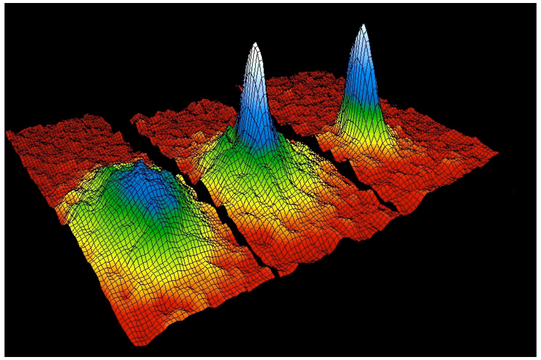
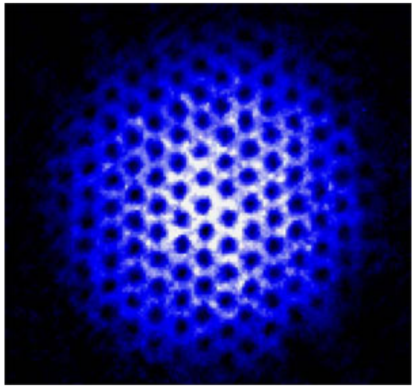
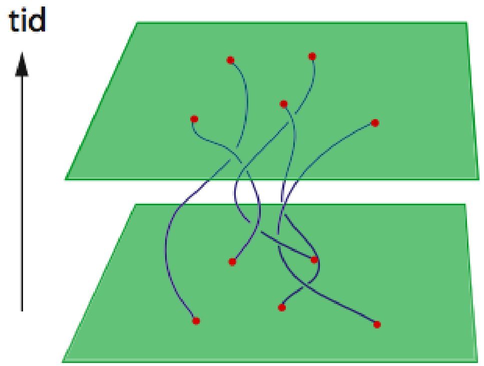
Strongly correlated states of

(Rotating) Bose condensates

Susanne Viefers, University of Oslo







Outline

- A glimpse of low-dimensional physics
- Quantum Hall ABC, anyonic quasiparticles
- Introduction to rotating Bose condensates
- Rotating bosons in the lowest Landau level
- Quantum Hall physics in rotating Bose condensates (theory)
  - Composite fermions, anyons, low L, two-species systems
- Experimental realization: Status, perspectives, alternatives
- Summary and references

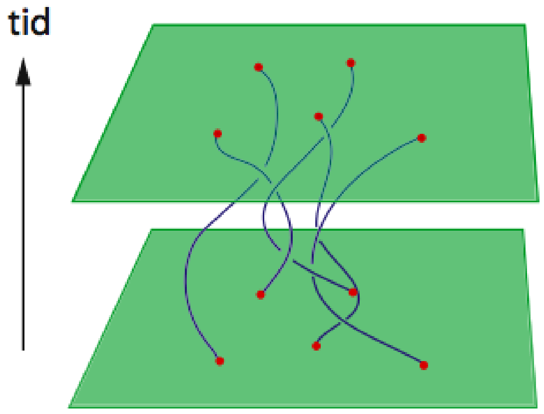
If the world were two-dimensional...

- Reduced dimensionality, i.e. quantum mechanics in 2d (or 1d) allows for "exotic" quantum phenomena which can never occur in 3d.

Anyons!

- Possible in 2D [Leinaas & Myrheim 77]
- Neither bosons nor fermions

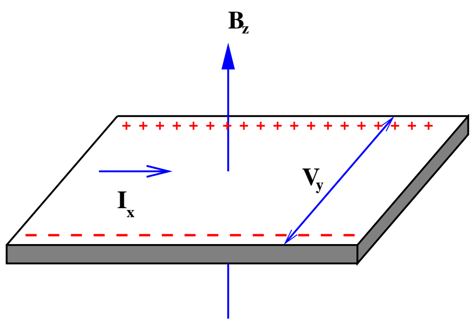
$$\psi(x_2, x_1) = e^{i\theta}\psi(x_1, x_2)$$

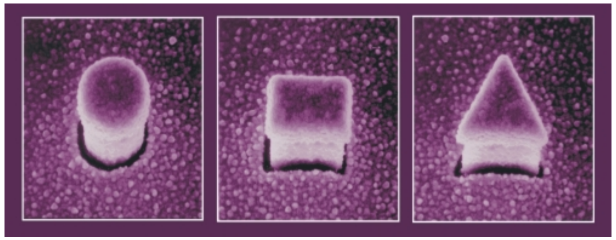


Occur in the systems we'll discuss today!

Low-dimensional physics

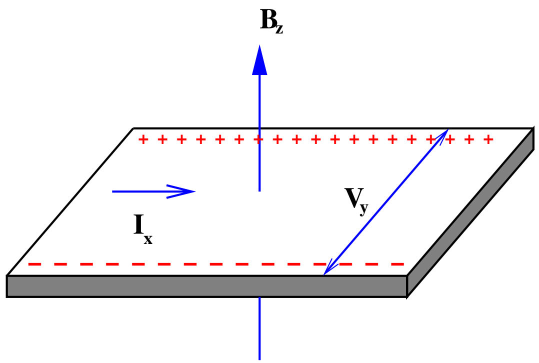
- Fundamentally new physics, interesting from basic research point of view
- Low-dimensional components can be realized in modern material technology
- Theoretical understanding is necessary for potential applications in future (nano)technology.





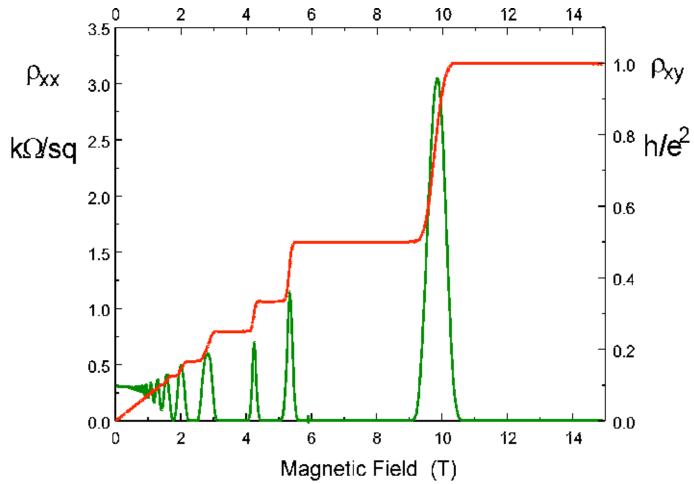
Quantum Hall ABC

Electrons are 'captured' at the (2D) interface between two semiconductor crystals, exposed to a strong magnetic field and cooled to ~ milli-Kelvin



Measure *transverse* (Hall) resistance. It is quantized! (Resistance standard!)

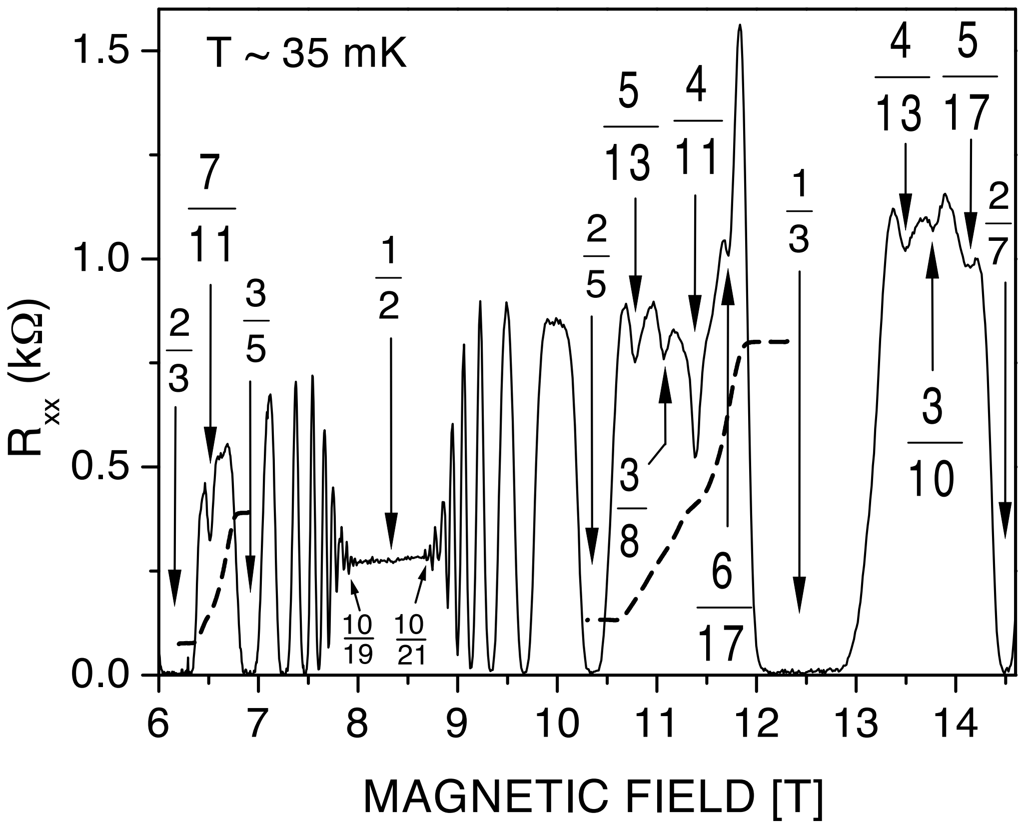
$$R_{xy} = \frac{V_y}{I_x} = \frac{1}{\nu}\frac{h}{e^2}$$



