Exact Numerical Study of Rotating Ideal Bose-Einstein Condensates

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Motivation: Semiclassical (SC) treatment of ideal Bose-Einstein condensates (BEC) was previously used [1] to describe results of recent experiments with fast-rotating ^{87}Rb atoms in an anharmonic trap [2]. We apply newly developed abitrary-order short-time propagator expansion technique [3] to exactly calculate static and dynamic properties of such condensates and to estimate the quality of SC results. We also show that first correction to SC result for T_c diverges for critical rotation, where only the presented exact approach can be used to assess SC results.

Properties of ideal BECs

- **Grand-canonical ensemble**
  - ⋆ Cumulant expansion for the free energy:

$$\mathcal{F} = -\frac{1}{\beta}\ln\mathcal{Z} = \frac{1}{\beta}\sum_k \ln(1-e^{-\beta(E_k-\mu)}) = -\frac{1}{\beta}\sum_{m=1}^{\infty}\frac{e^{m\beta\mu}}{m}\mathcal{Z}_1(m\beta)$$

  where $\mathcal{Z}_1(m\beta)$ is one-particle partition function
  - ⋆ Total number of particles is sum of the ground-state occupancy N_0 and the number of thermal particles, given by $-\partial\mathcal{F}/\partial\mu$
  - ⋆ In BEC phase we set $\mu = E_0$, while in the gas phase $N_0 = 0$
- **Static properties**
  - ⋆ The condensate temperature T_c is defined by $\mu = E_0$ and $N_0 = 0$, where:

$$N_0 = N + \partial\mathcal{F}/\partial\mu = N - \sum_{m=1}^{\infty}(e^{m\beta\mu}\mathcal{Z}_1(m\beta) - 1)$$

  - ⋆ This also determines the condensate fraction N_0/N for $T < T_c$
  - ⋆ Diagonal density matrix elements $\langle\hat\Psi^\dagger(\vec r)\hat\Psi(\vec r)\rangle$ give the density profile

$$n(\vec r) = N_0|\psi_0(\vec r)|^2 + \sum_{n\geq 1} N_n|\psi_n(\vec r)|^2$$

  where thermal occupancies N_n are given by the Bose-Einstein distribution
- **Dynamic properties**
  - ⋆ Time-of-flight absorption pictures are obtained by switching off the trapping potential and allowing the gas to freely expand
  - ⋆ The density profile after time t is given by

$$n(\vec r, t) = N_0|\psi_0(\vec r, t)|^2 + \sum_{n\geq 1} N_n|\psi_n(\vec r, t)|^2$$

  where $\psi_n(\vec r, t) = \frac{1}{(2\pi)^3}\int d^3\vec k\, d^3\vec R\, e^{-i\omega_k t + i\vec k\cdot\vec r - i\vec k\cdot\vec R}\psi_n(\vec R)$
- **SC approximation**
  - ⋆ Static and some dynamic properties of BECs can be calculated in SC approach [1]
  - ⋆ First correction to SC result for T_c is known for harmonic [3] trap and can be calculated also for other trapping potentials
  - ⋆ However, for critically rotating ideal BEC in a quartic anharmonic trap $V_{\rm BEC}$ [1, 2], first correction

$$\frac{\Delta T_c}{T_c^0} = -\frac{2\,\zeta(3/2)\,E_0}{5\,\zeta^{3/5}(5/2)}\left(\frac{M^2\pi}{4k\hbar^6\omega_z^2}\right)^{1/5}\frac{1}{N^{2/5}}$$

  diverges for small anharmonicity k and cannot be used to assess SC result

Arbitrary-order short-time propagator expansion

- **Discretized effective potential**
  - ⋆ Recently developed approach [4] allows analytic calculation of the arbitrary-order short-time expansion of the non-relativistic many-body propagator, expressed in terms of the discretized effective potential W:

$$A(a,b;\,\epsilon) = \frac{1}{(2\pi\epsilon)^{Md/2}}\exp\left\{-\frac{(b-a)^2}{2\epsilon} - \epsilon\, W\left(\frac{a+b}{2}, \frac{b-a}{2};\,\epsilon\right)\right\}$$

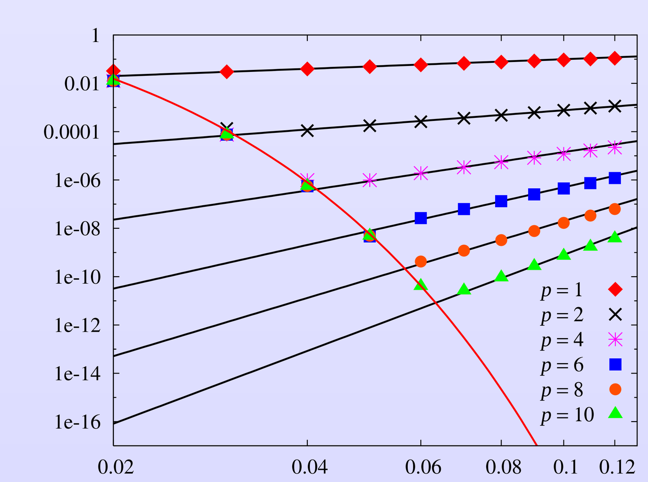
  where M is the number of particles in d spatial dimensions
  - ⋆ The expansion of the effective potential up to order ϵ^{p-1} yields propagator values correct to order ϵ^p
- **Calculation of one-particle eigenvalues and eigenstates**
  - ⋆ Approach based on the exact numerical diagonalization of the space-discretized evolution operator matrix [5, 6]

$$A_{nm}(\epsilon,\Delta) = \langle n\Delta|e^{-\epsilon\hat H}|m\Delta\rangle\cdot\Delta$$

