Anyons in 1d optical lattices by time periodic forcing

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Motivation: Anyons in 1d

Anyons are (quasi-) particles with fractional exchange statistics, i.e. they have commutation relations interpolating between bosons and fermions:

$$\hat{a}_j \hat{a}_k^\dagger - e^{-i\theta \mathrm{sgn}(j-k)} \hat{a}_k^\dagger \hat{a}_j = \delta_{jk}$$

In one dimension, anyons are to a large extent still unexplored.

It has been proposed (see [1]) to realize anyons via a transformation $\hat{a}_j = \hat{b}_j \exp\left(i\theta \sum_{l=j+1}^{M} \hat{n}_l\right)$ *to bosons.*

A simple Hubbard model describing anyons in one dimension

$$H = -J \sum_{j=1}^{M} \left(\hat{a}_j^\dagger \hat{a}_{j+1} + \mathrm{h.c.}\right) + \frac{U}{2} \sum_{j=1}^{M} \hat{n}_j (\hat{n}_j - 1)$$

then is a bosonic model with complex + parity breaking tunneling

$$H = -J \sum_{j=1}^{M} \left(\hat{b}_{j+1}^\dagger \hat{b}_j e^{i\theta \hat{n}_{j+1}} + \mathrm{h.c.}\right) + \frac{U}{2} \sum_{j=1}^{M} \hat{n}_j (\hat{n}_j - 1) \qquad (1)$$

Proposal for experimental realization

It has been suggested to realize the anyonic Hubbard model by using several internal atomic states that are coupled via asstisted Raman tunneling [see 2,3].

We propose a simpler scheme, where the complex + parity breaking tunneling is accomplished by a lattice tilt and a lattice shaking:

$$H(t) = -J_0 \sum_{j=1}^{M} \left(b_j^\dagger b_{j+1} + \mathrm{h.c.}\right) + \sum_{j=1}^{M} \left[\Delta + F(t,\omega)\right] j \hat{n}_j + \frac{U}{2} \sum_{j=1}^{M} \hat{n}_j (\hat{n}_j - 1)$$

For high frequency forcing, the dynamics is described by an effective Hamiltonian similar to (1):

$$H = -\sum_{j=1}^{M} \left(\hat{b}_{j+1}^\dagger \hat{b}_j J_{\mathrm{eff}}(\hat{n}_j, \hat{n}_{j+1}) + \mathrm{h.c.}\right) + \frac{U_{\mathrm{eff}}}{2} \sum_{j=1}^{M} \hat{n}_j (\hat{n}_j - 1) \qquad (2)$$

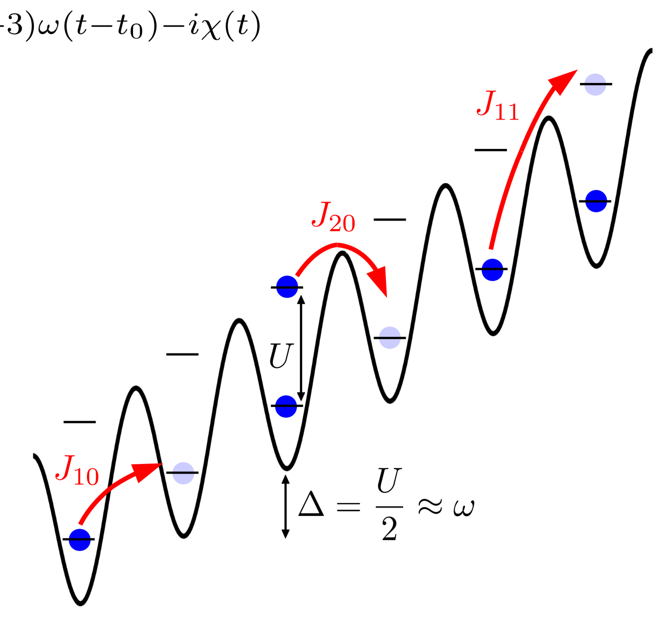
It then has effective, occupation dependent tunneling:

$$J_{\mathrm{eff}}(\hat{n}_l, \hat{n}_{l+1}) = \frac{J_0}{T} \int_0^T dt \; e^{i(2\hat{n}_{l+1} - 2\hat{n}_l + 3)\omega(t-t_0) - i\chi(t)}$$

with $\frac{d}{dt}\chi(t) = F(t,\omega)$

In the low density limit, where we often have only two particles per adjacent sites, we thus have to fulfill the condition