ν: Landau level filling fraction at center of plateau. Takes integer and fractional values (e.g. 1/3, 2/5, 4/11,...)

Zoo of quantum Hall states...

nu=4/11 paper, Fig.1



[Pan et al PRL 2003]

Landau levels

- QM problem of a single electron in 2D exposed to a magnetic field
- Harmonic oscillator in disguise, kinetic energy quantized: Landau levels

$$H = \frac{1}{2m}\left(\mathbf{p} - e\mathbf{A}\right)^2$$

- Landau gauge: $\boldsymbol{A} = B\,(-y,0,0)$
- Symmetric gauge: $\boldsymbol{A} = B/2\,(-y,x,0)$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c \qquad \left(\omega_c = \frac{eB}{m}\right)$$

- Symmetric gauge, $z=x+iy$: $\quad \eta_{nl} \propto e^{-|z|^2/4}\, z^l\, L_n^l\left(\frac{|z|^2}{2}\right)$

- $n=0$: $\quad \boxed{\eta_{0l} \propto z^l\, e^{-|z|^2/4}}$ Lowest Landau level (LLL)

Landau levels (2)

- Degeneracy per unit area: One state per flux quantum per LL

$$G = \frac{B}{\phi_0} = \frac{eB}{2\pi\hbar}$$

- ***Filling factor:***

$$\boxed{\nu = \frac{\rho}{G} = \frac{\rho}{B/\phi_0}}$$

  - Number of electrons per flux quantum
  - = number of occupied LLs for non-interacting electrons.
  - Note: Filling fraction ~ 1/B.

Lowest Landau level wave functions

- 2-dimensional electron gas in a strong perpendicular magnetic field at low T
- Electrons residing (mainly) in the lowest Landau level (LLL).

Single particle basis states: $$|l\rangle = N_l z^l e^{-|z|^2/4}, \quad z = x + iy$$

N-particle (trial) wave functions constructed as *antisymmetric combinations* of these, i.e. homogeneous polynomials. Total angular momentum = degree of polynomial.

Construction of explicit trial wave functions by various schemes (in particular Laughlin, composite fermions) has proven very successful in exploring QH physics. Not exact but do capture essential properties.

Fractional quantum Hall effect

The ground state at $\nu=1/m$ is an *incompressible quantum fluid*, described by Laughlin´s wave function (Nobel prize 1998):

$$\psi_N(z_1, ...z_N) = \prod_{i<j}(z_i - z_j)^m e^{-\sum |z_i|^2/4} \qquad (z = x + iy)$$

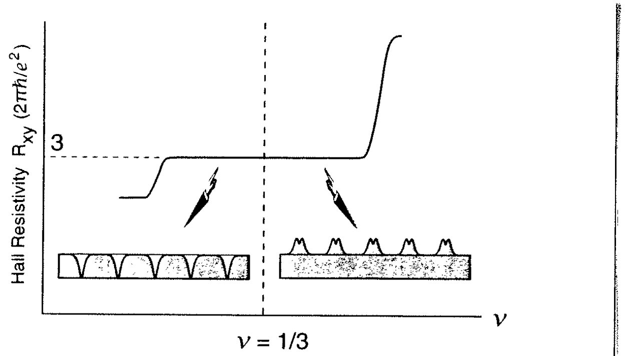
(Lowest Landau level, m = 3,5,...)

Quasiparticle excitations with *fractional charge and - statistics -- anyons!*

Quasihole at z_0: Multiply Laughlin wf by $\prod_i (z_i - z_0)$

Quantum Hall quasiparticles

- Change B away from plateau center: Generate *quasiparticles*
- They are the fundamental charged excitations of the state (gapped).



ig. 10.4 Quasielectrons are excited for $\nu > 1/3$ and quasiholes are excited for $< 1/3$. They are trapped by impurities and do not contribute to the Hall current. he Hall current remains unchanged in the vicinity of $\nu = 1/m$, as is the origin of ie Hall plateau.

[Ezawa, QHE. Copyright World Scientific]

- **Quasielectrons & quasiholes.**
- **Fractional, localized charge e/m**
- **They are anyons!**
- **Exist at all plateaux**

Can be decribed by trial wave functions, e.g. derived using methods from conformal field theory [work with T.H. Hansson and others]

Anyon properties of QH quasiparticles

- Laughlin ($\nu = 1/m$): Quasiholes have charge 1/m and anyon statistics (explicit Berry phase calculation, Arovas et al 1984)

$$\theta_{qh} = \pi/m$$

- General arguments that $\theta_{qh} = \theta_{qe}$

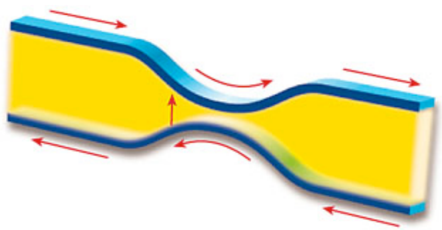
- Hierarchy states ($\nu=p/q$): Typically *several* different fundamental quasiholes with $\theta = r\pi/q$, integer r.

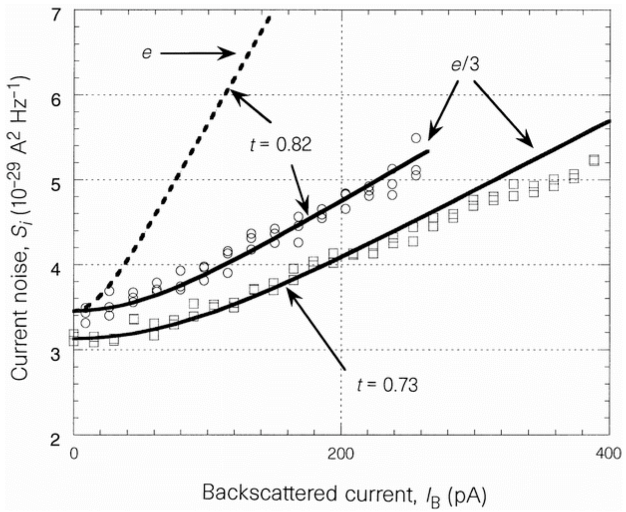
- Several lines of argument: Composite fermions, CFT / Wen classification scheme, clustering arguments, numerical Berry phase calculations..

- Example $\nu = 4/11$: Fundamental holes with statistics 3/11 and 5/11, Laughlin hole with statistics 4/11.

Are these anyonic quasiparticles real?

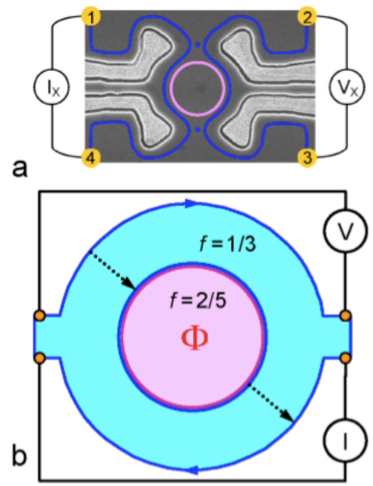
- Fractional charge has been measured!





