



Motivation Within a variational approach we describe the physical properties of a Bose-Einstein condensate in a cylinder-symmetric harmonic trap with a single vortex in the center. At first we analyze the equilibrium configuration and determine the vortex size as well as the Thomas-Fermi radii. Then we calculate the critical rotation frequency for the emergence of the vortex and compare our findings with the literature. Finally, we investigate how the presence of the vortex changes the collective excitation frequencies. All results are obtained analytically in form of an asymptotic series in the limit of strong two-particle interactions.

Vortex BEC

• Gross-Pitaevskii (GP) Equation

 \star Time-independent GP equation [1,2]

$$\mu \Psi(\mathbf{r}) = \left\{ -\frac{\hbar^2}{2M} \Delta + V(\mathbf{r}) + gN \left| \Psi(\mathbf{r}) \right|^2 \right\}$$

where $V(\mathbf{r}) = \frac{1}{2}M\omega_{\rho}^{2}\left(\rho^{2} + \lambda^{2}z^{2}\right)$ is a trap with anisotr λ , and $g = 4N\pi\hbar^2 a/m$ is the interaction strength def scattering length a and number of atoms N.

- Thomas-Fermi (TF) Approximation
- * Density profile in TF: $n(\mathbf{r}) = |\Psi(\mathbf{r})|^2 = \begin{cases} \frac{\mu V(\mathbf{r})}{g}, & \text{for } \mu \\ 0, & \text{otherw} \end{cases}$

* TF radii:
$$R_{\perp} = \sqrt{\frac{2\mu}{M\omega_{\rho}^2}}$$
, $R_{\parallel} = \sqrt{\frac{2\mu}{M(\omega_{\rho}\lambda)^2}}$

* Chemical potential: $\mu = \left[\frac{15\omega_{\rho}^{3}\lambda Ng}{8\pi}\left(\frac{M}{2}\right)^{3/2}\right]^{2/5}$

• Quantization of circulation

- * Madelung transformation [3]: $\Psi(\mathbf{r},t) = \sqrt{n(\mathbf{r},t)}e^{iS(\mathbf{r},t)}$
- * Superfluid velocity $\mathbf{v} = \frac{\hbar}{M} \bigtriangledown S(\mathbf{r}, t)$
- ***** Velocity field is irrotational $\nabla \times \mathbf{v} = \mathbf{0}$
- * Circulation around a closed contour $k = 2\pi l \frac{\hbar}{M}$, l is an integer number.
- Structure of Single Vortex
- ★ Solution ansatz $f(\rho, z)e^{il\phi}$ for GP equation [2,4]:

$$-\frac{\hbar^2}{2M} \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) + \frac{\partial^2 f}{\partial z^2} \right] + \frac{\hbar^2}{2M\rho^2} l^2 f + \frac{M}{2} \omega_\rho^2 \left(\rho^2 + \lambda^2 z^2 \right) f + g f^3 = \mu f$$

 \star Healing length

$$\xi = \sqrt{\frac{\hbar^2}{2Mgf^2}} = \frac{1}{\sqrt{8\pi af^2}}$$

 \star For $\rho \ll \xi$ dominant term arises from kinetic energy:

$$-\frac{\hbar^2}{2M} \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) \right] + \frac{\hbar^2}{2M\rho^2} l^2 f = 0$$

Solving for f yields

$$f = c\rho^l, \ n = c^2\rho^{2l}, \ \rho \ll \xi, \ \text{where } c \text{ is a suitable}$$

 \star For $\xi \ll \rho < R_{\perp}$ kinetic energy can be neglected and we have Thomas-Fermi solution:

$$n_{\rm TF}(\rho, z) = \frac{\mu}{g} \left[1 - \left(\frac{\rho}{R_{\rho}}\right)^2 - \left(\frac{z}{R_z}\right)^2 \right]$$

