Evolution of Bose-Einstein Condensates in a Gravitational Cavity

Javed Akram[1] and Axel Pelster[2]

[1]Freie Universität Berlin, Germany

[2]Technische Universität Kaiserslautern, Germany

Abstract

We investigate both the static and the dynamic properties of weakly interacting Bose-Einstein condensates (BEC) in a one-dimensional gravitational cavity [1,2]. There the effect of gravity is compensated by an exponentially decaying potential, which is created by the total internal reflection of an incident laser beam from the surface of a dielectric serving as a mirror for the atoms [3,4]. By solving the underlying Gross-Pitaevskii equation with a variational Gaussian condensate wave function [5], we derive a coupled set of differential equations for both the width and the height of the condensate. By considering small deflections around the respective equilibrium positions, we determine the collective excitations of the BEC. Furthermore, we analyze how the BEC cloud expands ballistically due to gravity after switching off the evanescent laser field.

Gross-Pitaevskii (GP) Equation

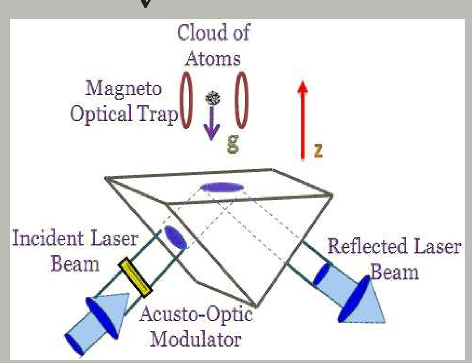
- The dynamics of one dimensional BEC at zero temperature is determined by the time dependent GP Equation

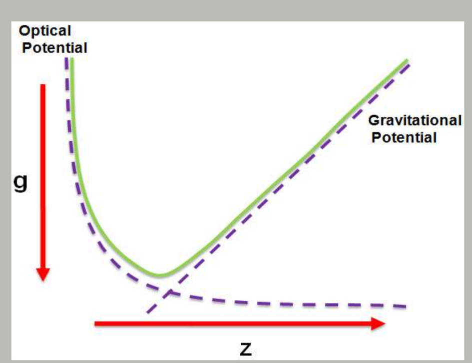
$$\iota\hbar\frac{\partial}{\partial t}\Psi(z,t) = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + V(z) + G\|\Psi(z,t)\|^2\right\}\Psi(z,t)$$

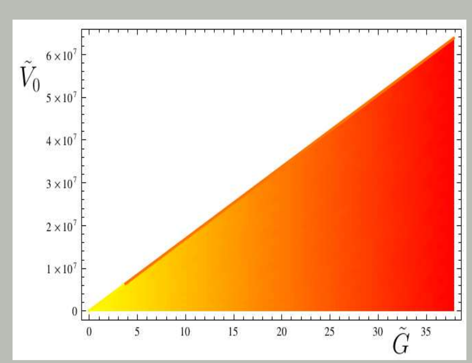
- The last term represents the two-particle interaction of BEC atoms, where its strength $G = 2a\hbar\omega_r$ is related to the s-wave scattering length a, ω_r denotes a radial frequency,
- $V(z) = V_0 e^{-\kappa z} + mgz$ is the potential energy,
- Here, we consider the one-dimensional Gaussian trial function as

$$\Psi(z,t) = \sqrt{\frac{2N}{\sqrt{\pi}A(t)\left[1+\mathrm{Erf}\left(\frac{z_0(t)}{A(t)}\right)\right]}} \times \exp\left\{-\frac{[z-z_0(t)]^2}{2A(t)^2} - iR(t)\left[z-z_0(t)\right]^2 - i\alpha(t)\left[z-z_0(t)\right]\right\}$$

- Here $z_0(t)$ is the mean height of the BEC from the prism surface
- $A(t)$ stands for the width of the BEC
- $R(t)$ and $\alpha(t)$ represent variational parameters
- $\mathrm{Erf}(y) = \frac{2}{\sqrt{\pi}}\int_0^y e^{-x^2}dx$ denotes the error function