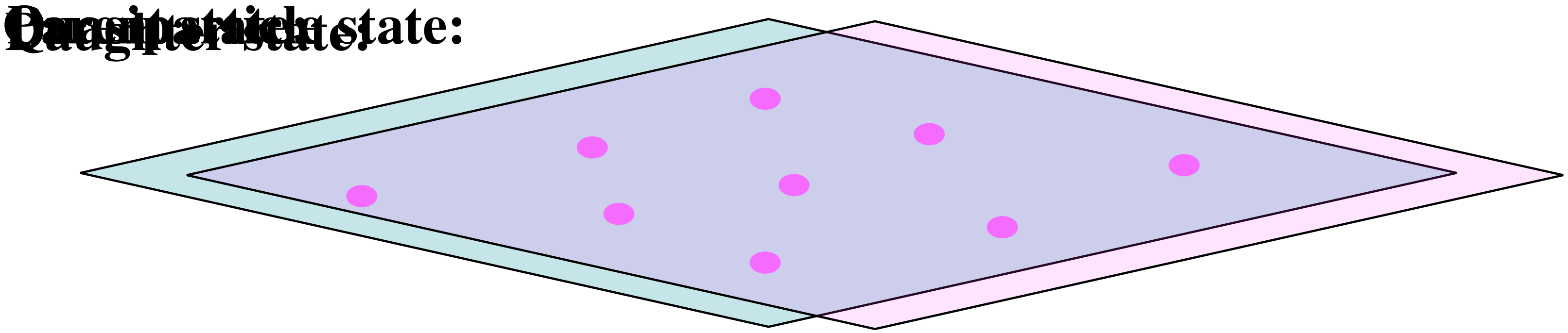
[de Picciotto et al]

- Experiments claiming the direct observation of anyon statistics (interferometry). Still under debate.



[V. Goldman]

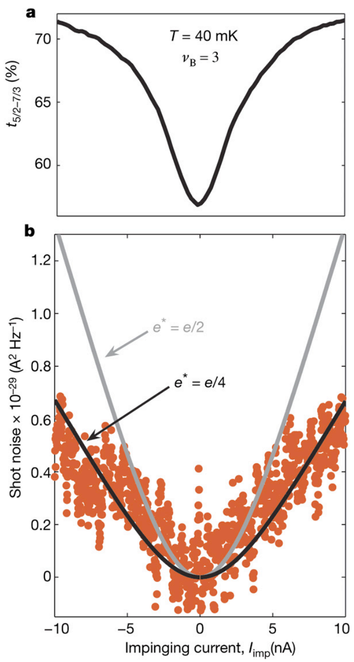
Beyond Laughlin states: hierarchy



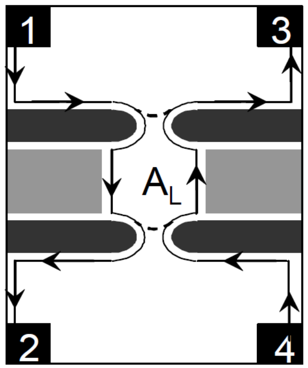
- Wave functions for these general states more complicated than Laughlin. No explicit trial WFs known until recently.
- Using methods from CFT, we have found explicit candidate wave functions for *all* hierarchy states -- qh condensates, qe condensates and mixtures of these. (Work with T.H. Hansson, A. Karlhede et al)

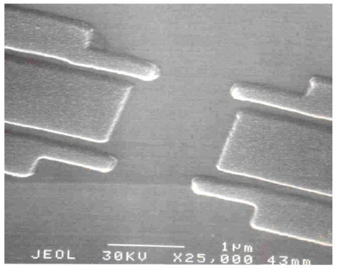
Even weirder quantum Hall states...

- *ν* = 5/2 plateau: Believed to be described by *Pfaffian* wave function. Paired state.
- Quasiparticles: *Non-Abelian anyons.* $\psi_\alpha \to \rho^{(ij)}_{\alpha\beta}\psi_\beta$



[Heiblum group]





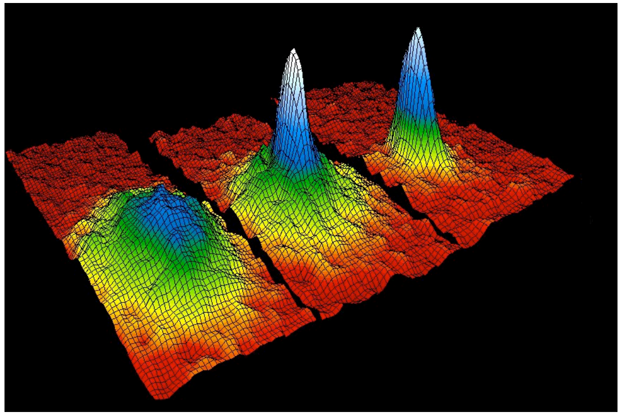
Willett et al 08

Fantasy: Use these quasiparticles to build a topologically protected quantum computer...

(cfr Hans Hansson's lecture)

And now for something completely different...

Atomic Bose condensates

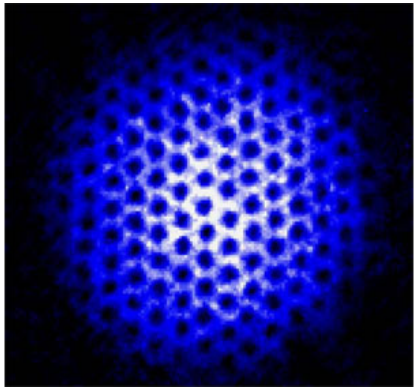


Alkali atoms in magnetic traps

Experimental traps well approximated by harmonic oscillator potential

$$T \leq \mu K, \quad N \approx 10^3 - 10^6$$

Rotating BEC (stirring): Angular momentum carried by *vortices* (vortex lattice). Several hundred vortices observed in experiment



[JILA]

Rotating Bose condensates

- 1995: First atomic Bose condensate
- 1999: First vortex in rotating BEC (JILA, Paris)
- 2004: Abrikosov lattice in lowest Landau level (200 vortices).

Increasing rotation: Cloud flattens out (pancake shape), density decreases → weaker interaction → lowest Landau level.

Eventually: Vortex lattice predicted to melt, so system enters quantum Hall regime. (More on experimental status later!)

Bose condensate in the lowest Landau level

Neutral, rotating bosons behaving like charged fermions (electrons) in a magnetic field?!

Mathematical equivalence in 2D between rotation (harmonic oscillator) and perpendicular magnetic field:

$$\mathcal{H} = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + \frac{1}{2}m\omega^2(x^2+y^2)$$
$$= \frac{1}{2m}\left(\mathbf{p} - \mathbf{A}\right)^2 + \omega L_z$$

$$L_z = xp_y - yp_x$$

$$\mathbf{A} = m\omega\,(-y, x) \;\rightarrow\; \mathbf{B} \equiv \nabla \times \mathbf{A} = 2m\omega\hat{z}$$

Eigenstates: Landau levels in the effective ‘magnetic’ field **B**.