A SINGLE VORTEX IN A BOSE-EINSTEIN CONDENSATE Hamid Al-Jibbouri¹, Vanderlei S. Bagnato², and Axel Pelster³ ¹Institut für Theoretische Physik, Freie Universität Berlin, Germany ²Instituto de Física de São Carlos, Universidade de São Paulo, Brazil ³Fachbereich Physik und Forschungszentrum OPTIMAS, Technische Universität Kaiserslautern, Germany

$$\begin{aligned} \textbf{Variational Approach} \\ \text{ariational ansatz [5-8]:} \\ \Psi(\rho, z, \phi) &= C_{\sqrt{1}} \left(\frac{\rho^2}{(\rho^2 + R_{\mu}^2 \beta^2)} \right)^{1} \left[1 - \left(\frac{\rho}{R_{\mu}} \right)^2 - \left(\frac{z}{R_{\nu}} \right)^2 \right] e^{i(\phi - iB_{\mu}\beta^2) iB_{\nu}z^2} \\ \text{ariational functions in dimensionless form with } P &= Na/l, \, l = \sqrt{\hbar/M\omega_{\mu}} \\ \bar{r}_{z} + r_{z} \left[\lambda^2 + G_{0}(\beta)\beta^2 + G_{1}(\beta)\beta \right] - \frac{G_{2}(\beta)P}{r_{p}^2 r_{z}^2} + 2G_{1}(\beta)\dot{r}_{z}\beta = 0 \\ \bar{r}_{\mu} + r_{\mu} \left[1 + G_{3}(\beta)\beta^2 + G_{1}(\beta)\beta \right] - \frac{G_{2}(\beta)}{r_{p}^2} - \frac{G_{1}(\beta)P}{r_{p}^2 r_{z}^2} + 2G_{1}(\beta)\dot{r}_{p}\beta = 0 \\ (\beta)r_{p}^2 - G_{13}(\beta)r_{z}^2 \right] \bar{\beta} = -5r_{\mu}^2 + r_{z}^2 \lambda^2 - 5r_{\mu}\dot{r}_{\mu} + r_{z}\ddot{r}_{z} + \frac{G_{1}(\beta)}{r_{p}^2} + \frac{G_{3}(\beta)P}{r_{p}^2 r_{z}^2} \\ (\beta)r_{\mu}^2 - G_{10}(\beta)r_{z}^2 + G_{11}(\beta)r_{\mu}\dot{r}_{\mu}\dot{\beta} + G_{12}(\beta)\dot{\beta} + G_{13}(\beta)r_{\mu}^2\beta^2 + G_{12}(\beta)r_{z}^2\beta^2 = 0 \\ \delta_{1}(\beta) - G_{10}(\beta) \text{ are functions which depend on relative vortex core size } \beta \text{ and ave been calculated by using Mathematica.} \\ \textbf{tationary solutions are determined by:} \\ \sigma - \frac{G_{2}P}{r_{p}^2 r_{0}^2} = 0, r_{\mu} - \frac{G_{\mu}P}{r_{\mu}^2 r_{2}^2} - \frac{G_{1}}{r_{\mu}^2} = 0, 5r_{\mu}^2 - \lambda^2 r_{20}^2 - \frac{G_{1}}{r_{\mu}^2} - \frac{G_{2}P}{r_{\mu}^2 r_{20}} = 0 \\ \sigma - \frac{r_{\mu}^2 r_{\mu}^2 r_{0}^2}{r_{\mu}^2 r_{0}^2} - \frac{G_{1}}{r_{\mu}^2 r_{20}^2} - \frac{G_{1}}{r_{\mu}^2 r_{20}^2} - \frac{G_{1}}{r_{\mu}^2 r_{20}^2} - \frac{G_{1}}{r_{\mu}^2 r_{20}^2} = 0 \\ \sigma - \frac{r_{\mu}^2 r_{\mu}^2 r_{\mu}^2 r_{\mu}^2 - \frac{G_{1}}{r_{\mu}^2 r_{20}^2} - \frac{$$