Dimensionless Parameters and Soft-wall Approximation

- Dimensionless time $\tau = \omega_z t$ with the classical frequency $\omega_z = \sqrt{g\kappa}$
- $\tilde A(\tau) = \kappa A(t)$ as a dimensionless width of the BEC
- $\tilde z_0(\tau) = \kappa z_0(t)$ as a dimensionless mean height of the BEC
- $\tilde R(\tau) = R(t)/\kappa^2$ and $\tilde\alpha(\tau) = \alpha(t)/\kappa$ as dimensionless variational parameters for the BEC dynamics
- $\tilde\omega = \hbar\kappa\omega_z/gm$ as a dimensionless frequency
- $\tilde V_0 = \kappa V_0/gm$ as a dimensionless strength of the evanescent field
- $\tilde k = \hbar^2\kappa^3/gm^2$ as a dimensionless kinetic energy
- $\tilde G = \tilde\omega_r\tilde a$ with $\tilde a = a\kappa$ being a dimensionless s-wave scattering length
- The soft-wall approximation provided that the mean height $\tilde z_0$ is much larger than its width $\tilde A$
- We can achieve this approximation by tuning correspondingly $\tilde V_0$ and $\tilde G$, which can be experimentally adjusted by varying the EW real intensity and by using a Feshbach resonance, respectively
- With this approximation $\tilde z_{0\mathrm{eq}} \gg \tilde A_{\mathrm{eq}}$ the dimensionless Lagrangian reduces to

$$\tilde L \simeq -\frac{N}{16}\left\{8\sqrt{\frac{2}{\pi}}\frac{N\tilde G}{\tilde A(\tau)} + 16\left[\frac{e^{-\frac{\tilde z_0(\tau)^2}{\tilde A(\tau)^2}}\tilde V_0}{\tilde A(\tau)} + \tilde z_0(\tau)\right] - 8\tilde\omega\left[\tilde A(\tau)^2\tilde R'(\tau) - 2\tilde\alpha(\tau)\tilde z_0'(\tau)\right] + \frac{4\tilde k}{\tilde A(\tau)^2}\left[1 + 4\tilde A(\tau)^4\tilde R(\tau)^2 + 2\tilde A(\tau)^2\tilde\alpha(\tau)^2\right]\right\}.$$

Euler-Lagrange Equations

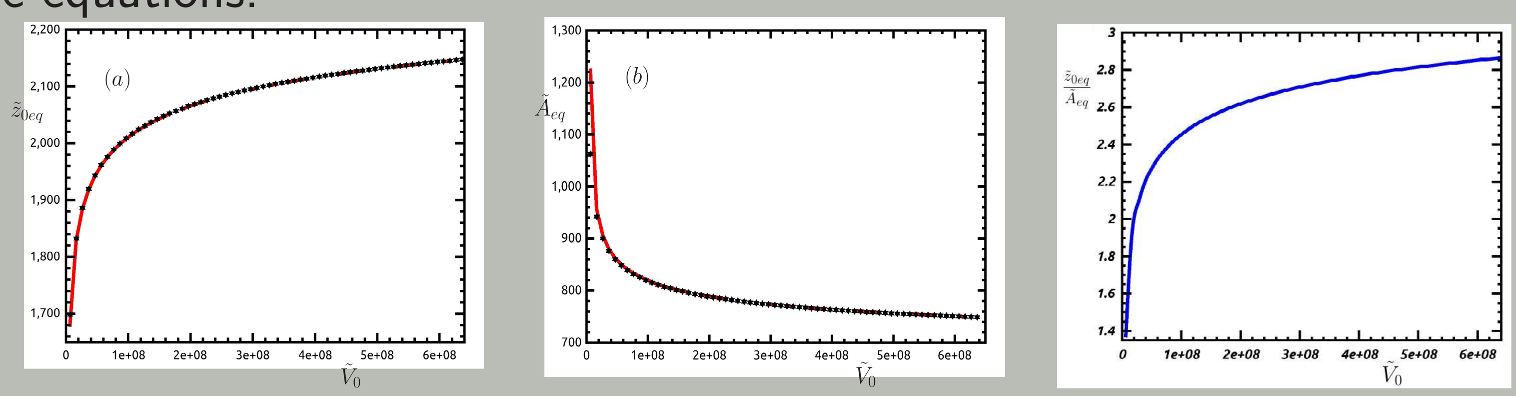
- Finally equations of motion are

$$\tilde A''(\tau) - \frac{2\tilde V_0 e^{-\frac{\tilde z_0(\tau)^2}{\tilde A(\tau)^2}}\left[\tilde A(\tau)^2 - 2\tilde z_0(\tau)^2\right]}{\sqrt{\pi}\tilde A(\tau)^4} = \frac{\tilde k}{\tilde A(\tau)^3} + \sqrt{\frac{2}{\pi}}\frac{\tilde G N}{\tilde A(\tau)^2}$$

$$\tilde z_0''(\tau) + 1 = \frac{2\tilde V_0\tilde z_0(\tau)e^{\frac{-\tilde z_0(\tau)^2}{\tilde A(\tau)^2}}}{\sqrt{\pi}\tilde A(\tau)^3}$$

