Interacting Anyons in a One-Dimensional Optical Lattice

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Theory of Condensed Matter and Many Body Systems

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Abstract We analyze in detail the properties of the one-dimensional Anyon-Hubbard model, which can be mapped to a corresponding Bose-Hubbard model with a density-dependent Peierls phase via a generalized Jordan-Wigner transformation [1]. At first we extend the modified version of the classical Gutzwiller-mean-field ansatz of Ref. [2] in order to obtain the pair-correlation function for both the bosonic and the anyonic system. A comparison of the resulting quasi-momentum distributions with high-precision DMRG calculations reveals in general a parity breaking, which is due to anyonic statistics. Afterwards, we determine how the boundary of the superfluid-Mott quantum phase transition changes with the statistical parameter. We find in accordance with Ref. [1] that the statistical interaction has the tendency to destroy superfluid coherence.

Anyon-Hubbard Model

✗ Anyon-Hubbard model in 1D lattice [1]: $\hat{H} = \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) - J\sum_i \left(\hat{a}_i^\dagger \hat{a}_{i+1} + \text{h.c.}\right) - \mu \sum_i \hat{n}_i$

✗ Deformed commutation relations: $\hat{a}_i \hat{a}_j^\dagger - \exp\left(i\theta \operatorname{sgn}[i-j]\right)\hat{a}_j^\dagger \hat{a}_i = \delta_{i,j}\hat{I}$

✗ Bose-Hubbard model with correlated hopping:

$$\hat{H}^{\text{OBC}} = \frac{U}{2}\sum_{i=1}^{L}\hat{n}_i(\hat{n}_i-1) - J\sum_{i=1}^{L-1}\left[\hat{b}_i^\dagger \hat{b}_{i+1}\exp(i\theta\hat{n}_i) + \text{h.c.}\right] - \mu\sum_{i=1}^{L}\hat{n}_i$$

$$\hat{H}^{\text{PBC}} = \hat{H}^{\text{OBC}} - J\left[\exp(i\theta(\hat{N}-1))\hat{b}_L^\dagger \hat{b}_1 \exp(i\theta\hat{n}_L) + \text{h.c.}\right]$$

via a generalized Jordan-Wigner transformation $\hat{a}_i = \hat{b}_i \exp\left(i\theta\sum_{k<i}\hat{n}_k\right)$ with $\left[\hat{b}_i, \hat{b}_j^\dagger\right]_- = \delta_{ij}$

Symmetries

✗ Particle number conservation: $\left[\hat{H}, \hat{N}\right]_- = 0 \quad \longrightarrow \quad \text{SU}(L)$

✗ Translational (PBC): $\left[\hat{H}^{\text{PBC}}, \hat{U}\right]_- \neq 0 \quad \longrightarrow \quad \text{if} \quad \theta \neq 0, \frac{2\pi}{N-1}, \pi$

✗ Parity: $\left[\hat{H}, \hat{P}\right]_- \neq 0 \quad \longrightarrow \quad \text{if} \quad \theta \neq 0, \pi$

✗ Time inversion: $\left[\hat{H}, \hat{T}\right]_- \neq 0 \quad \longrightarrow \quad \text{if} \quad \theta \neq 0, \pi$

✗ Local gauge: $\left[\hat{H}, \hat{S}\right]_- \neq 0, \quad \hat{S}^\dagger \hat{b}_i \hat{S} = (-1)^i \hat{b}_i, \quad \text{if} \quad \text{OBC} \quad \text{or} \quad \text{PBC} \quad L = \text{even}$

$\Longrightarrow$ JW-anyons generally break discrete symmetries as well as translational invariance

✗ Invariance for combined discrete symmetries: $\left[\hat{H}, \hat{P}\hat{T}\right]_- = \left[\hat{H}, \hat{S}\hat{T}\right]_- = \left[\hat{H}, \hat{P}\hat{S}\right]_- = 0$

$\Longrightarrow$ If the discrete symmetries anti-commute with $\hat{H}_{\text{hop}}$ for some parameter values