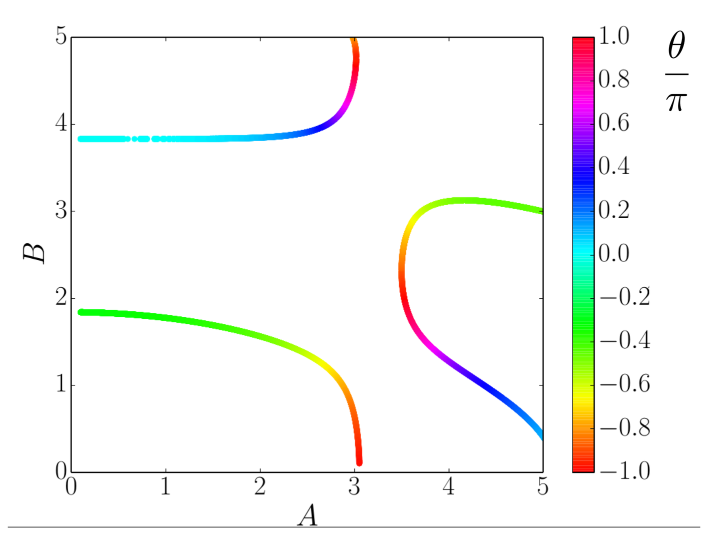
$$J_{10} = J_{20} = J_{11} e^{i\theta} \qquad (3)$$



We achieve this by choosing the tilt as half of the interaction strength and the driving function

$$\chi(t) = A\cos(\omega t) + B\cos(2\omega t)$$

Then we find values A, B for which condition (3) is fulfilled, giving us statistical angles θ in the full range [0,2π]. Thus a anyonic Hubbard model is realized for low densities.



Bogoliubov Theory

In the low-interaction limit, Bose condensates can be described by the Bogoliubov approximation. This theory has been generalized to also describe quasi-condensates in 1d [see 4]. We further generalize the Bogoliubov theory to approximately solve the anyonic Hubbard model in 1d (1):

We split $\hat{b}_j = e^{i\hat{\Phi}_j}\sqrt{\hat{n}_j}$ *in phase and density parts and look at fluctuations*

$$\begin{cases} \hat{\Phi}_j = \langle \phi_j \rangle + \hat{\phi}_j \\ \hat{n}_j = n_0 (1 + \hat{\epsilon}_j) \end{cases}$$

. In a quasi-condensate we expect $|\hat{\phi}_j - \hat{\phi}_{j+1}| \ll 1$, $|\hat{\epsilon}_j| \ll 1$

and can expand the Hamiltonian in these small variables up to 2nd order.

Defining new bosonic quasi-particles $\hat{\delta}_j = \sqrt{n_0}\left(\frac{\hat{\epsilon}_j}{2} + i\hat{\phi}_j\right)$ *, the Hamiltonian becomes in momentum representation:*

$$H = \sum_k \left(2a(k)\delta_k^\dagger \delta_k + b(k)\delta_k^\dagger \delta_{-k}^\dagger + b^*(k)\delta_k \delta_{-k}\right) \qquad (4)$$

with functions

$$a(k) = J\left(1 + \theta^2 n_0^2 - \cos(ak) - \theta n_0 \sin(ak)\right) + \frac{U n_0}{2}$$

$$b(k) = J\left(i\theta n_0 \cos(ak) + \theta^2 n_0^2 - i\theta n_0\right) + \frac{U n_0}{2}$$

As in the ordinary Bogoliubov theory, we now rotate the basis $\hat{c}_k = u_k \hat{\delta}_k + v_k \hat{\delta}_{-k}^\dagger$ *such that the Hamiltonian takes a simple oscillator form:*

$$H = \sum_k \left[\lambda(k) \hat{c}_k^\dagger \hat{c}_k + \alpha(k)\right] \qquad (5)$$

$$u_k = \frac{1}{\sqrt{1 - |l_k|^2}}, \quad v_k = \frac{l_k}{\sqrt{1 - |l_k|^2}}, \quad l_k = \frac{2a(k) - \lambda(k)}{2b^*(k)}$$

$$\lambda(k) = a(k) - a(-k) + \sqrt{(a(k) + a(-k))^2 - 4|b(k)|^2}, \quad \alpha_k = -\lambda(k)|v_k|^2$$