Static Solutions

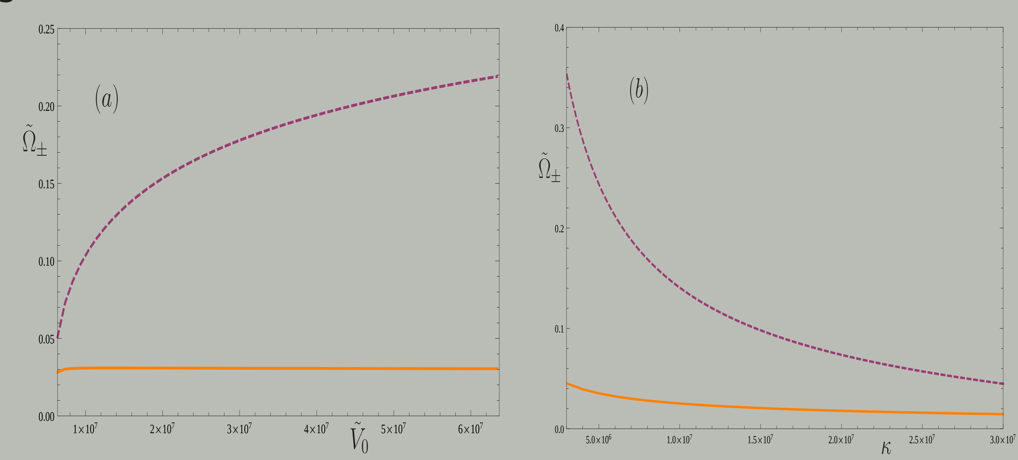
- In order to determine the static solution $\tilde A_{\mathrm{eq}}$, $\tilde z_{0\mathrm{eq}}$, we neglect the derivatives in above equations.



- In order to make our proposed model experimentally realizable i.e. $\tilde z_{0\mathrm{eq}} \gg \tilde A_{\mathrm{eq}}$, we need $\tilde V_0 \gg 4.07\times 10^7$ which is realized in the GOST experiment [2].

Collective Oscillations

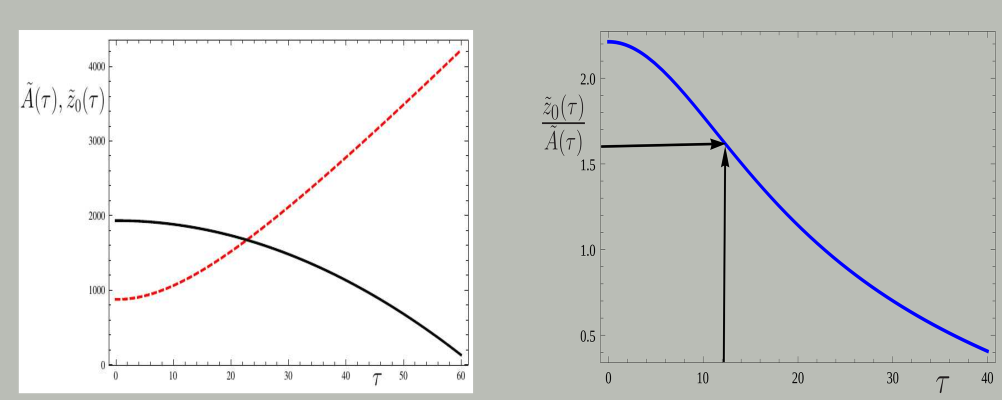
- In order to determine the dynamics for small deflections around the equilibrium position, let us assume for the width of the BEC $\tilde A(\tau) = \tilde A_{\mathrm{eq}} + \delta\tilde A(\tau)$ and for the mean position of the BEC $\tilde z_0(\tau) = \tilde z_{0\mathrm{eq}} + \delta\tilde z_0(\tau)$. If we insert this ansatz for $\tilde A(\tau)$ and $\tilde z_0(\tau)$ in equations of motion, it leads after a linearization to two equations



- Dimensionless collective excitation frequencies $\tilde\Omega_+$ (dashed) and $\tilde\Omega_-$ (solid):
  - (a) as a function of the dimensionless optical decaying strength $\tilde V_0$ for an inverse decaying length $\kappa = 6.67\times 10^6\ \mathrm{m}^{-1}$
  - (b) as a function of the inverse decay length κ while the strength of the EW is $V_0 = 0.96\times k_B\ \mathrm{K}$.

Time-of-flight Expansion

- The standard observation of the BEC is based on suddenly turning off the trapping fields and allowing the atoms to expand ballistically.
- We read off that our approximation $\tilde z_{0\mathrm{eq}}/\tilde A_{\mathrm{eq}} \gg 1$ for a large enough value of $\tilde V_0$ is valid up to the expansion time $t = \tau/\omega_z = 2.7$ ms.



References and Acknowledgements

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axel.pelster@physik.uni-kl.de

javedakram@daad-alumni.de