Bosonic Quasi-Momentum Distribution

✗ Modified Gutzwiller Ansatz: $|G\rangle = \prod_j \left(\sum_{n=0}^{n_{\max}} f_n^{(j)}|n\rangle\right) \qquad f_n^{(j)} = |A_n|\exp(ij\beta_n + i\gamma_n)$

✗ Quasi-momentum distribution: $\langle \hat{n}_k^{(\text{b})}\rangle = \frac{1}{L}\sum_{ij} e^{ik(x_i - x_j)}\langle \hat{b}_i^\dagger \hat{b}_j\rangle$

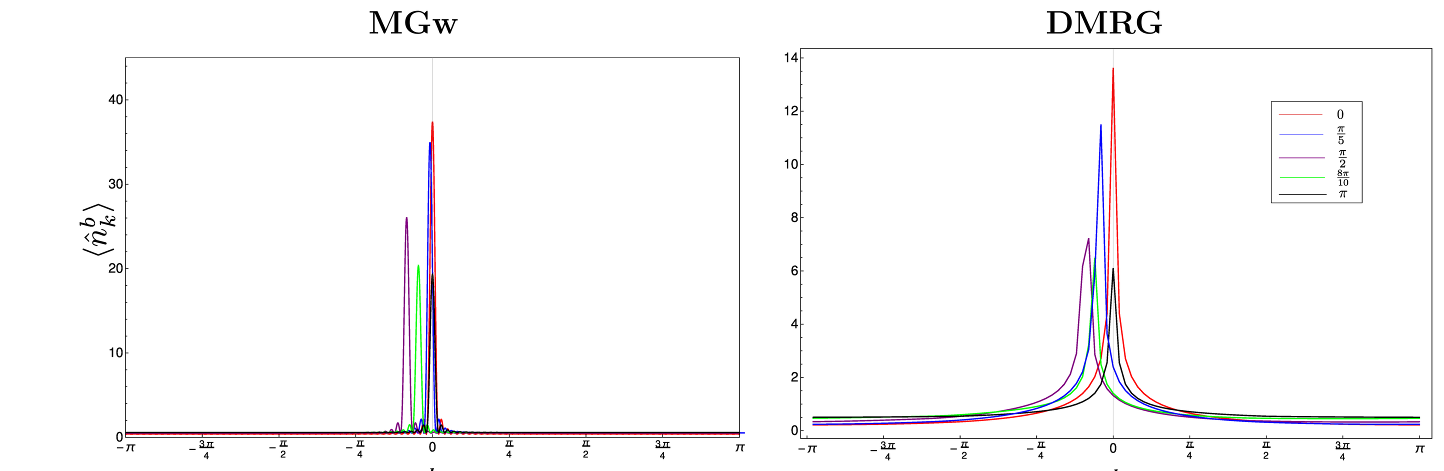
✗ Finite system ($n_{\max} = 2$):

$$\langle \hat{n}_k^b\rangle = n_0 - |A_1|^2\left[|A_0|^2 + 2\sqrt{2}|A_0||A_2|\cos\left(\frac{\theta}{2}\right) + 2|A_2|^2\right]$$
$$+\frac{1}{L}|A_1|^2\left[|A_0|^2 + 2\sqrt{2}|A_0||A_2|\cos\left(\frac{\theta}{2}\right) + 2|A_2|^2\right]\frac{1-\cos\left[(k+\Delta\beta_0)L\right]}{1-\cos\left[k+\Delta\beta_0\right]}$$