Preliminary Results

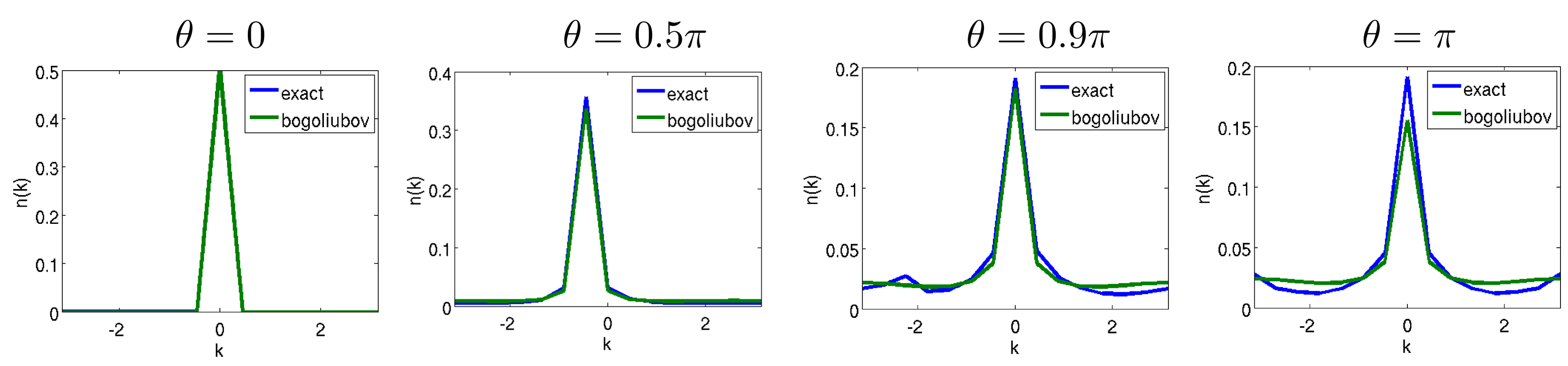
We look the at momentum distribution of the anyonic Hubbard model (1,2) to investigate the effect of the statistical angle. For this, we compare Bogoliubov theory with exact diagonalization for small systems, small particle densities and low interaction.

In the Bogoliubov theory, the momentum distribution is

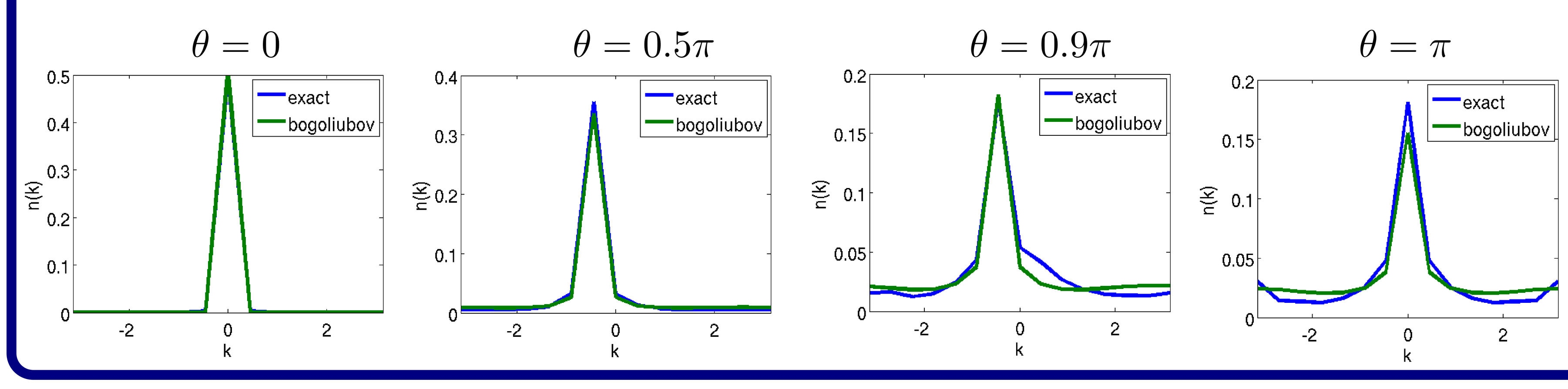
$$n(k) = \langle \hat{b}_k^\dagger \hat{b}_k \rangle = \frac{1}{n_0} \sum_{m=1}^{M} \exp\left[-\frac{1}{n_0 M} \sum_{q \neq 0} (1 - \cos(qm)) |v_q|^2 - ikm\right]$$

We solve the model for N=7 particles on M=14 sites, i.e. n=0.5.

$\underline{U = 0:}$
- *looking at free anyons, we see that the statistical angle broadens the momentum distribution, i.e. has a similar effect as an interaction*
- *the condensation peak is shifted for intermediate θ*
- *n(k) is asymmetric, which cannot be reproduced by the Bogoliubov theory*



$\underline{U = 0.25J:}$ - *a finite but low interaction strength has a stronger effect for larger θ*



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