Bose condensate in the lowest Landau level

Full 3D many-body Hamiltonian in rotating frame

$$\mathcal{H} = \sum_{i=1}^{N} \left[\frac{\mathbf{p_i^2}}{2m} + \frac{1}{2} m\omega^2 \mathbf{r_i^2} \right] - \Omega L_z + H_I$$

Sufficiently weak interaction compared to harmonic oscillator gap:
Restrict to lowest Landau level in the effective 'magnetic' field:

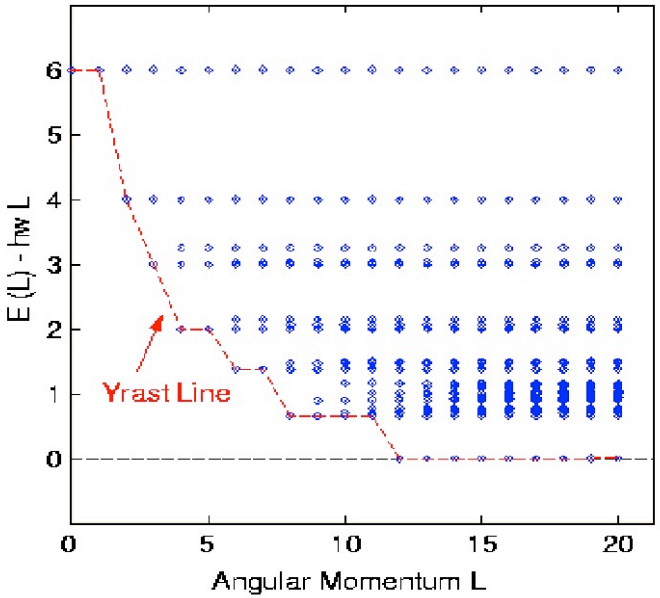
$$\mathcal{H} = (\omega - \Omega)\, L_z + g \sum_{i<j} \delta^2(\mathbf{r_i} - \mathbf{r_j}))$$

Ideal limit, degenerate Landau levels:

$$\Omega = \omega$$

The Yrast spectrum

In the absence of interaction: Lowest N-body state *with given L* is highly degenerate. The interaction lifts this degeneracy and selects the lowest ("yrast") state. (Yrast = "most dizzy")



Recall single particle basis in LLL:

$$|l\rangle = N_l z^l e^{-|z|^2/4}, \qquad z = x + iy$$

N-particle states are constructed as *symmetric combinations* of these, i.e. homogeneous polynomials. Total angular momentum = degree of polynomial.

Laughlin-type wave functions

Like in the QHE, there are particularly well correlated wave functions of the Jastrow form (suppressing the exponential for simplicity)

$$\psi_N = \prod_{i<j} (z_i - z_j)^2 \, f(z)$$

where *f(z)* is a symmetric polynomial in the coordinates. For all wave functions of this type, the delta function interaction energy is zero.
=> Yrast line for L=N(N-1) and above

In particular, L = N(N-1): m=2 Laughlin state [Wilkin et al 1998]

$$\boxed{\psi_N = \prod_{i<j} (z_i - z_j)^2}$$

Composite fermions

Statistical transmutation in 2D : Bosons can be turned into fermions and vice versa by ”attaching” an odd number of flux quanta to the particle. (Due to the additional Aharonov-Bohm phase). Attaching an *even* number of flux quanta conserves the statistics.

Technically, “attaching a flux quantum” corresponds to multiplying the wave function by a Jastrow factor ,

$$\prod_{i<j}(z_i - z_j)$$

Jain very successfully constructed trial wave functions for the FQHE by mapping the strongly interacting electrons to weakly interacting composite fermions (each “swallowing” 2 quanta of external flux), thus moving in a reduced external magnetic field.

Rotating bosons as composite fermions

Modify Jain´s construction by attaching *one* flux quantum to each particle, thus mapping $\nu = 1/2$ bosons into $\nu^* = 1$ composite fermions.

$$\psi(z_1, ..., z_N) = \mathcal{P}\left[e^{-\sum_i |z_i|^2/4} f_S(z, \bar{z}) \prod_{i<j}(z_i - z_j)\right]$$

$f_S(z, \bar{z})$ Slater determinant of single particle wave functions

$\mathcal{P}$: Projection to LLL. Essentially $\bar{z}_i \rightarrow \partial/\partial z_i$

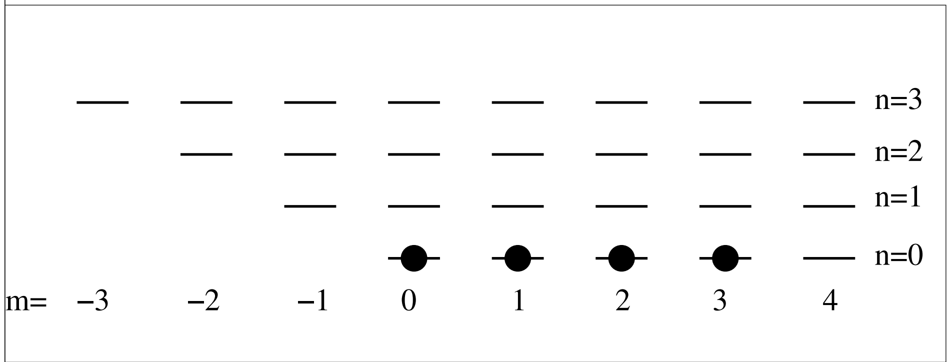
Overlap with exact wave functions in the QH regime of the bosonic Yrast spectrum: Typically 98% or more.

[Cooper & Wilkin (99), Wilkin & Gunn (2000), Viefers et al PRA (2000), Chang et al PRA (2005)]

Rotating bosons as composite fermions

Example 1: Laughlin state, L=N(N-1)

Slater determinant:



$$f_T = \begin{vmatrix} 1 & 1 & 1 & 1 \\ z_1 & z_2 & z_3 & z_4 \\ z_1^2 & z_2^3 & z_3^2 & z_4^2 \\ z_1^3 & z_2^3 & z_3^3 & z_4^3 \end{vmatrix} = \prod_{i<j}(z_i - z_j)$$

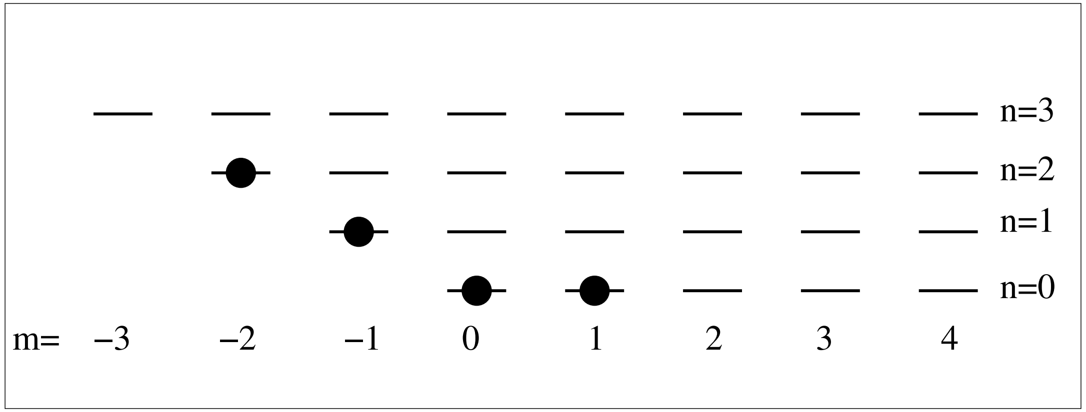
Wave function:

$$\psi = \prod_{i<j}(z_i - z_j)^2 e^{-\sum_i |z_i|^2/4}$$

Rotating bosons as composite fermions

Example 2: Single vortex, L=N

Slater determinant:



$$f_T = \begin{vmatrix} \bar{z}_1^2 & \bar{z}_2^2 & \bar{z}_3^2 & \bar{z}_4^2 \\ \bar{z}_1 & \bar{z}_2 & \bar{z}_3 & \bar{z}_4 \\ 1 & 1 & 1 & 1 \\ z_1 & z_2 & z_3 & z_4 \end{vmatrix}$$

LLL projection:

$$\bar{z}_i \longrightarrow \partial/\partial z_i$$

Anyonic excitations

Provided bosonic incompressible QH states can be fabricated, their qp excitations would be anyons. E.g. semionic quasiholes of the bosonic Laughlin state:

$$\psi = \prod_i (z_i - z_0) \prod_{i<j} (z_i - z_j)^2$$

Like the atoms themselves, these are *charge-neutral.* There have been proposals how to create, detect and manipulate such quasiparticles. [Paredes et al 2001]

Non-Abelian states

Non-Abelian states in conventional QH systems have received much attention due to proposals to use their anyonic quasiparticles for topologically protected quantum computing.

Cold atom systems are generally much easier to manipulate and tune, and thus may eventually be better candidates for this.

