DRIVEN BOSE-HUBBARD MODEL WITH IMPURITY

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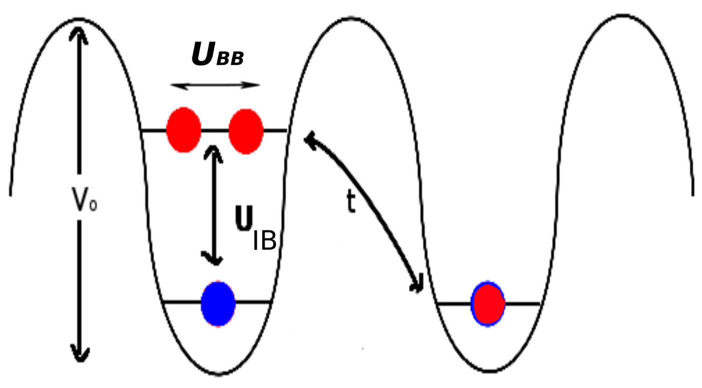
Abstract

We consider two aspects of a single-band Bose-Hubbard Hamiltonian with an additional impurity fixed at a lattice site.In a time-independent model we generalize the atomic limit [1] to the case that the impurity dislocates an arbitary number of bosons depending on the boson-impurity interaction strength.Afterwards, we study a time-dependent generalization where the periodic modulation of the boson-impurity interaction strength and the boson-boson interaction leads effectively to a conditional hopping.

Time-independent model

$$\hat{\mathcal{H}} = \sum_i f_i(\hat{n}_{i,b}) + \tilde{f}_R(\hat{n}_{R,b},\hat{n}_{R,c}) - t\sum_{\langle ij\rangle}\hat{b}_i^\dagger\hat{b}_j$$

- $f_i(\hat{n}_{i,b}) = \frac{U_{BB}}{2}\left(\hat{n}_{i,b}^2 - \hat{n}_{i,b}\right) - \mu\hat{n}_{i,b}$
- bosons: $\hat{n}_{i,b} = \hat{b}_i^\dagger\hat{b}_i$
- impurity: $\hat{n}_{i,c} = \hat{c}_i^\dagger\hat{c}_i$
- $\tilde{f}_R(\hat{n}_{R,b},\hat{n}_{R,c}) = U_{IB}\hat{n}_{R,b}\hat{n}_{R,c}$



1. Extremal distributed states

Mott ground state without impurity: $|\{n_b\}\rangle = \prod_l |n_b\rangle_l$

Possible ground states with impurity:

$$|R,\{r_j\}\rangle = \frac{1}{C}\prod_{j=1}^{k}\hat{b}_{R+r_j}^\dagger\left(\hat{b}_R\right)^k\hat{c}_R^\dagger|\{n_b\}\rangle, \quad C = \sqrt{\frac{n_b!}{(n_b-k)!}(n_b+1)!}$$

$$|R,r\rangle = \frac{1}{C'}\left(\hat{b}_{R+r}^\dagger\right)^k\left(\hat{b}_R\right)^k\hat{c}_R^\dagger|\{n_b\}\rangle, \quad C' = \sqrt{\frac{n_b!}{(n_b-k)!}\left[k! + \frac{n_b!}{(n_b-k)!}\right]}$$

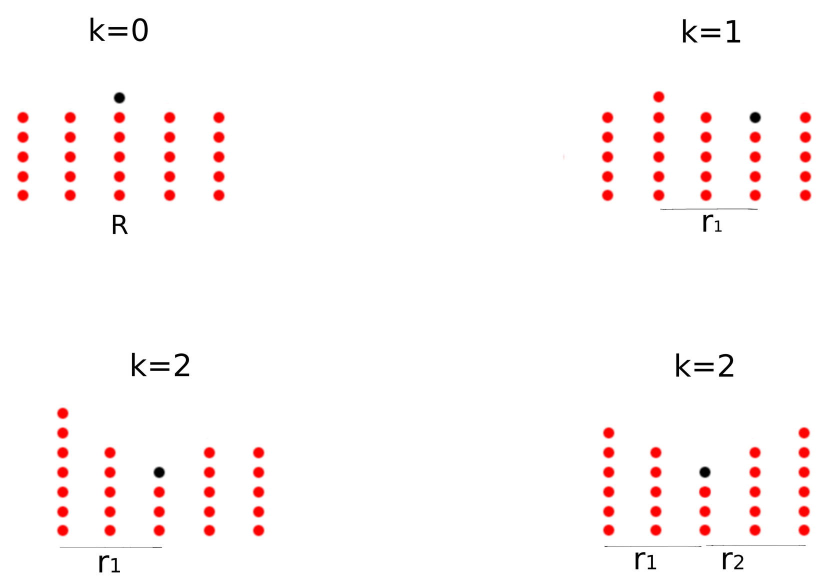


FIG. 1: Schematic diagramm of possible distributions for dislocated bosons, according to different values of k [1].

In the atomic limit:

$$\hat{\mathcal{H}}|R,\{r_j\}\rangle = \left[E_0 + \frac{U_{BB}}{2}k(k+1) + U_{IB}(n-k)\right]|R,\{r_j\}\rangle$$

$$\hat{\mathcal{H}}|R,r\rangle = \left[E_0 + U_{BB}k^2 + U_{IB}(n-k)\right]|R,r\rangle$$

with

$$E_0 = N\frac{U_{BB}}{2}n(n-1) - \mu n N$$

Result:

In the atomic limit equipartition is always the minimum, which can be proved by linear optimization. While the impurity-boson interaction is the same in both cases, the boson-boson interaction is strictly smaller in the case of uniform distribution for all values of k bigger than one.