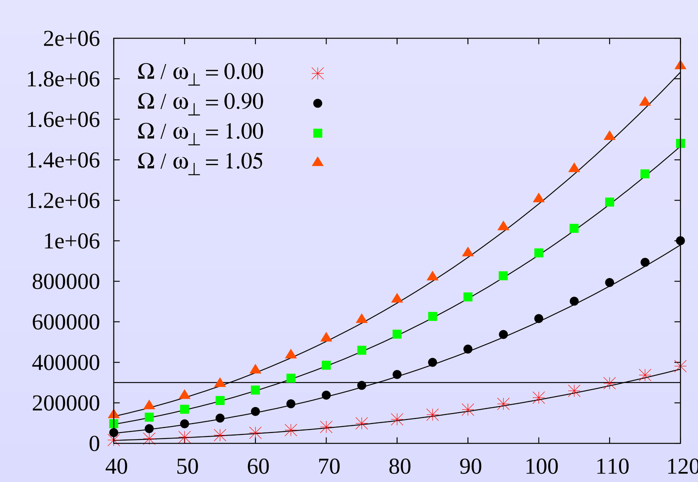
  - ⋆ Eigenvectors of matrix A_{nm} are space-discretized eigenvectors of the Hamiltonian $\hat H$, while eigenvalues are $e^{-\epsilon E_n}$

Numerical results: Static properties of BECs

- **Calculation of energy eigenvalues and T_c**

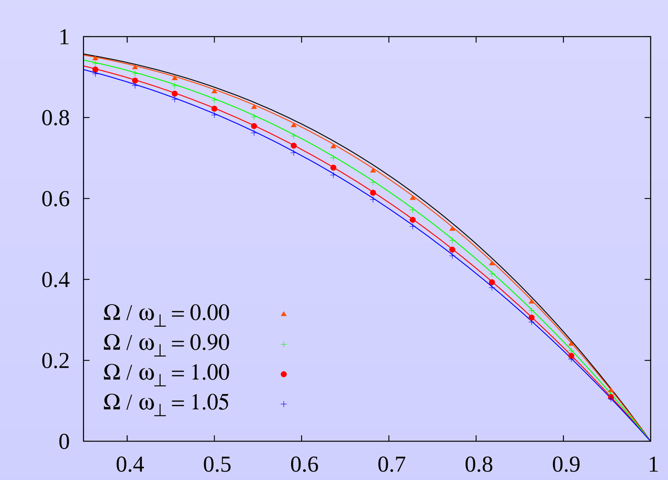


Errors of numerically calculated E_0 vs. diagonalization parameter ϵ for critical rotation $\Omega/\omega_\perp = 1$

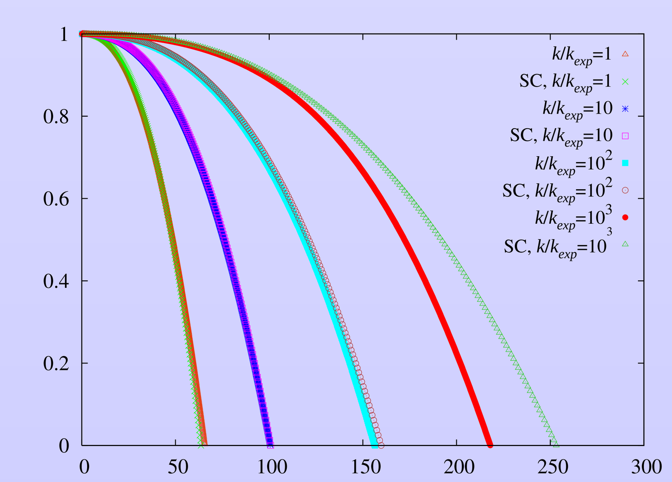


Number of particles vs. T_c [nK] for different rotation frequencies, obtained with $p = 18$ effective action

- **Condensate fraction**



Ground-state occupancy N_0/N as a function of T/T_c^0 for different rotation frequencies, obtained with $p = 18$ effective action, with $T_c^0 = 110$ nK