Variational Approach
•Variational ansatz 5-8:

$$\Psi(\rho, z, \phi) = C_{\sqrt{\left(\frac{\rho^2}{\rho^2 + R_{\mu}^2 \beta^2\right)}}^{1} \left[1 + \left(\frac{\rho}{R_{\nu}}\right)^2 - \left(\frac{z}{R_{z}}\right)^2\right]} e^{\beta(\phi + (B_{\mu}^2 + iB_{\nu}^2)}$$
•Variational functions in dimensionless form with $P = Na/l$, $l = \sqrt{h/M\omega_{\rho}}$
 $\vec{r}_{z} + r_{z} \left[\lambda^2 + G_{5}(\beta)\beta^2 + G_{1}(\beta)\beta\right] - \frac{G_{5}(\beta)}{r_{\mu}^2} + 2G_{1}(\beta)r_{\mu}\beta = 0$
 $\vec{r}_{\rho} + r_{\rho} \left[1 + G_{3}(\beta)\beta^2 + G_{4}(\beta)\beta\right] - \frac{G_{5}(\beta)}{r_{\mu}^2} - \frac{G_{5}(\beta)}{r_{\mu}^2 r_{z}} + 2G_{4}(\beta)r_{\mu}\beta^2 = 0$
 $G_{1}(\beta)r_{\rho}^2 - G_{10}(\beta)r_{z}^2 + G_{11}(\beta)r_{\mu}r_{\mu}\beta - G_{12}(\beta)\beta - G_{3}(\beta)r_{\mu}\beta^2 + G_{11}(\beta)r_{\nu}\beta^2 - 0$
 $G_{2}(\beta)-G_{2}(\beta)$ are functions which depend on relative vortex core size β and have been calculated by using Mathematica.
Equilibrium Positions
•Stationary solutions are determined by:
 $2r_{\mu} - \frac{G_{\nu}P}{r_{\mu}^2 r_{20}^2} - 0$, $r_{\mu} - \frac{G_{\mu}P}{r_{\mu}^2 r_{20}} - \frac{G_{\tau}}{r_{\mu}^2} - \beta^2 r_{20}^2 - \frac{G_{\tau}}{r_{\mu}^2} - \frac{G_{\tau}}{r_{\mu}^2} - \frac{G_{\tau}}{r_{\mu}^2} - \frac{G_{\tau}P}{r_{\mu}^2 r_{20}^2} - 0$
Analytical approximation in limit $P \to \infty$:
 $r_{20} - \frac{r_{\lambda}}{R_{\lambda}}, r_{\mu} - (15P\lambda)^{1/5}, \beta_{0} - \frac{\sqrt{r_{\mu}^2}}{r_{\mu}^2 r_{20}^2} + \frac{M_{\mu}^2}{r_{\mu}^2 r_{20}^2} - 0$
For analytical approximation in limit $P \to \infty$:
 $r_{20} - \frac{r_{\lambda}}{R_{\lambda}}, r_{\mu} - (15P\lambda)^{1/5}, \beta_{0} - \frac{\sqrt{r_{\mu}^2}}{r_{\mu}^2 r_{20}^2} + \frac{M_{\mu}^2}{r_{\mu}^2 r_{20}^2} + \frac{M_{\mu}^2}{r$

Variational Approach
• Variational ansatz [5-8]:

$$\Psi(\rho, z, \phi) = C_{\sqrt{\left(\frac{\rho^2}{\rho^2 + R_x^2 \beta^2}\right)^l} \left[1 - \left(\frac{\rho}{R_p}\right)^2 - \left(\frac{z}{R_z}\right)^2\right]} e^{i(\phi + i2\omega^2 + i2\omega^2)}$$
• Variational functions in dimensionless form with $P = Na/l$, $l = \sqrt{h/M\omega_p}$
 $\vec{r}_x + r_z \left[\lambda^2 + G_0(\beta)\beta^2 + G_1(\beta)\beta\right] - \frac{G_2(\beta)P}{r_x^2} + 2G_1(\beta)\vec{r}_x\beta = 0$
 $\vec{r}_p + r_p \left[1 - G_3(\beta)\beta^2 + G_1(\beta)\beta\right] - \frac{G_1(\beta)}{r_x^2} - \frac{G_0(\beta)P}{r_x^2 + 2} + 2G_1(\beta)\vec{r}_x\beta = 0$
 $G_0(\beta)r_p^2 - G_0(\beta)r_z^2 + G_1(\beta)r_pr_p\beta - G_3(\beta)\beta + G_0(\beta)r_x^2\beta^2 + G_1(\beta)r_z^2\beta^2 = 0$
• $G_0(\beta)-G_{12}(\beta)$ are functions which depend on relative vortex core size β and have been calculated by using Molthematica.
Equilibrium Positions
• Stationary solutions are determined by:
 $N^2r_a - \frac{G_2P}{r_x^2}g_a^2 = 0$, $r_{p0} - \frac{G_0P}{r_p^2 r_y^2} - \frac{G_5}{r_{p0}^2} - \lambda^2 r_{p0}^2 - \frac{G_2}{r_{p0}^2} - \frac{G_2P}{r_{p0}^2 r_{p0}} = 0$
• Analytical approximation in limit $P \to \infty$:
 $r_{g0} = \frac{G_2P}{r_x}$, $r_{g0} = (15P\lambda)^{1/5}$, $\beta_0 = \frac{4^{1/6}}{(2^{1/2})^{1/2}}$
Deregies and Critical Botational Frequencies
• Total energy of condensate is given by:
 $E_{neal} = \int d^3r_{1} \left[\frac{\hbar^3}{2M} \bigtriangledown \Psi(\mathbf{r}, t) \bigtriangledown \Psi'(\mathbf{r}, t) + V(\mathbf{r}) |\Psi(\mathbf{r}, t)|^2 + \frac{g}{2} |\Psi(\mathbf{r}, t)|^2 \right]$
• Energies $M^2(r_1 - G_2 + 2SA_1 + 63A_1/\delta_0^2) = 8A_1\left(-30r_{g0}^2 + 35r_{g0}^2 A_1r_0^2$
 $= 6\lambda^2 r_{g0}^2 - 7A_1\lambda^2r_{g0}^2 - 7A_1\lambda^2r_{g0}^2 - 7A_1\beta^2\lambda^2r_{g0}^2 - \frac{175A_1(26 + 33\beta_2)}{r_{g0}^2}$



Pariational Approach
all ansatz [5-8]:

$$\hat{\phi}) = C_{\sqrt{1}} \left(\frac{\rho^2}{(\rho^2 + R_{\mu}^2 \beta^2)} \right)^{l} \left[1 - \left(\frac{\rho}{R_{\mu}} \right)^2 - \left(\frac{z}{R_z} \right)^2 \right]} e^{il\phi(i)B_{\mu}\phi^2(i)B_{\nu}z^2}$$
all functions in dimensionless form with $P = Na/l$, $l = \sqrt{h/M\omega_{\mu}}$
 $\bar{r}_z + r_z \left[\lambda^2 + G_0(\beta)\dot{\beta}^2 + G_1(\beta)\ddot{\beta} \right] - \frac{G_1(\beta)P}{r_{\mu}^2 z} + 2G_1(\beta)\dot{r}_z\dot{\beta} = 0$
 $J \left[1 + G_3(\beta)\dot{\beta}^2 + G_1(\beta)\ddot{\beta} \right] - \frac{G_1(\beta)}{r_{\mu}^2} - \frac{G_0(\beta)P}{r_{\mu}^2 z} + 2G_1(\beta)\dot{r}_{\mu}\dot{\beta} - 0$
 $-G_{12}(\beta)r_z^2 \right] \dot{\beta} = -5r_{\mu}^2 + r_z^2\lambda^2 - 5r_{\mu}\dot{r}_{\mu} - r_z\dot{r}_z + \frac{G_7(\beta)}{r_{\mu}^2} + \frac{G_8(\beta)P}{G_1(\beta)r_{\mu}^2}\dot{\beta}^2 = 0$
 $h_5(\beta)$ are functions which depend on relative vortex core size β and
in calculated by using Mathematica.
Equilibrium Positions
ry solutions are determined by:
 $\frac{P}{r_{30}^2} = 0, r_{\mu 0} - \frac{G_0P}{r_{\mu 0}^2 r_{30}} - \frac{G_5}{r_{\mu 0}^2} - \lambda^2 r_{30}^2 - \frac{G_7}{r_{\mu 0}^2} - \frac{G_8P}{r_{\mu 0}^2 r_{30}} = 0$
all approximation in finit $P \to \infty$:
 $r_{20} = \frac{r_{24}}{r_{\mu}}, r_{\mu 0} = (15P\lambda)^{1/5}, \beta_0 = \frac{5^{2/9}}{\sqrt{2}(4P\lambda)^{2/6}}$
Energies and Critical Rotational Frequencies
regy of condensate is given by:
 $\int d^3r \left[\frac{\hbar^2}{2M} \lor \Psi(r, t) \lor \Psi^*(r, t) + V(r) |\Psi(r, t)|^2 + \frac{g}{2} |\Psi(r, t)|^4 \right]$
of concensate with central vortex in TF-limit:
 $\frac{\omega_{\mu}}{M_1} \left[\frac{300P(-54+28A_1-63A_7\beta_0^2)}{r_{\mu}^2 r_{20}^2} - 8A_1(-30r_{\mu}^2 + 35r_{\mu}^2 A_1\beta_0^2)^2 - R_{\mu}^2 A_1\beta_0^2 + 35r_{\mu}^2 A_1\beta_0^2 + R_{\mu}^2 A_1\beta_0^2 + R_{\mu}^2$