Bosonic non-Abelian states are predicted to appear in the lowest Landau level, while only realized in higher LLs in conventional QHE. In particular, Pfaffian wins over Fermi sea at ν=1.

$$\psi_{Pf} = \mathcal{S}\left[\prod_{i=1}^{N/2}(z_i - z_j)^2 \prod_{k=N/2+1}^{N}(z_k - z_l)^2\right]$$

Summary: Fermionic vs bosonic QHE

- ☑ Incompressible states, Abelian and non-Abelian hierarchies
- ☑ Anyonic excitations, Abelian and non-Abelian

$$\psi = \prod_{i<j} (z_i - z_j)^2$$

Bose-Laughlin state

- Qualitative differences, e.g. ground state at $\nu = 1$ is *Pfaffian*
- Although major challenges remain to realize these states in experiment, they may eventually be superior to conventional QH systems -- high tunability of experimental parameters and less decoherence. (More on this later)

Low angular momenta (L<N)

Exact, analytic yrast wave functions are known for the angular momentum regime L=0, 1, ..., N [Wilkin et al, Bertsch & Papenbrock].

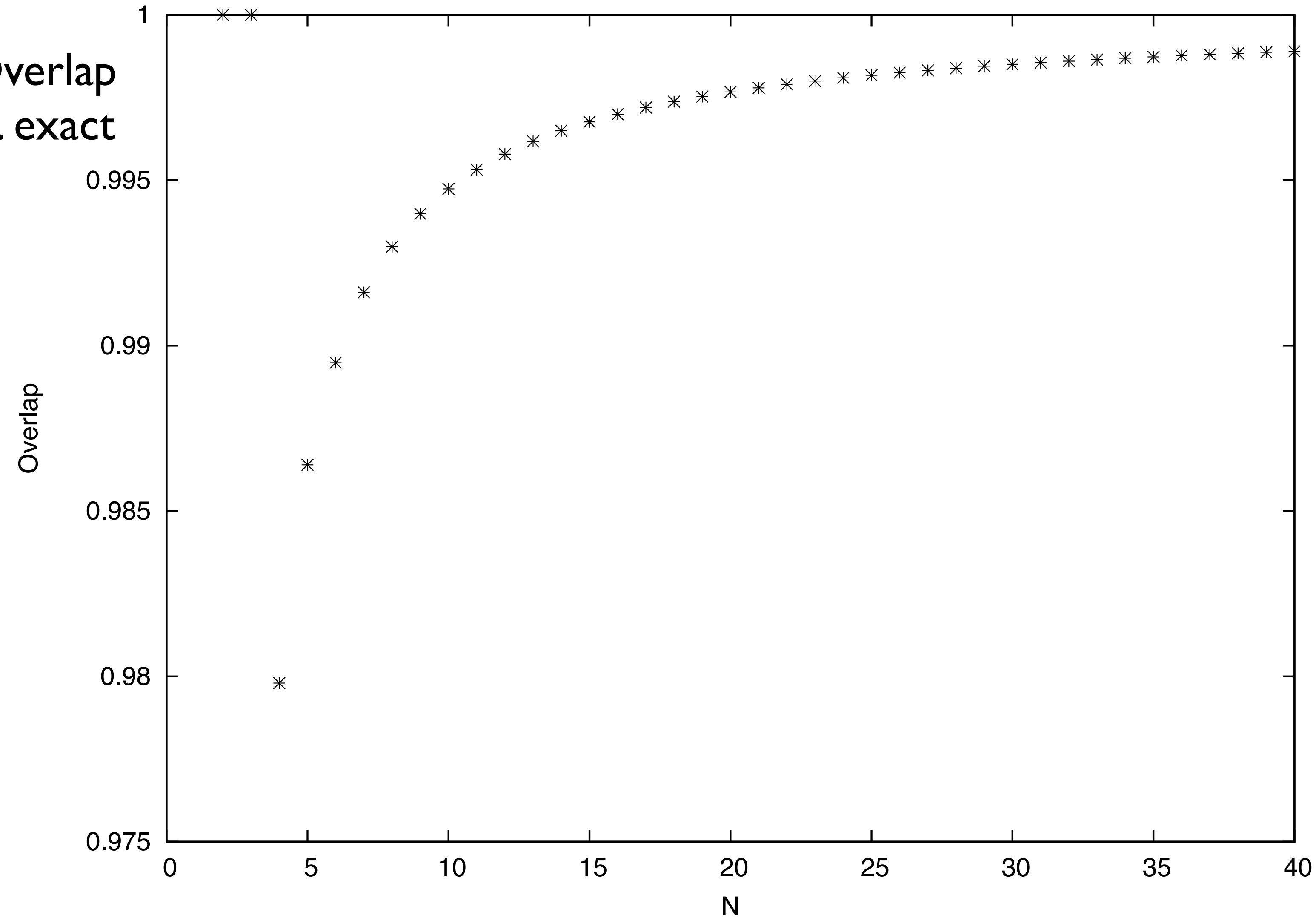
CF phenomenology is, *a priory,* not expected to work for angular momenta this low, far below the QH regime.

Surprise: Applying the CF construction to this regime gives analytical trial wave functions which are strikingly similar in structure to the exact ones!
[Viefers & Kristiansen, (2007), Korslund & Viefers (2006), Viefers & Taillefumier (2010)]

Overlaps between CF- and exact wave functions increase with N. Numerical and analytical evidence that CF trial wave functions in this angular momentum regime thus become *exact in the thermodynamic limit.* [Viefers & Kristiansen, (2007), Korslund & Viefers (2006), Viefers & Taillefumier (2010)]

The single vortex: overlap

Overlap
w. exact



N

Two-component systems

- Much recent interest in (rotational) properties of *two-species* Bose systems.
- E.g. mixture of two types of atoms, two isotopes of the same atom, two hyperfine states of the same atom (all experimentally realized)
- Several parameters can in principle be varied -- inter- vs intraspecies interaction, particle numbers, masses..
- Rich physics. E.g. miscible to immiscible phase transition (non-rotating) as interspecies interaction gets large; defects such as coreless vortex lattices (square, triangular).
- Convenient language: "Pseudospin" 1/2 (at least for homogeneous interaction), label the species "up" and "down".

Two-component systems: QH regime

- Two-species Bose gases in the quantum Hall regime: Several recent theoretical studies. [Ueda group, Jain group]
- Proposed (under discussion): NASS state at *ν=4/3*

$$\psi = \mathcal{S}_{group}\left[\tilde{\psi}^{221}\tilde{\psi}^{221}\right] \qquad \tilde{\psi}^{221} = (\uparrow - \uparrow)^2(\downarrow - \downarrow)^2(\uparrow - \downarrow)$$

- Fundamental quasiholes: charge e/3 non-Abelian anyons
- State at integer filling *ν=2:"Symmetry-protected topological"* (SPT) state. No 'intrinsic' topological order, no fractional excitations, but non-trivial (gapped bulk state, gapless edge modes,quantized hall conductance), provided SU(2) pseudospin symmetry (and particle number conservation) are not violated.

Two-component systems: Slow rotation

[M.L. Meyer, G.J. Sreejith, SV]

- Recent work [Papenbrock et al] identified a class of analytically exact many-body eigenstates for low angular momenta $L \leq M \quad (M \geq N)$
- We study low L regime in terms of composite fermions, exploiting (pseudo)spin analogy for homogeneous interaction.
- CF basis states: Products of two Slater determinants, one for each species,