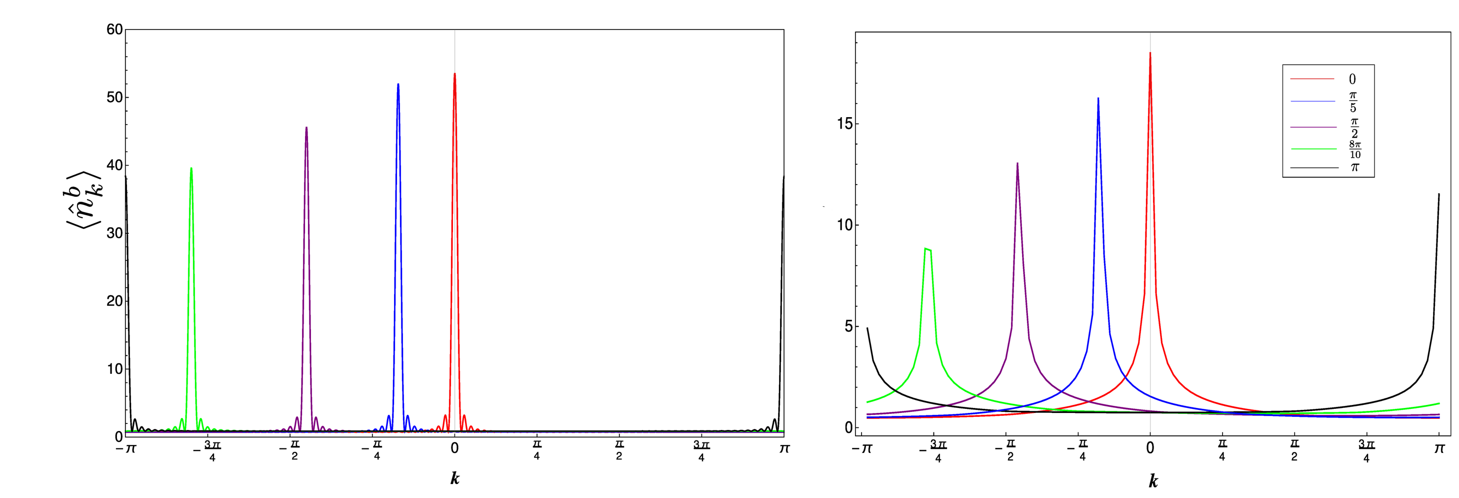
✗ Thermodynamic limit ($n_{\max} = 2$):

$$\langle \hat{n}_k^b\rangle_{L\to\infty} = n_0 - |A_1|^2\left[|A_0|^2 + 2\sqrt{2}|A_0||A_2|\cos\left(\frac{\theta}{2}\right) + 2|A_2|^2\right]$$
$$+|A_1|^2\left[|A_0|^2 + 2\sqrt{2}|A_0||A_2|\cos\left(\frac{\theta}{2}\right) + 2|A_2|^2\right]\delta(k + \Delta\beta_0)$$

✗ Example ($L = 100, \quad \frac{U}{J} = 10$):