2. Tuning of occupation number and number of dislocated bosons

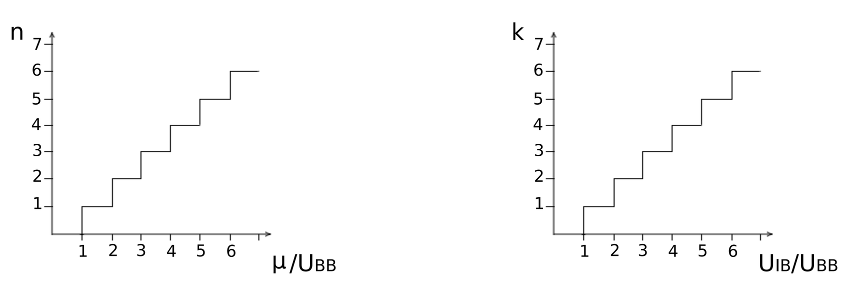


FIG. 2: Variation of particle number per lattice site n and number of dislocated bosons k respectively, by tuning the system parameters.

Result:

The particle number n per site becomes independent of the boson-impurity interaction strength in the thermodynamic limit, as expected for a local effect. The number of dislocated bosons k depends on the ratio of the interaction strengths, which is Feshbach tuneable.

Outlook

- quantitative spatial distribution of bosons dislocated due to presence of impurity
- calculation of ground-state energy beyond atomic limit
- impact of impurity on boundary of quantum phase transition
- size and effective mass of Bose-polaron [1]

Time-dependent model

Bose-Hubbard Hamiltonian with a single impurity and a time-periodic driving on a D-dimensional lattice

$$\hat{\mathcal{H}}(t) = \sum_i f_i(\hat{n}_{i,b}) + \tilde{f}_R(\hat{n}_{R,b},\hat{n}_{R,c}) + \cos(\omega t)\left[\sum_i g_i(\hat{n}_{i,b}) + \tilde{g}_R(\hat{n}_{R,b},\hat{n}_{R,c})\right] - t\sum_{\langle ij\rangle}\hat{b}_i^\dagger\hat{b}_j$$

Using the condition $U_{BB}, U_{IB}, t \ll \hbar\omega \ll \delta$ and time averaging over driving period [2 – 4], with the Floquet states

$$|\{R,\{r_j\}\},m\rangle\rangle = \frac{1}{C}\exp\{\mathrm{i}m\omega t\}\prod_{j=1}^{k}\hat{b}_{R+r_j}^\dagger\left(\hat{b}_R\right)^k\hat{c}_R^\dagger\prod_l\exp\left\{-\frac{\mathrm{i}}{\hbar\omega}h_l(\hat{n}_{l,b})\sin(\omega t)\right\}|\{n_b\}\rangle$$

with

$$\sum_l h_l(\hat{n}_{l,b}) = g_R(\hat{n}_{R,b}-k) + \sum_{j=1}^{k} g_{r_j}\left(\hat{n}_{r_j,b}+1\right) + \sum_{i\neq R,\{r_j\}} g_i(\hat{n}_{i,b}) + \tilde{g}_R(\hat{n}_{R,b}-k,1)$$

and the modified scalar product

$$\langle\langle u_1(t)|u_2(t)\rangle\rangle = \frac{1}{T}\int_0^T\langle u_1(t)|u_2(t)\rangle\,dt$$

yields

$$\langle\langle\{R,\{r_j'\}\},m'|\hat{\mathcal{H}}(t) - \mathrm{i}\hbar\frac{\partial}{\partial t}|\{R,\{r_j\}\},m\rangle\rangle = \delta_{m,m'}\langle\{n_b'\}|\hat{\mathcal{H}}_{\mathrm{eff}}|\{n_b\}\rangle + \mathrm{H.O.T}$$

with an effective time-independent Bose-Hubbard Hamiltonian with conditional hopping

$$\hat{\mathcal{H}}_{\mathrm{eff}} = \sum_i f_i(\hat{n}_{i,b}) + \tilde{f}_R(\hat{n}_{R,b},1) - t\sum_{\langle ij\rangle}\hat{b}_i^\dagger J_0\left(G_{ij}(\hat{n}_{i,b},\hat{n}_{j,b})\right)\hat{b}_j$$

with

$$G_{ij}(\hat{n}_{i,b},\hat{n}_{j,b}) = \frac{h_j(\hat{n}_{j,b}+1) - h_j(\hat{n}_{j,b}) + h_i(\hat{n}_{i,b}) - h_i(\hat{n}_{i,b}+1)}{\hbar\omega}$$

Example: periodic modulation of s-wave scattering length [2 – 5]

- $g_e(\hat{n}_{e,b}) = \frac{A}{2}\left(\hat{n}_{e,b}^2 - \hat{n}_{e,b}\right)$ with $e = R, r_j, i$
- $\tilde{g}_R(\hat{n}_{R,b},\hat{n}_{R,c}) = B\hat{n}_{R,b}\hat{n}_{R,c}$

Outlook

- impact of impurity on boundary of quantum phase transition
- higher order calculation of effective Hamiltonian
- multi-periodic driving of Bose-Einstein-condensates with impurities
- impurity quench: equilibrium or non-equilibrium in long-time limit [6]

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