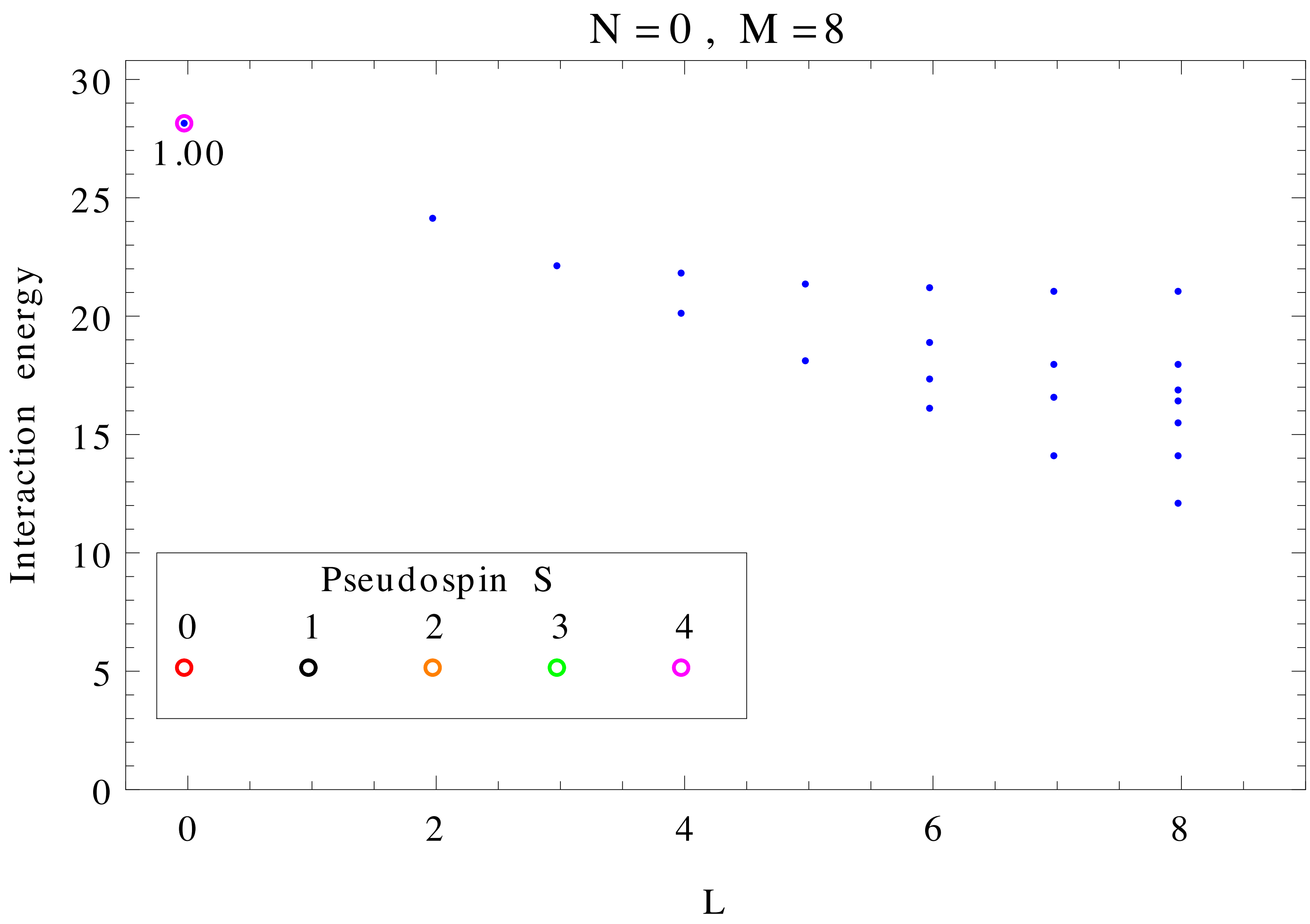
$$\psi_{CF} = P_{LLL} \left[\Phi_z \Phi_w J_{N,M} \right]$$

- a) CF diagonalization in compact states
- b) Consider only *simple states* -- candidate CF states with at most one CF per lambda level

Two-component systems: Slow rotation

[M.L. Meyer, G.J. Sreejith, SV]

- CF diagonalization in compact "highest weight" states: Reproduces all Papenbrock states, and some additional ones exactly, CF basis smaller than dimension of Hilbert space.
- *Simple* state diagonalization: Ground states and some low-lying states. Exact or very high overlap (on average 98% for eight particles). Including states *outside* Papenbrock regime.
- Number of simple states *much* smaller than dimension of Hilbert space (1-2 orders of magnitude for eight particles), can find very good approximation to low-lying states dramatically faster than with full numerical diagonalisation.
- Gemelke experiments (single component, see below) are in this regime. Future experiments?



N = 0 , M = 8
Interaction energy
L
1.00
Pseudospin S
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3
4
0
5
10
15
20
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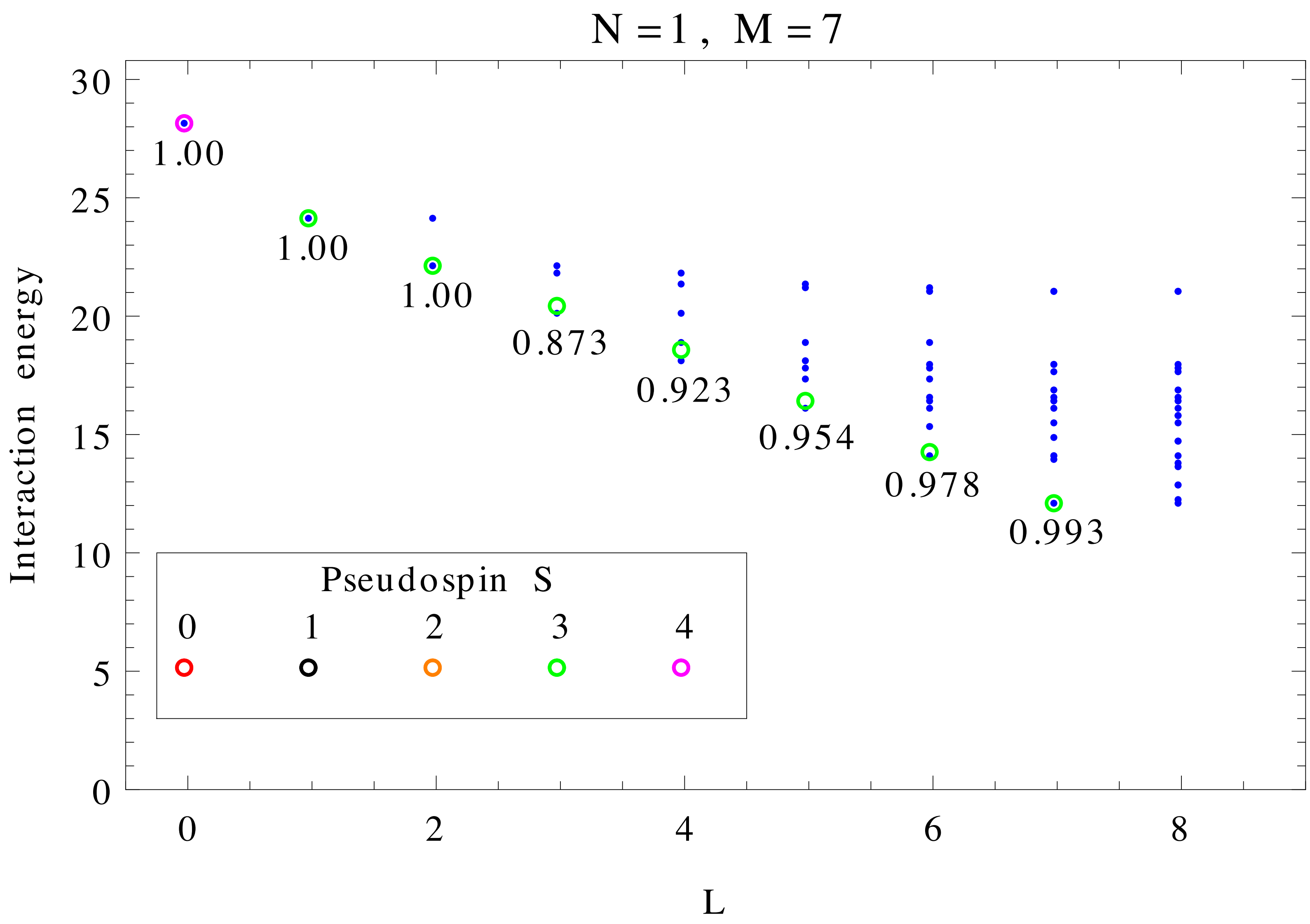
0
2
4
6
8