$n_0 = 0.75$



$n_0 = 1.25$

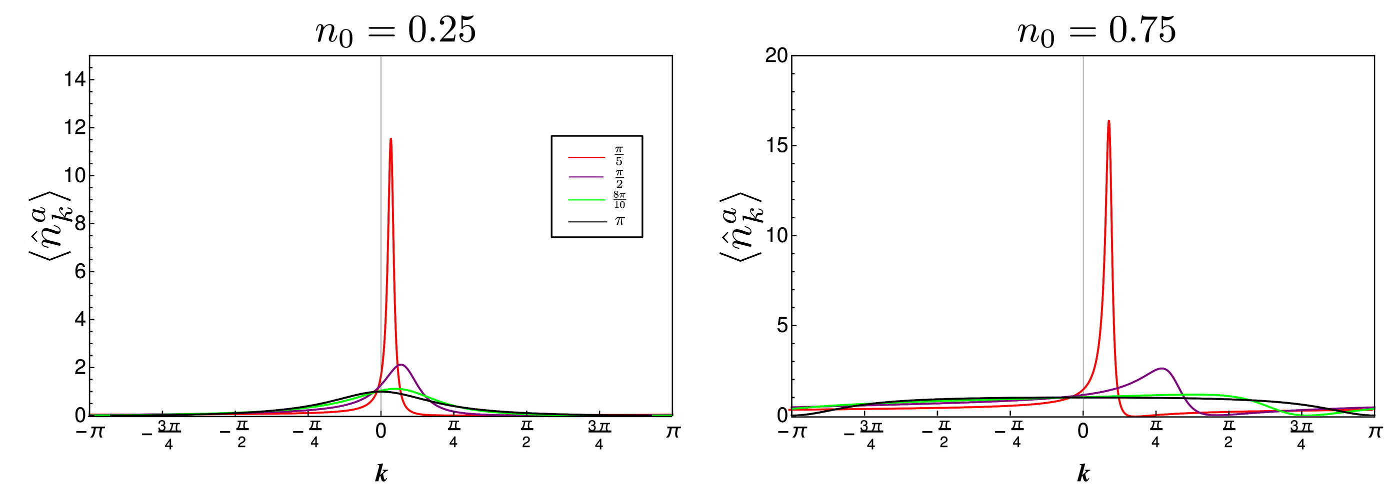
Anyonic Quasi-Momentum Distributions

✗ Quasi-momentum distribution: $\langle \hat{n}_k^{(\text{a})}\rangle = \frac{1}{L}\sum_{ij} e^{ik(x_i - x_j)}\langle \hat{a}_i^\dagger \hat{a}_j\rangle$

✗ Thermodynamic limit:

$$\langle \hat{n}_k^a\rangle_{L\to\infty} = n_0 - 2|C|\left\{\frac{|z|\cos\left[\text{Arg}(C) - \text{Arg}(z)\right] - \cos\left[k + \Delta\beta_0 - \text{Arg}(C)\right]}{|z|^2 - 2|z|\cos\left[k + \Delta\beta_0 - \text{Arg}(C)\right] + 1}\right\}$$
$$C = |A_1|^2\left(|A_0|^2 + \sqrt{2}|A_0||A_2|\left\{\exp\left[i\Delta\gamma\right] + \exp\left[-i(\Delta\gamma - \theta)\right]\right\} + 2|A_2|^2\exp\left[i\theta\right]\right)$$
$$z = |A_0|^2 + |A_1|^2\exp\left[i\theta\right] + |A_2|^2\exp\left[2i\theta\right]$$

✗ Example for MGw ($L = 100, \quad \frac{U}{J} = 10$):



$\Longrightarrow$ Deformation to smoothened Fermi edge is observable in limit $\theta \to \pi$

Mean-Field Decoupling

✗ Mean-field scheme: $\hat{a}_i \longrightarrow \langle \hat{a}_j\rangle + \delta\hat{a}_j, \qquad \langle \hat{a}_j\rangle = \alpha_j \longrightarrow \min_{\alpha_j}\left[E_{\text{MF}}(\alpha_j)\right]$

✗ Classical/Modified ansatz: $\alpha_j \longrightarrow \alpha, \qquad \alpha_j \longrightarrow \text{Abs}(\alpha)\exp[ij\text{Arg}(\alpha)]$

✗ Bi-partite lattice:

$$\hat{H} = \frac{U}{2}\sum_{j=1}^{L}\hat{n}_j(\hat{n}_j - 1) - J\sum_{j=1}^{L/2}\left(\hat{c}_{2j-1}^\dagger \hat{b}_{2j} + \hat{c}_{2j}^\dagger \hat{b}_{2j+1} + \text{h.c.}\right) - \mu\sum_{j=1}^{L}\hat{n}_j$$
$$\hat{c}_j^\dagger = \hat{b}_j^\dagger \exp(i\theta\hat{n}_j)$$