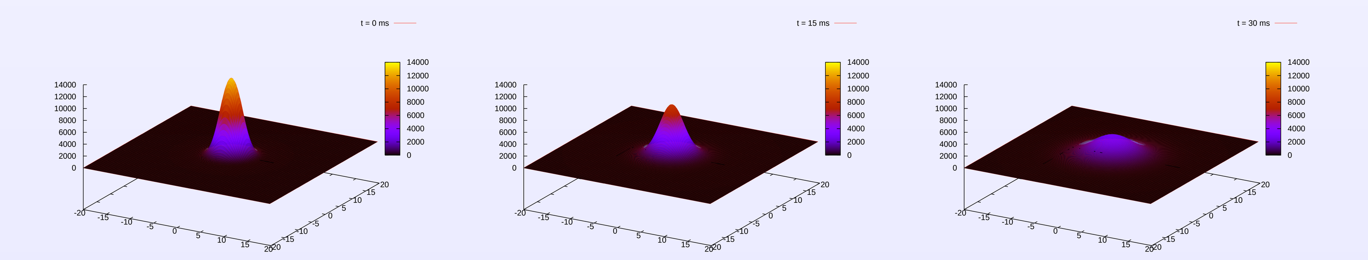
Comparison of N_0/N vs. T for critical rotation, obtained in SC approximation and using the presented approach, for several values of $k/k_{\rm exp}$.

Experimental setup: Fast-rotating BEC

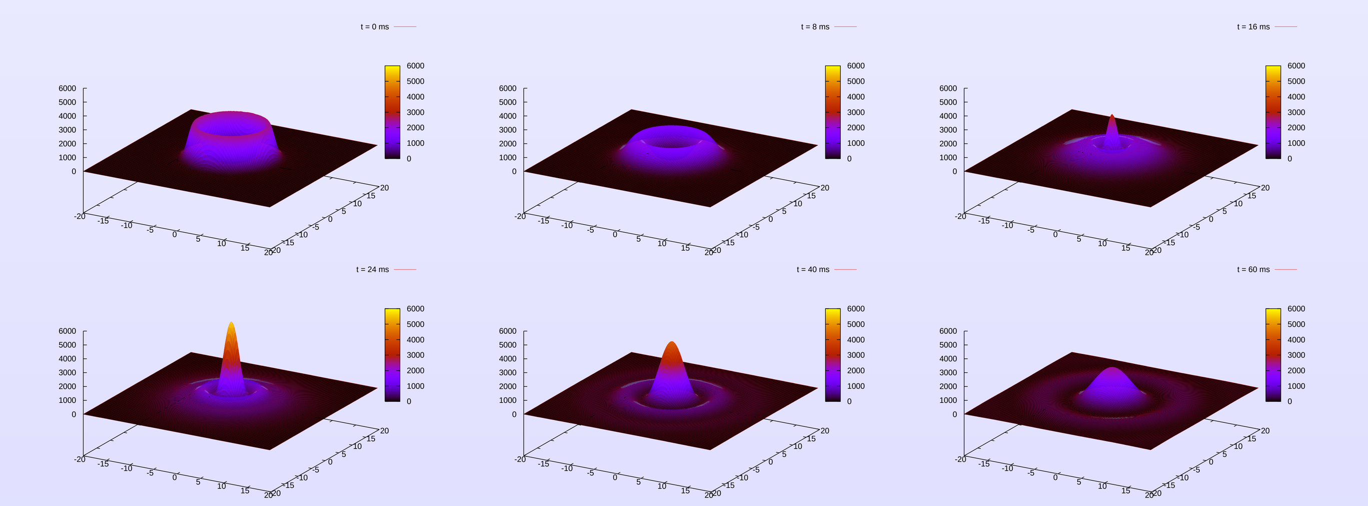
- ⋆ Paris group of J. Dalibard recently realized critically rotating BEC of $3\cdot 10^5$ atoms of ^{87}Rb in an axially symmetric trap [2]
- ⋆ The small quartic anharmonicity in $x - y$ plane was used to keep the condensate trapped even at the critical rotation frequency
- ⋆ Effective trapping potential: $V_{\rm BEC} = \frac{1}{2}M(\omega_\perp^2 - \Omega^2)r_\perp^2 + \frac{1}{2}M\omega_z^2 z^2 + \frac{k_{\rm exp}}{4}r_\perp^4$, where $\omega_\perp = 2\pi\times 64.8$ Hz, $\omega_z = 2\pi\times 11.0$ Hz, $k_{\rm exp} = 2.6\times 10^{-11}$ Jm^{-4}

Numerical results: Dynamic properties of BECs

- **Critical rotation:** Time-of-flight absorption graphs at $T = 10$ nK, obtained with $p = 21$ effective action. Linear profile size is 54 μm.



- **Overcritical rotation:** Time-of-flight graphs for $\Omega/\omega_\perp = 1.05$ at $T = 10$ nK, obtained with $p = 21$ effective action. Linear profile size is 54 μm.



Summary and outlook

- ⋆ Exact numerical treatment used to asses quality of SC approximation
- ⋆ SC results have accuracy of 1-3% in the considered range of parameters
- ⋆ Numerical approach necessary to assess SC results near the critical rotation
- ⋆ Formation and evolution of vortices requires interactions to be introduced

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