N = 1 , M = 7
Interaction energy
L
1.00
1.00
1.00
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0.923
0.954
0.978
0.993
Pseudospin S
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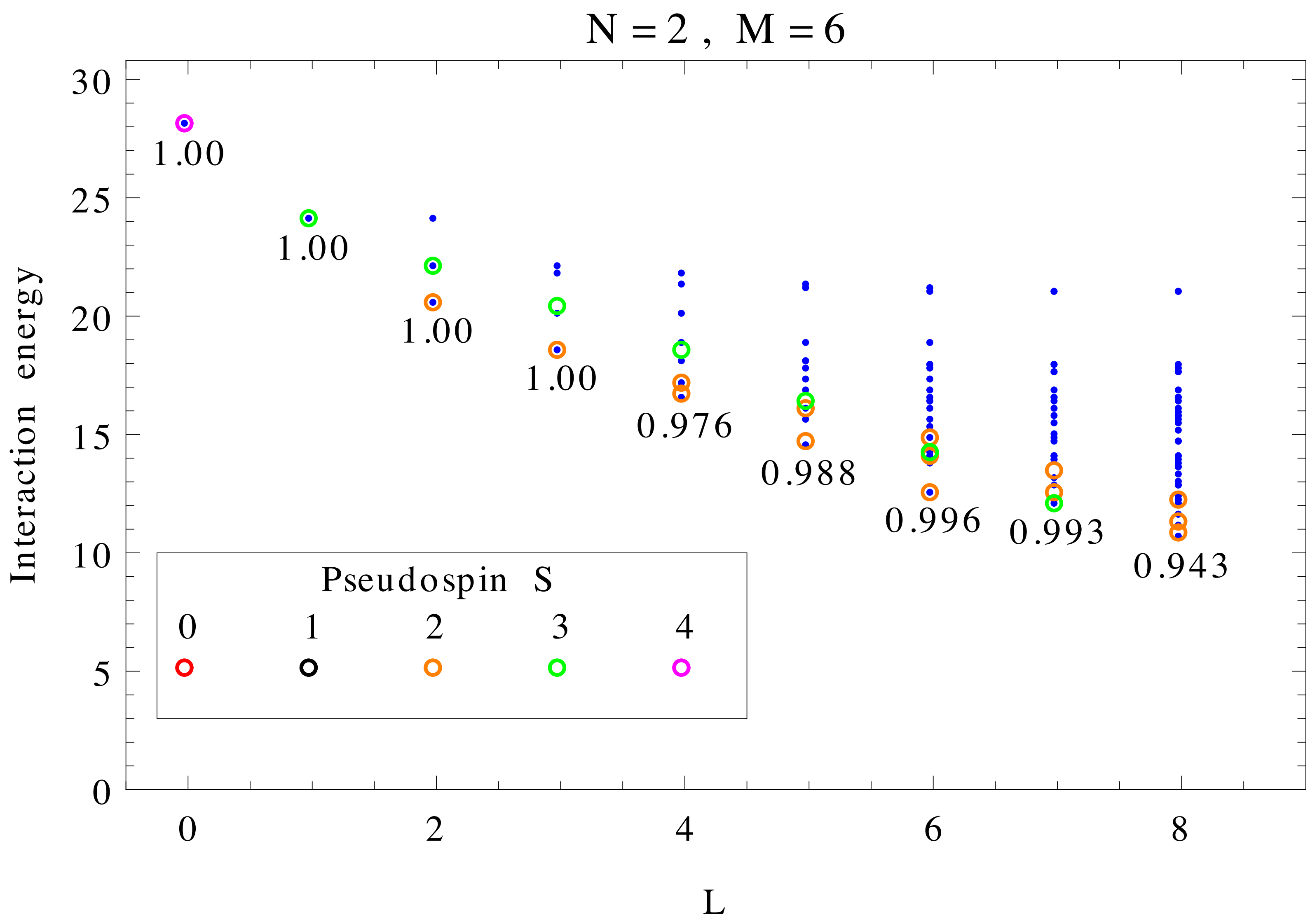
N = 2 , M = 6
Interaction energy
L
0
2
4
6
8
0
5
10
15
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30
1.00
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0.993
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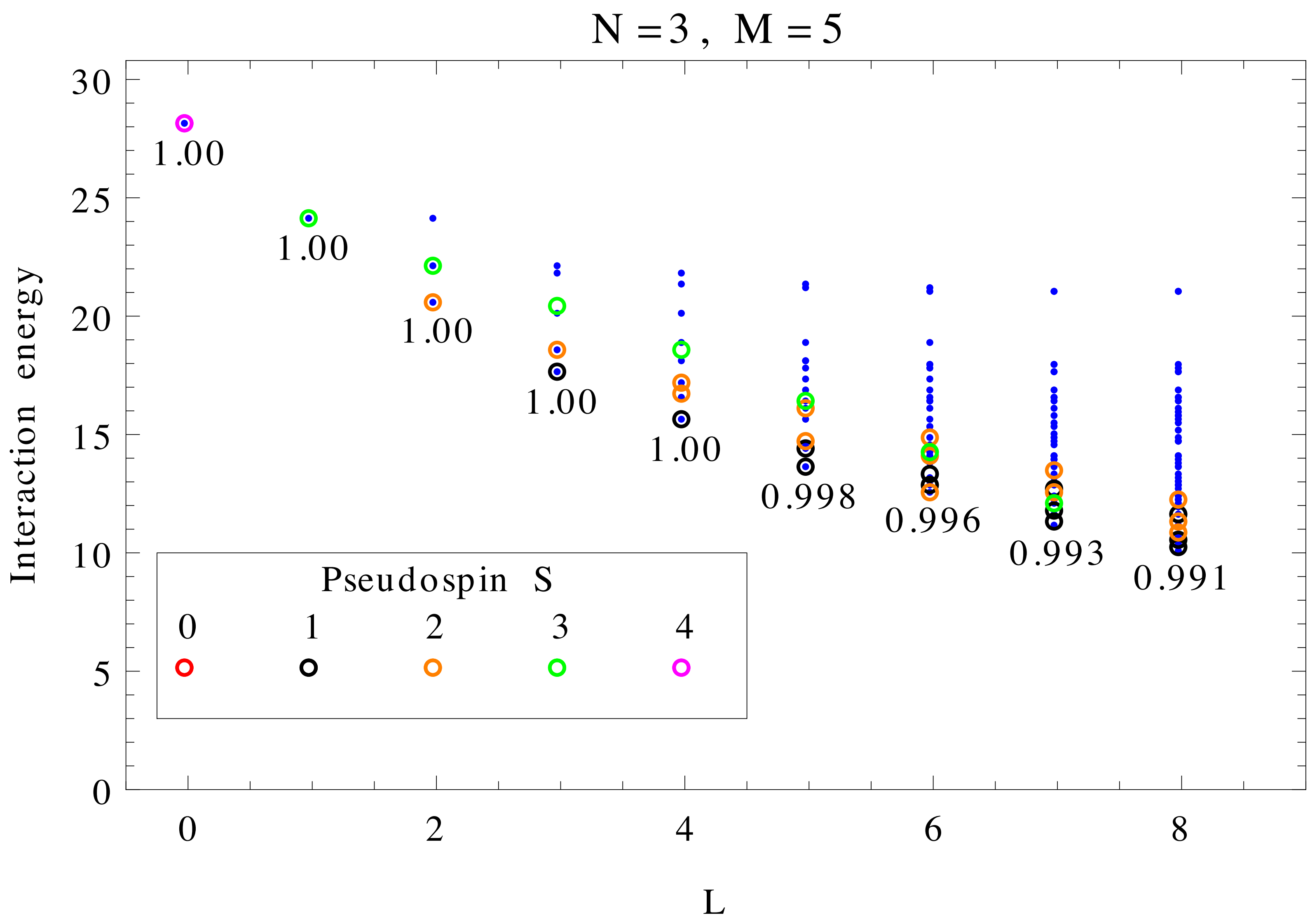
N = 3, M = 5
Interaction energy
L
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Pseudospin S
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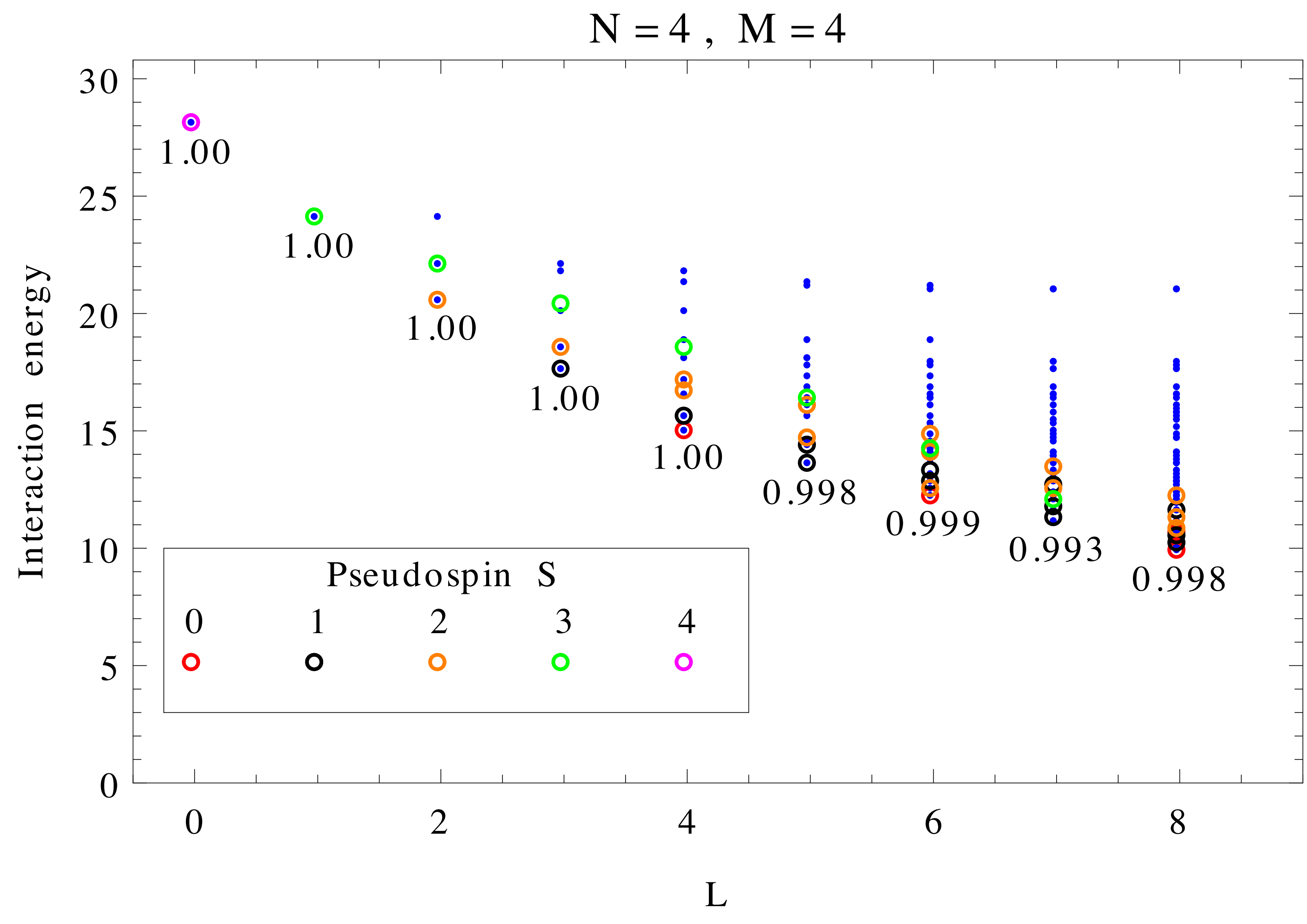
N = 4 , M = 4
Interaction energy
L
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25
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15
10
5
0
0
2
4
6
8
1.00
1.00
1.00
1.00
1.00
0.998
0.999
0.993
0.998
Pseudospin S
0
1
2
3
4