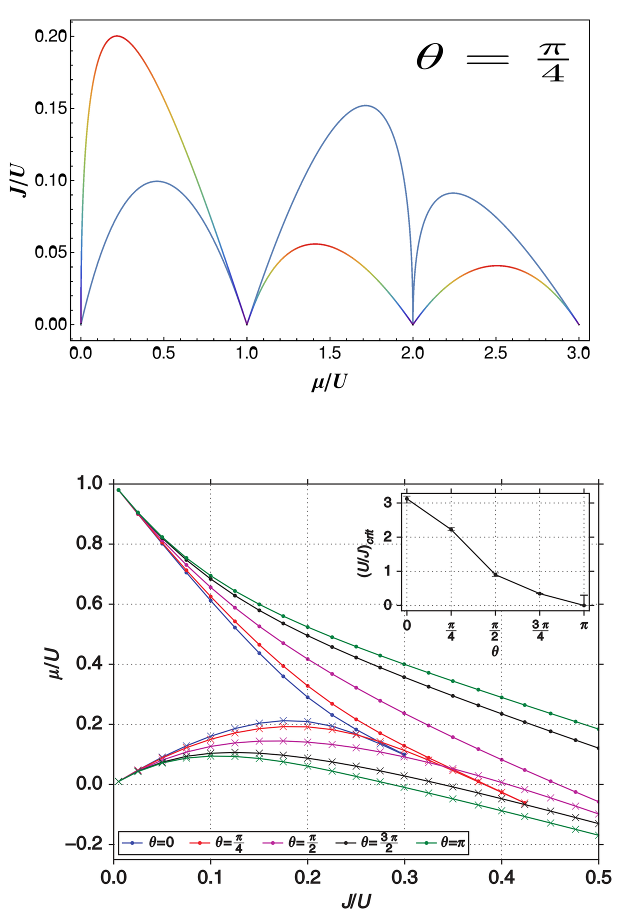
✗ Local Hamiltonian: $\hat{H}_{\text{MF}} = \hat{H}_0 - J\sum_{j=1}^{L/2}\left[B_o^*\left(\eta_o, \nu_e, \hat{n}_{2j}\right)\hat{b}_{2j} + A_e^*\left(\eta_e, \nu_o, \hat{n}_{2j+1}\right)\hat{b}_{2j+1} + \text{h.c.}\right]$

✗ Self-consistency equations motivated by [1]:

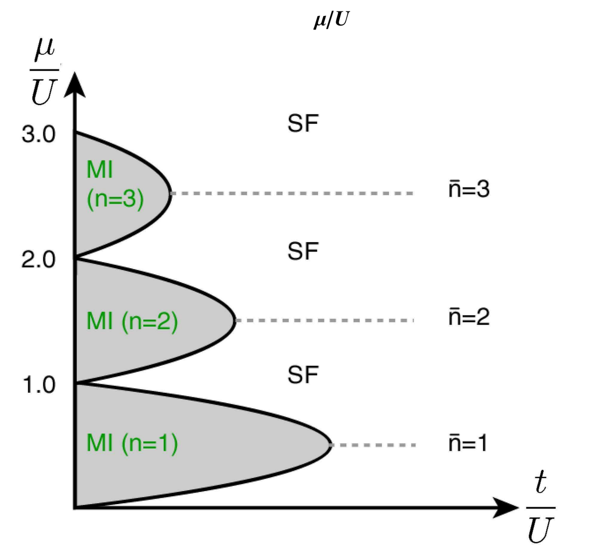
$$\langle \hat{b}_{2j}\rangle = \text{Abs}(b), \qquad \langle \hat{c}_{2j}\rangle = \text{Abs}(c)$$
$$\langle \hat{b}_{2j+1}\rangle = \text{Abs}(b)(-1)^{n_2 - n_1}, \qquad \langle \hat{c}_{2j+1}\rangle = \text{Abs}(c)(-1)^{n_2 + n_1}$$

$\longrightarrow$ Two types of superfluid depending on $n_1, n_2 \in \mathbb{Z}$ [3]

✗ Quantum phase diagram:



see [1]



see [4]

Conclusions

✗ Smooth transmutation between bosonic and fermionic exchange statistics

✗ Consistent modified Gutzwiller approach [2]

✗ Superfluid condensation at finite momentum

✗ Non-trivial deformation of Mott lobes [1]

✗ Emergence of new superfluid phases [3]

References

[1] T. Keilmann, S. Lanzmich, I. McCulloch, and M. Roncaglia, "Statistically induced phase transitions and anyons in 1d optical lattices," *Nat. Commun.*, vol. 2, p. 361, 2011.

[2] G. Tang, S. Eggert, and A. Pelster, "Ground-state properties of anyons in a one-dimensional lattice," *New J. Phys.*, vol. 17, no. 12, p. 123016, 2015.

[3] W. Zhang, E. Fan, T. C. Scott, and Y. Zhang, "Beats, broken-symmetry superfluid on a one dimensional Anyon Hubbard model," *arXiv:1511.01712*, 2015.

[4] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, "Boson localization and the superfluid-insulator transition," *Phys. Rev. B*, vol. 40, pp. 546–570, 1989.