Variational Approach
Variational ansatz [5:8]:

$$\Psi(\rho, z, \theta) = C \sqrt{\left(\frac{\rho^2}{(\rho^2 + R_{\beta}^2\beta)}\right)^{\ell}} \left[1 - \left(\frac{\rho}{R_{c}}\right)^{2} - \left(\frac{z}{R_{c}}\right)^{2}\right]} e^{i(z+\beta)l_{\rho}^{2} - (l_{\sigma}z)^{2}}$$
Variational functions in dimensionless form with $P = No/l$, $l = \sqrt{h/M\omega_{\rho}}$
 $\bar{r}_{\rho} + r_{\rho} \left[\lambda^{2} + G_{0}(\beta)\beta^{2} + G_{1}(\beta)\beta\right] - \frac{G_{2}(\beta)P}{r_{\rho}^{2}} - 2G_{1}(\beta)r_{\rho}\beta = 0$
 $\bar{r}_{\rho} - r_{\rho} \left[1 + G_{3}(\beta)\beta^{2} + G_{1}(\beta)\beta\right] - \frac{G_{2}(\beta)}{r_{\rho}^{2}} - \frac{G_{3}(\beta)P}{r_{\rho}^{2}r_{c}^{2}} + 2G_{4}(\beta)r_{\rho}\beta = 0$
 $\bar{r}_{0}(\beta)r_{\rho}^{2} - G_{5}(\beta)r_{c}^{2}\beta = -5r_{\rho}^{2} + r_{c}^{2}\lambda^{2} - 5r_{\rho}\bar{r}_{\rho} + r_{s}\bar{r}_{c}} + \frac{G_{1}(\beta)}{r_{\rho}^{2}} + \frac{G_{3}(\beta)P}{r_{\rho}^{2}} - \frac{G_{3}(\beta)P}{r_{\rho}^{2}} - \frac{G_{3}(\beta)P}{r_{\rho}^{2}} - \frac{G_{3}(\beta)P}{r_{\rho}^{2}} - \frac{G_{3}(\beta)P}{r_{\rho}^{2}} + \frac{G_{3}(\beta)P}}{r_{\rho}^{2}} + \frac{G_{3}(\beta)P}}{r_{\rho}^{$

Variational Approach
• Variational ansatz [5–8]:

$$\psi(\rho, z, \phi) = C_{\sqrt{\left(\frac{\rho^2}{\rho^2 + R_{\mu}^2 \beta^2\right)}} \left[1 - \left(\frac{\rho}{R_{\nu}}\right)^2 - \left(\frac{z}{R_{\nu}}\right)^2\right]} e^{i(\phi + iR_{\mu}^2 + iR_{\nu}^2)}$$
• Variational functions in dimensionless form with $P = Na/l$, $l = \sqrt{h/M\omega_P}$
 $\vec{r}_{\nu} + r_{\nu} \left[1 - G_{5}(\beta)\beta^2 + G_{4}(\beta)\beta\right] - \frac{G_{5}(\beta)}{r_{\mu}^2 - 2} - G_{1}(\beta)r_{\nu}\beta = 0$
 $\vec{r}_{\nu} + r_{\mu} \left[1 - G_{5}(\beta)\beta^2 + G_{4}(\beta)\beta\right] - \frac{G_{5}(\beta)}{r_{\mu}^2} - \frac{G_{5}(\beta)P}{r_{\mu}^2 - 2} - 2G_{1}(\beta)r_{\nu}\beta = 0$
 $[5G_{11}(\beta)r_{\mu}^2 - G_{12}(\beta)r_{\nu}^2] \vec{\beta} = -5r_{\mu}^2 + r_{\nu}^2\lambda^2 - 5r_{\mu}\beta_{\mu} + r_{\nu}r_{\nu}^2 - \frac{G_{7}(\beta)}{r_{\mu}^2} + \frac{G_{5}(\beta)P}{r_{\mu}^2 r_{\nu}} - 0$
 $(5G_{1}(\beta)r_{\mu}^2 - G_{12}(\beta)r_{\nu}^2] \vec{\beta} = -5r_{\mu}^2 + r_{\nu}^2\lambda^2 - 5r_{\mu}\beta_{\mu} + r_{\nu}r_{\nu}^2 - \frac{G_{7}(\beta)}{r_{\mu}^2} + \frac{G_{5}(\beta)P}{r_{\mu}^2 r_{\nu}} - 0$
 $(5G_{1}(\beta)r_{\mu}^2 - G_{12}(\beta)r_{\nu}^2 + G_{11}(\beta)r_{\mu}r_{\mu}\beta + G_{12}(\beta)\beta + G_{21}(\beta)r_{\mu}^2\beta^2 + G_{41}(\beta)r_{\nu}^2\beta^2 - 0$
 $(5G_{6}(\beta) - G_{12}(\beta))$ are functions which depend on relative vortex core size β and have been calculated by using Mathematica.
Equilibrium Positions
• Stationary solutions are determined by:
 $\lambda^2 r_{20} - \frac{G_{2}P}{r_{\mu}^2 r_{20}^3} - 0$, $r_{\mu q} - \frac{G_{0}P}{r_{\mu}^2 r_{\mu}^2 - 0}$, $5r_{\mu}^2 - \lambda^2 r_{\mu}^2 - \frac{G_{2}r_{\mu}}{\sqrt{2}(2P)^{2}}$
 $- \frac{1}{\sqrt{2}(2P)^{2}r_{\mu}^2}$
 $- \frac{1}{\sqrt{2}(2P)^{2}r_{\mu}$

$$\Psi(\mathbf{r})$$

$$u - V(\mathbf{r}) \ge 0$$

erwise.

e constant.



motion around equilibrium positions:

$$\delta \ddot{r}_{z1}(t) + m_0 \delta eta_1(t) \\ \delta \ddot{r}_{
ho 1}(t) + m_4 \delta \ddot{eta}_1(t)$$

$$m_8 \delta \ddot{\beta}_1(t) + 5r_{\rho 0} \delta \ddot{r}_{\rho 1}(t) - r_{z 0} \delta \ddot{z}_{z 1}(t)$$

• Frequencies of collective modes are given

$$\omega_{1,2}^2 = \frac{-z_2}{-z_2}$$

• In the limit P going to infinity,



- tional approach.
- analytical approximation for the stationary points of the system.
- \star We studied frequencies of collective modes.
- Acknowledgment German Academic Exchange Service (DAAD)

• Frequencies of collective modes are determined by linearizing equations of

 $(1) + m_1 \delta r_{\rho 1}(t) + m_2 \delta u_{z1}(t) + m_3 \delta \beta_1(t) = 0,$) + $m_5 \delta r_{\rho 1}(t) + m_6 \delta r_{z1}(t) + m_7 \delta \beta_1(t) = 0$, $+ m_9 \delta r_{\rho 1}(t) + m_{10} \delta r_{z1}(t) + m_{11} \delta \beta_1(t) = 0$

 $r_{2} \pm \sqrt{z_{2}^{2} - 4z_{1}z_{3}}$

*We studied a Bose-Einstein condensate with a central vortex using a varia-

 \star We discussed the equilibrium positions of the condensate, we also found an

 \star We studied the energy of a condensate with a central vortex as well as the critical frequency of a rotating trap at which the vortex state becomes stable.



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A SINGLE VORTEX IN A BOSE-EINSTEIN CONDENSATE

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