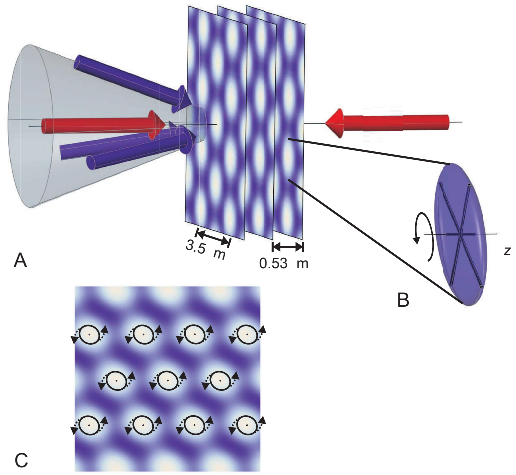
Experimental realization of QH states?

- Experiments have reached LLL $(\omega > 0.99\,\Omega)$ but not the vortex melting transition.
- For harmonic confinement, cloud will explode at $\omega = \Omega$!
- Some suggested solutions:

1. *Start from rotating ring, adiabatically transform to oscillator [Dalibard group 2011]*
2. *Add small quartic component to V(r)? Has been done. Recent theoretical claim: Bose-Laughlin state attainable with presently accessible rotation rates, but requires fine-tuning.*
3. *Co-rotating optical lattice (theoretical proposal)?*
4. *Non-rotating optical lattice, B-field simulated by laser-induced hopping?*

Experiments: Recent developments

- Reports of *small* systems (N<10, ensemble measurements) reaching FQH regime in novel type of optical lattice with local rotation of each site. [Gemelke et al, arXiv:1007:2677]

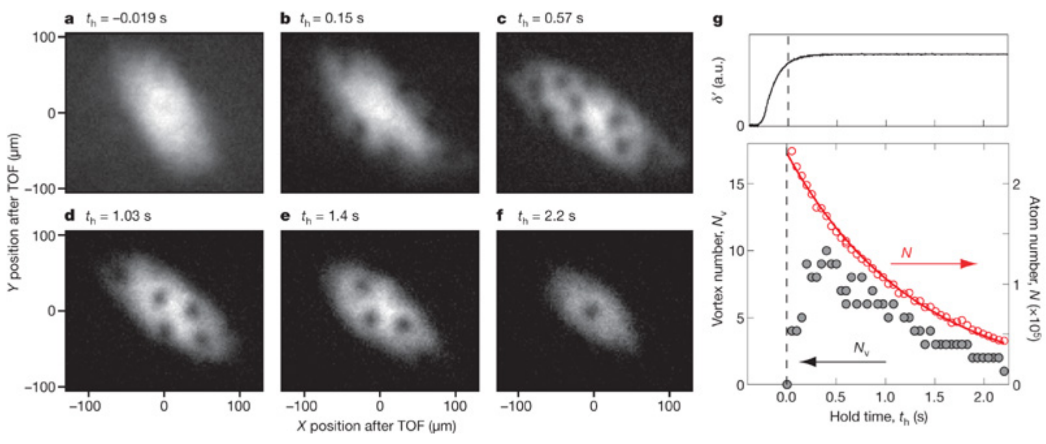


- Correlation- and momentum distribution measurements compared to numerics, seem to indicate strongly correlated quantum Hall-type states.

- Still under debate.

Experiments: Recent developments

- Synthetic (artificial) gauge fields instead of rotation.
- Use atom-light interactions to simulate the effect of a gauge field
- Vortices observed, not yet in QH regime
- Detailed theoretical proposals exist, eg Juliá-Diáz et al, NJP (2012)



Summary

- Exotic quantum phenomema in low-dim systems interesting from both fundamental and technological point of view.
- Quantum Hall effect: Realization of incompressible quantum liquid with anyonic quasiparticles.
- Fractional charge observed, fractional statistics *maybe* observed
- Very similar physics (purer) in rapidly rotating Bose condensates, but experimental challenges remain...
- Graphene (FQHE observed spring/summer 2009!)
- Fantasy of topological quantum computing far into future, but drives lots of basic research activities, has spurred renewed interest in (non-Abelian) anyons.

Some review papers:

N. R. Cooper, Adv. Phys. **57**, 539 (2008)

SV, J.Phys.: Cond. Mat. **20**, 123202 (2008)

A. Fetter, Rev. Mod. Phys. **81**, 647 (2009)

I. Bloch et al, Nature Physics **8**, 267 (2012)